A DELAY-TIME MODEL WITH IMPERFECT INSPECTIONS FOR MULTI-UNIT SYSTEMS

MODEL OPÓŹNIEŃ CZASOWYCH Z NIEPERFEKCYJNĄ DIAGNOZĄ STANU SYSTEMU DLA SYSTEMÓW WIELOELEMENTOWYCH

Anna Jodejko-Pietruczuk, Sylwia Werbińska-Wojciechowska

Wroclaw University of Technology, Wroclaw, Poland,
e-mails: anna.jodejko@pwr.wroc.pl; sylwia.werbinska@pwr.wroc.pl

Abstract: In this paper, the authors’ research work is focused on imperfect inspection policy investigation, when not all defects are identified during inspection action performance. They are interested in Block Inspection Policy performance for multi-unit systems, the maintenance policy which is one of the most commonly used in practice. As a result, at the beginning, few words about delay time modelling approach and a brief literature overview is given. Later, the model of Block-Inspection Policy is provided. The numerical example with the use of QNU Octave program is given. In the next Section, the sensitivity analysis of the developed model is characterised. The article ends up with summary and directions for further research.

Keywords: delay-time modelling, maintenance, multi-unit systems


Słowa kluczowe: modelowanie relacji czasowych, utrzymanie w stanie zdatności, systemy wieloelementowe
1. Introduction

Maintenance problems of real systems, like production plant, vehicle fleet, an offshore structure, a housing estate or a transportation system, has become very complicated recently and regard new more sophisticated solution methods implementation. On the one hand, there is a problem of optimal time between maintenance action performance definition, on the other – logistical problems of supplying the correct spares and materials with competent fitters and correct instructions (logistical support, scheduling, or spares provisioning issues) occur. Thus, when there is analysed the relationship between the performance of equipment and maintenance intervention, the conventional reliability analysis of time to first failure or time between failures is not sufficient [8]. The presented interaction can be captured with the use of Delay Time (DT) concept implementation.

The mentioned approach was developed by Christer et al. (see e.g. [9, 10, 13, 14, 17]). The basic idea rests on an observation that a failure does not usually occur suddenly, but is preceded by a detectable fault for some time prior to actual failure, the delay time [11]. So, the delay time $h$ is defined as the time lapse from the moment when a fault could first be noticed till the moment when a subsequent failure occurs, if left unattended [8, 11]. During period $h$ there is an opportunity to identify and prevent failure [30]. For more information see e.g. [24, 25, 30].

A literature review, in which delay-time models are investigated along with other PM models are given in [20, 21, 22, 29, 32, 35, 36]. The state of art works, dedicated strictly to DT modelling are given in [1, 3, 6, 7, 8, 11, 12, 28, 30, 33, 37]. Some of problems connected with delay time modelling for technical systems are investigated by authors in [24, 25, 26, 41]. In work [24] authors investigate the effectiveness of a system when defined maintenance strategy is one of two kinds. First, the typical group maintenance policy is considered. Second one regards to delay time model. Authors assume that the system is a three-component with a $k$-out-of-$n$ reliability structure. To make a comparison, the basic assumptions are defined to be the same for both the investigated models. Next, in [25] authors focus on investigation of Block-Inspection Policy performance level with economical and availability point of view. Later, in [26] authors analyse the problem connected with model parameters estimation process and its influence on Block Inspection Policy results. In the last work [41] author investigates the possibilities of basic DT model implementation in the area of logistic system of sixteen forklifts performance analysis.

However, all the presented works assume perfect inspections performance. In real-world situation, inspections may not reveal all defects present in a system, especially for large complex systems. Moreover, the quality of performed inspections depends on inspection techniques used, inspection training, inspection practices imposed or the nature of any supervision. As a result, the focus of this paper is to investigate the Block Inspection Policy model with imperfect inspections and to define their influence on maintenance policy results.
Following the introduction, the paper is organized as follows: In the introduction Section, few words about delay time modelling approach and a brief literature overview is given. Later, the model of Block-Inspection Policy is provided. The numerical example with the use of QNU Octave program is given. In the next Section, the sensitivity analysis of the developed model is characterised. The article ends up with summary and directions for further research.

