SHIP’S ROLLING AMPLITUDE AS A SIGNIFICANT FACTOR INFLUENCING LIQUID SLOSHING IN PARTLY FILLED TANKS

AMPLITUDA KOŁYSAŃ STATKU JAKO ISTOTNY PARAMETR W BADANIACH ZJAWISKA SLOSHINGU W NIEPEŁNYCH ZBIORNIKACH OKRĘTOWYCH

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Abstract: The study considers ship’s rolling amplitude as one of the key parameters influencing liquid sloshing in partly filled tanks during sea voyage. This issue is strictly related to the safety of navigation matters and belongs to the group of non-linear hydrodynamic phenomena. The presented investigation is focused on the estimation of typical and extreme rolling amplitude of a vessel in terms of dynamic approach towards liquid motion onboard ships. The number of exemplary numerical simulations of liquid sloshing taking place in moving tanks is carried out and the heeling moment due to liquid sloshing is obtained.

Keywords: liquid sloshing in ship’s tank, free surface effect, ship’s rolling amplitude, dynamic stability of ships, safety of navigation

Streszczenie: Przedstawione w artykule dociekania dotyczą amplitudy kołysań statku morskiego, będącej jednym z kluczowych parametrów wpływających na zjawisko sloshingu w częściowo zapełnionych zbiornikach okrętowych. Zagadnienie jest ściśle związane z bezpieczeństwem żeglugi poprzez cechy statecznościowe statków. Badania nakierowane były na określenie typowych oraz granicznych amplitud kołysania statków morskich w kontekście uwzględniania dynamiki przelewania się cieczy w niepełnych zbiornikach. Wykonano przykładowe obliczenia symulacyjne zjawiska sloshingu dla uzyskanych amplitud kołysania statku.

Słowa kluczowe: zjawisko sloshing’u w zbiornikach okrętowych, swobodne powierzchnie cieczy, amplituda kołysań statku, stateczność dynamiczna statku, bezpieczeństwo żeglugi
1. Introduction

Ship stability is a term used to describe the tendency of a ship to return back to her equilibrium when she is inclined from an upright position [10]. Since the initial position of a ship is not always upright one, the more practical definition states that the stability is a feature enabling to perform, when remaining in determined position, the task she is constructed for [10]. The complementary definitions lead to point out that the stability of a ship is an element of her operational safety qualifying factors under.

The seagoing vessel’s stability calculation and evaluation made onboard nowadays is based on the prescriptive stability criteria published by the ship’s classification societies [10]. These criteria are mainly based on the A749(18) Resolution of International Maritime Organization. The resolution and their later amendments are known as the Intact Stability Code [8].

The criteria qualify the shape of the righting arm curve. In addition, the weather criterion is to ensure the sufficient stability of a ship to withstand the severe wind guests during rolling [10]. Although the weather criterion reflects a very simple model of dynamic ship’s behavior, the static stability curve is used. Anyway, the weather criterion is the only, which is partly based on the model of heeling phenomenon not only on the statistic data, while the rest of criteria are based on the statistics of historical disasters only [6]. The modern and still developing approach towards ship stability qualification is an implementation of performance-based stability criteria in the future. They are based mainly on the risk assessment [10].

Regardless the approach towards ship stability evaluation, the physical background of phenomena taking place onboard ought to be taken into account. In case of contemporary prescriptive stability standards, the righting and heeling arms need to be obtained and compared. Then the work of the righting arm enabling accumulation and then dissipation of the energy could be compared to the energy provided to the ship by external forces which is called the energy balance method for dynamic stability calculation [10]. The balance of righting arm (righting moment) and heeling arm (heeling moment) has to comprise all significant components of each moment and among others the heeling moment due to liquid sloshing in a partly filled moving tank too.

In case of an application of risk assessment methods the practical approach requires carrying out a large set of numerical simulations of ship motion on seaway [10]. Every single ship motion simulation, or in a simplified approach ship rolling simulation, requires obtaining the force and moment matrix at every time step of the computation. The forces and moment need to cover all significant components due to different phenomena. Thus, the heeling moment due to liquid sloshing in ship tanks ought to be involved as well.

In the light of ship stability related concepts, the accuracy of ship’s transverse stability assessment is an important problem in vessels’ operation process. Both practical approaches towards ship stability assessment known nowadays call for...
characteristics of heeling moment due to liquid sloshing in tanks. This need justifies the research program focused on the liquid sloshing phenomenon.

2. Liquid Sloshing Phenomenon

Liquid sloshing phenomenon taking place in partly filled ship’s tanks contribute to the overall stability performance of a vessel among many other phenomena. According to the referred energy balance approach, the work of ship’s righting moment needs to be compared to the total rolling energy including all its significant elements. One of such element is the moment due to liquid sloshing in tanks. As a tank moves, it supplies energy to induce and sustain a fluid motion. Under external large amplitude excitations or an excitation near the natural frequency of sloshing, the liquid inside a tank is in violent oscillations which is of great practical importance to the safety of the liquid transport [13]. Both the liquid motion and its effects are called sloshing. The interaction between ship’s tank structure and water sloshing inside the tank consists in the constant transmission of energy [1].