In conclusion, this article is a continuation of the delay time modelling problems being investigated in [24, 25, 26, 30, 41].

2. Delay-time models with imperfect maintenance – state of art

The basic model of inspection assumes that a visible defect is always found in a system if it is there. However, performance process of real technical systems like transportation systems indicates, that this assumption is not always valid. Thus, the imperfect inspection case should be investigated. The problem of imperfect inspection is analysed and overviewed e.g. in [1, 6, 8, 19, 27, 28, 33, 34, 37].

Ones of the first works which investigate the delay-time model with imperfect inspection are [12, 13]. In these works the basic inspection model for industrial plant maintenance is provided. In the next work [14], authors present the variation of this imperfect-inspection model for a vehicle fleet maintenance.

Inspection models for single-component system are presented in [31, 42]. In [31], author develops a method for determining the discrete time points of inspection for a deteriorating single-unit system under condition-based maintenance. The delay-time model is here utilized to describe the transition of the system’s states. In the next work, [42], authors investigate the model to evaluate the reliability and optimise the inspection schedule for a multi-defect component. There is also considered the situation of non-constant inspection intervals. Following this, in [4], there is analysed the repairable machine that may fail or suffer breakdown many times during the course of its service lifetime, and is inspected for visible faults at intervals. The authors mostly focus on the problem of model parameter estimation with the use of maximum likelihood method and Akaike information criterion, providing also a model for imperfect inspections performance.

An inspection-replacement model for a multi-component system is proposed in [15]. In this model, authors assume also that system is inspected not only on a planned basis, but also when a component fails. The model let the total expected cost per unit time minimize with respect to the inspection intervals and the system replacement time.

Delay-time based models are also investigated for various types of technical objects, e.g. production plant maintenance processes development. In [16] authors present a Preventive Maintenance (PM) model applying the delay-time modelling technique to optimize the PM of the key machine in cooper products manufacturing company. An inspection model is developed to describe the relationship between the total downtime and the PM interval. Later, in [40] authors focus on optimising the preventive maintenance interval of a production plant. As in previous models,
A Delay-time model with imperfect inspections for multi-unit systems

Model opóźnień czasowych z nieperfekcyjną diagnozą stanu systemu

authors use likelihood formulation to model the problem. This problem is continued for the case of complex plant in [5]. The model is analysed for a case example of exploitation process of extrusion press working in Cooper Company. One step further goes author in [39]. In this work, author model the production process which may be subjected to two types of deteriorations. The first type of deterioration is a shift in product quality caused by minor process defects that may be identified and rectified by routine inspections and repair. Minor inspections are assumed to be perfect. The second deterioration type is a major defect caused by a major mechanical or electrical problem that can be observed only when the defect has led to a breakdown of the process or the defect is revealed by a major inspection followed by an appropriate major repair action at the time of the inspection. In the investigated model major inspections are assumed to be imperfect. The inspection model is focused on optimising the inspection intervals for both types of inspections. Later, the availability model based on delay-time modelling with imperfect maintenance is investigated in [38].

Other applications of delay-time models with imperfect maintenance regard to e.g. wind turbine maintenance [2]. The methodology for the application of delay time analysis via Monte Carlo Simulation is given in [18].

Following this, in the next Sections the Block Inspection (BI) Policy is investigated and its sensitivity for type of inspection action performance.

3. Block Inspection Policy Model

The investigated system is comprised of \( k \) identical elements, in a \( k\)-out-of-\( n \) (e.g. 2-out-of-3 in the Fig. 1) reliability structure, working independently under the same conditions. Moreover, components may be in one of three states: operating properly, operating with defects or down. They prone to become defective independently of each other when the system is in operating. The performed maintenance policy bases on Block Inspection policy which assumes, that inspections take place at regular time intervals of \( T \), and each requires constant time. The inspections are assumed to be imperfect. Thus, any component’s defect, which occurred in the system till the moment of inspection, will be unnoticed with probability \( p \) or correctly identified with probability \( 1-p \). All elements with identified defects will be replaced within the inspection period.

The performance of the investigated system, being illustrated in Fig. 1, is also defined by the additional assumptions:

- the system is a two state system where, over its service life, it can be either operating or down for necessary repair or planned maintenance,
- maintenance actions restores maintained components to as good as new condition,
- the system can remain functioning in an acceptable manner until breakdown (despite having elements’ defects),
- defects which may have arisen in the system, deteriorate over an operating time,
- the breakdown will be assumed to have been caused by $k$-out-of-$n$ defects which has deteriorated sufficiently to affect the operating performance of the system as a whole,
- failures of the system are identified immediately and repairs or replacements are made as soon as possible,
- system incurs costs of: new elements, when they are replaced, inspection costs, and some additional, consequence costs, when system fails,
- elements’ lifetime, repair time, replacement time and the length of the delay time before element’s failure are random and their probability distributions are known.

The system in Fig. 1 is inspected at $t_{PM}$ moments. Diagnosis of defect symptoms is imperfect thus elements 2 and 7 are allowed to further work although their defects might be noticed. Because of the fact, during one of following periods between inspections ($t_{PM1} - t_{PM2}$) two consecutive elements fail what causes a system failure.

On the other hand, some elements’ defects are properly diagnosed at the first possible inspection (elements: 1,4,5,6) and the components are preventively replaced but their potential lifetime is wasted.

The system presented above was modelled in GNU Octave software. The list of tested system parameters, which were used in the simulation model of the system exploitation process, is given in the Table 1.
A Delay-time model with imperfect inspections for multi-unit systems

Model opóźnień czasowych z nieperfekcyjną diagnozą stanu systemu

Table 1. Modelled system parameters

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Basic value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>number of elements of the system</td>
<td>3</td>
</tr>
<tr>
<td>$k$</td>
<td>minimal number of up-stated elements for having system in an operational state</td>
<td>2</td>
</tr>
<tr>
<td>$c_e$</td>
<td>the cost of a new element</td>
<td>1</td>
</tr>
<tr>
<td>$c_i$</td>
<td>the cost of an inspection</td>
<td>1</td>
</tr>
<tr>
<td>$c_f$</td>
<td>the cost of a system failure</td>
<td>1000</td>
</tr>
<tr>
<td>$T_i$</td>
<td>the time required for inspection</td>
<td>0</td>
</tr>
<tr>
<td>$T$</td>
<td>the constant period between inspections</td>
<td>[2,350]</td>
</tr>
<tr>
<td>$p$</td>
<td>probability that element defect will be unnoticed during inspection</td>
<td>[0,1]</td>
</tr>
<tr>
<td>$F(t)$</td>
<td>C.d.f. of single element’s lifetime</td>
<td>$F(t) = 1 - e^{-t(100)^{0.5}}$</td>
</tr>
<tr>
<td>$G_r(t)$</td>
<td>C.d.f. of single element’s replacement time when corrective action is taken</td>
<td>$G_r(t) = 1 - e^{-t(100)^{2.5}}$</td>
</tr>
<tr>
<td>$G_p(t)$</td>
<td>C.d.f. of single element’s replacement time when preventive action is taken</td>
<td>$G_p(t) = 1 - e^{-t(10)^{2.5}}$</td>
</tr>
<tr>
<td>$F(h)$</td>
<td>C.d.f. of delay time</td>
<td>$F(h) = 1 - e^{-t(35)^{0.3}}$</td>
</tr>
</tbody>
</table>

4. Imperfect inspection research – obtained analysis results

The aim of the paper is to develop findings given in the literature dealing with the Block Inspection Policy and especially to accentuate the dependency between inspection precision and policy performances. Inspection precision is understood as the ability of system to detect (and correctly interpret) its elements’ defects during inspection if their symptoms may be observable. This ability is given by $p$ factor describing the probability that an element defect will not be noticed by maintenance service. The greater value of $p$ means more difficult diagnosis of a real state of system during inspection.

The first step of the research aims to check which operational results of the BI policy are the most sensitive to imperfect inspection performance. Figures 2-9 present system performance for various levels of inspection accuracy and chosen vector of period between inspections. The analysis has proved the expected fact that all tested costs and availability ratio depend on inspection precision and the strength of this impact is much stronger in series structured system (Fig. 2, 9), more liable to system failures. Lower precision of inspection (higher value of $p$) increases the system failure cost but at the same time it decreases the summary cost of new elements that are used in the system for the both analysed reliability structures. Detailed analysis of the obtained results also shows that the parallel system cost is rather insensitive to imperfect inspection for very short periods between inspections $T$. Imperfect, but often performed, inspections allows to avoid system failures and make system cost independent on inspection precision. In
contrast, the series system, being much more liable to failures, is very sensitive to probability of defect omission \( p \) for short \( T \) periods. Extending the period between inspections causes similar effect as lower precision of inspection – decreases probability of correct preventive maintenance and increases the risk of system failures (Fig. 5). The same effect of decreasing chance to maintain system prophylactically and to prevent system failure for “long” \( T \) may be observable in figures presenting: the system availability ratio (\( A \)) (Fig. 8, 9), the per cent of preventively replaced elements as a part of all component used by the system (\( NR \)) (Fig. 4, 5) and the per cent of elements whose failures are discovered and eliminated during inspections (\( NF \)) without any further consequences for the system (Fig. 6, 7). The Fig. 7 presents the per cent of elements of the series system that are identified and replaced during inspection because of their failure but without causing a system breakdown. The value of zero results from a character of series reliability structure – every element failure makes a system fault.

---

**Fig. 2.** The total costs of the parallel system maintained according to BI policy

**Fig. 3.** The total costs of the series system maintained according to BI policy

**Fig. 4.** The per cent of preventively replaced components in the parallel system maintained according to BI policy

**Fig. 5.** The per cent of preventively replaced components in the series system maintained according to BI policy
A Delay-time model with imperfect inspections for multi-unit systems
Model opóźnień czasowych z nieperfekcyjną diagnozą stanu systemu

Fig. 6. The per cent of failed components in the parallel system replaced during inspection

Fig. 7. The per cent of failed components in the series system replaced during inspection

Fig. 8. The availability ratio in the parallel system replaced during inspection

Fig. 9. The availability ratio in the series system replaced during inspection

All the observed availability and cost results have given the ground to believe that inspection uncertainty may be neutralized by shortening the period between inspections (e.g. the defect of the element 1 in Fig. 1 might be noticed at $t_{PM1}$ or $t_{PM2}$ moment). This conclusion has determined further research direction. The next part of analysis is dedicated to finding out how the best period between inspections in the system changes when inspections become more unreliable.

The “near optimal” time between preventive inspections $T$, according to literature findings [24], depends mainly on: a system lifetime determined by lifetime of system components and its reliability structure as well as elements’ delay time. The period $T$ that assures low costs and high availability ratio of maintained system should fulfill following conditions in a series structured system:

$$4 \leq \frac{T_p}{T} \leq 6$$  
$$\frac{h}{T} \approx 2.$$  

(1)  
(2)
where: $T$ – the period between inspections,
$T_p$ – the $MTTF$ of system elements,
$h$ – the mean delay time value.

System with parallel reliability structure may be inspected even at longer time intervals because of its higher resistance to single element failures.

The expressions 1 and 2 have been found to be true for perfect inspected systems. The goal of the following analysis is to show how the above expressions should be changed when inspection becomes less precise. Figures 10-17 present chosen BI results obtained for various levels of inspection imprecision $p$ and various lengths of period $T$ in relation to mean element’s lifetime ($T_p/T$) and to mean delay time ($h/T$). To save the space of the paper, the outcomes are presented at the same time as the function of $p$, $T_p/T$, $h/T$.

Presented in Fig. 11 cost results are quite unambiguous for the analysed series reliability structured system – the period between inspections set according to formulas 1 and 2 generates low system costs when inspection is perfect. When diagnosis becomes less reliable (higher $p$ value) the system should be inspected much more often ($T_p/T > 4$, $h/T > 2$) but explicit optimum $T$ cannot be specified for any level of $p$. The result is the effect of significant role of the system failure consequence cost in the total cost of the maintained system and makes profitable every maintenance strategy with no system failure. The same effect of higher (but comparable) profitability of shorter periods $T$, is observable in 1-out-of-3 system (parallel) but for lower values of $T_p/T$ and $h/T$. Thus one cannot to precisely determine explicit optimum relation of period $T$ to elements’ lifetime or delay time.

The analysis of the availability ratio allows confirming the presented earlier thesis that lower precision of inspection may be counterbalanced by oftener diagnosis of system state. The parallel system inspected at long time intervals ($T_p/T < 1$) has low availability level independently upon inspection precision (Fig. X11). Reduction of period between inspections ($T_p/T \approx 2$) allows the system to increase its availability.

Generally, all the results in Fig. 12 may be divided into two groups:
1) periods between inspections for which the increase of inspection imprecision (growing value of $p$) causes the decrease of the system availability ratio because of higher number of unexpected system downtimes (left circle in Fig. 12),
2) too short periods $T$, which worsts availability results because too often preventive maintenance actions performance (right circle in Fig. 12). Higher value of $p$ (less reliable inspections) causes that some of excessed elements’ preventive replacements are omitted (because components’ defects are not recognized) and system does not lose the time for maintenance actions and returns quickly to upstate, still without its failure.

The contact line between this two areas may be found as optimum or “near optimum” period $T$ in relation to $T_p$ and $h$ for every level of inspection accuracy. The line marked by additional line in Fig. 12. The optimum relation of element’s lifetime and delay time to period between inspections changes in comparison to expressions 1-2. Less reliable inspection should force maintenance staff to shorten time between inspections in order to retain BI maintenance advantages. The same effect, but much more subtle, is noticeable and marked in Fig. 13 in the series reliability structured system.
A Delay-time model with imperfect inspections for multi-unit systems
Model opóźnień czasowych z nieperfekcyjną diagnozą stanu systemu

Fig. 10. The total costs of the parallel system for various levels of inspection precision $p$, relations $T_p/T$ and $h/T$

Fig. 11. The total costs of the series system for various levels of inspection precision $p$, relations $T_p/T$ and $h/T$

Fig. 12. The availability ratio of the parallel system for various levels of inspection precision $p$, relations $T_p/T$ and $h/T$

Fig. 13. The availability ratio of the series system for various levels of inspection precision $p$, relations $T_p/T$ and $h/T$

Fig. 14. The best simulated relations $T_p/T$ and $h/T$ for the parallel system for the sake of cost and availability ($c_c = 1000$)

Fig. 15. The best simulated relations $T_p/T$ and $h/T$ for the series system for the sake of cost and availability ($c_c = 1000$)
In order to assess this change of “optimal” $T$, all simulated variants of the BI policy were analysed for the sake of their best results. The cost and availability optimums, and – on this base – the best obtained relation of $T$ to $T_p$ and $h$, have been found. The effect of the analysis is presented in Fig. 14-17, where following indications define:

- $AA$ – relation $T_p/T$ that gives the highest system availability ratio,
- $AD$ – relation $h/T$ that gives the highest system availability ratio,
- $CA$ – the best relation of $T_p/T$ for the sake of the total cost,
- $CD$ – the best relation of $h/T$ for the sake of the total cost.

As one can see, independently on the system reliability structure, less accurate inspection (higher $p$ value) causes that the best relation $T_p/T$ and $h/T$ increase approximately linearly. It means that period between inspections should be shortened in relation to the case of BI perfect inspection. Gradients of trend lines that could describe results in Fig. 14-15 depend upon many factors of modelled system, e.g.: its reliability structure, number of system components, time of maintenance actions, costs components value and should be the base for another research. In order to expose one of exemplary factor influence, the optimum solutions were found for the modelled system whose unit cost of failure consequence ($c_c$) was reduced ten times in relation to the initial case. The effect may be seen in Fig. 16-17. The optimal time $T$, defined for availability constrain, does not change while the lowest cost is reached when inspections are rarer (lower optimum values of $T_p/T$ and $h/T$).

The second remark that is worth to comment is the fact of sudden decrease of the best relations of $T_p/T$ and $h/T$ in the series system when $p = 1$ (defect detection is improbable), especially in the availability-optimal case ($AA$, $AD$ in Fig. 14-17). System inspections are unprofitable in this case, because every of them make lower system availability and does not prevent system failures enough to balance availability decrease. Quite different situation is seen in parallel system, where inspections do not expose element’s defects but disclose their failures, thus prevent system failures. That is why any decrease of the best solution is not noticeable in Fig. 14 and 16.
The last purpose of this research was to answer the question if the BI maintenance policy is still more profitable in comparison to other maintenance policies when its inspection precision drops. For this reason the system has been tested assuming BI policy, corrective maintenance policy (CM) \((T \to \infty)\) and BP policy \((h_i = T_{pi}, p = 0)\). The results are presented in Fig. 18-21.

As one can see, the results show some general rules valid for the modelled system:

- the corrective maintenance policy is the worst of all the analysed policies (marked by the extra circle for clarity of figures).
- cost results of the BP policy obtained in the research are better than BI effects (or mostly the same) for every tested case of inspection precision. It happens because preventive replacement of all system elements ensures the lowest consequence cost, which is dominant cost component. Another relation of new element and system failure costs may make the BI policy more profitable in comparison to the BP policy.
- availability ratio of system maintained in accordance with the BP policy is higher than the others only in the cases when the period between inspections \(T\) is shorter than the one given in formulas 1-2. Independently on inspection precision, if the period between inspections \(T\) is determined correctly, the BI policy displays to be the better solution.
5. Conclusions

The presented sensitivity analysis of investigated BI policy model gives the possibility to obtain some rules for definition of the principal relations between the system performance under given PM policy with imperfect maintenance and chosen PM policy parameters.

In the presented paper authors develop simple DT model with imperfect maintenance. However, most of the imperfect inspection maintenance models assume, that the probability $p$ of a defect not detection during inspection action performance is constant, what may not be valid for real-life technical objects like means of transport. Thus, in the next step, authors plan to develop DT model with imperfect maintenance when the probability $p$ is not a constant but is randomly distributed with chosen probability distribution function. Later, there will be made an effort to define some rules how to choose a PM policy from an engineering point of view.

6. References

A Delay-time model with imperfect inspections for multi-unit systems
Model opóźnień czasowych z nieperfekcyjną diagnozą stanu systemu


A Delay-time model with imperfect inspections for multi-unit systems
Model opóźnień czasowych z nieperfekcyjną diagnozą stanu systemu


**PhD. Anna Jodejko-Pietruczuk**, Wroclaw University of Technology, Division of Logistics and Transportation Systems; Ph.D. work concerns economic, reliability and safety consequences generated by spare parts provisioning policy of a technical system; now she works and publishes papers in the area of maintenance modelling and optimization, particularly applied in logistics and production systems.

**PhD. Sylwia Werbińska-Wojciechowska**, Wroclaw University of Technology, Division of Logistics and Transportation Systems; PhD. work concerns the problems of logistic support systems for technical objects exploitation process performance modelling; now she is interested in delay-time modelling, especially concerned with application issues for transportation systems maintenance performance.