In the course of ship stability assessment carried out according to the IMO Intact Stability Code the righting lever curve should be corrected for the effect of free surfaces of liquids in tanks [8]. Generally the correction reflects moment of quasi-static fluid transfer in a tank and the liquid surface is always assumed flat and depends only on an angle of ship’s heel. Since the actual shape of free surface of liquid in a moving partly filled tank is time-variable and non-linear which was shown in numerous experiments worldwide (Fig. 1).

![Fig. 1. Exemplary shape of disturbed free surface in a model tank: Korean experiment from 2007 [9] (left) own experiment from 2012 (right)](image)

The photo on right hand side presented in the figure 1 was taken during an experimental research carried out in the Faculty of Navigation at the Gdynia Maritime University in 2012. The main purposes of the experiment is to verify the results of numerical simulations of sloshing phenomenon performed with the use of CFD technique and to systematically observe the shape of the free surface during the consecutive oscillations of the model tank.
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The numerical simulations of liquid sloshing in ship’s tank requires a set of initial assumptions regarding significant factors influencing the analyzed phenomenon. The world’s literature review provides the list of most important element to be defined [1], [3]:

- tank geometry;
- filling level (liquid depth in a tank);
- location of a tank in ship’s hull expressed by the shift from an axis of roll;
- characteristics of external excitation, e.g. ship motion parameters comprising mainly rolling period and amplitude.

This study is focused on the rolling amplitude of a vessel sailing in rough sea.

3. Ship Rolling Amplitude

As a ship motion on rough sea is very complex hydrodynamic phenomenon, numerous attempts for its estimation was performed worldwide. The rolling motion being an essential component of general motion is especially emphasized in the course of many researches [12].

The simplest approach towards calculation of ship rolling amplitude is given in the IMO Intact Stability Code [8]. It deals with a resonant mode of rolling and consists of rough estimation of rolling amplitude for a standard ship which is corrected for a couple of factors [10]. The calculation formula given in the IS Code is following:

\[ \varphi_a = 109 \cdot k \cdot X_1 \cdot X_2 \cdot \sqrt{r \cdot s} \ [^\circ] \]  

(1)

where:
- \( k \) – correction factor for bilge keels presence;
- \( X_1 \) – correction factor related to the quotient beam to draft;
- \( X_2 \) – correction factor related to ship block coefficient;
- \( s \) – factor related to the natural period of ship’s roll;
- \( r \) – effective slope factor calculated according to the formula:

\[ r = 0.73 + 0.6 \cdot \frac{z_G - T}{T} \]  

(2)

where:
- \( z_G \) – vertical center of gravity;
- \( T \) – ship mean draft.

The values of listed factors can be read from the IS Code while the natural rolling period of a ship could be obtained from the sally test or calculated according to the formula:

\[ T_v = \frac{2 \cdot \pi \cdot f}{\sqrt{g \cdot GM}} \approx \frac{2 \cdot f \cdot B}{\sqrt{GM}} \]  

(3)

where:
- \( f \) – transverse gyration radius of a ship;
- \( g \) – gravity acceleration;
- \( GM \) – ship’s transverse metacentric height;
The above described procedure for rolling amplitude calculation was implemented into the SOLAS international convention in 2008. Prior to that date the classification societies utilized their own variants of the procedure slightly varying from each other, however the results were very similar. On the basis of the simplest formulas recommended by IMO, the exemplary analysis of typical rolling amplitude was carried out for a general cargo vessel project B-354. The ship is rather flexible in term of cargo carriage and she can be loaded by break bulk, containers and even a bit of liquid cargo, for instance a pulp. Thus, the great number of typical loading condition is presented in her stability booklet. The histogram of typical values of her transverse metacentric height is shown in Fig. 2.

Following the described procedure for rolling amplitude calculation the values of this amplitude reflecting typical loading conditions of ship B-354 were obtained. The result is presented in Fig. 3. Histograms plotted in the figure 2 reveals the marginal value of rolling amplitude about 20 degrees for the ship sailing without parametric rolling (parametric rolling is out of the range of formula 1). According to the histogram in the figure 2 about 40% of typical loading conditions (given ex ante in ship’s stability booklet) reflects the metacentric height ranging from 0,25m to 0,50m or 60% ranging from 0,25m to 0,75m. Such expected by an architect values of GM seems to be rather lower than actually noticed in practice. If it is really so, the resultant rolling amplitude would be greater as well.
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Fig. 3. Histogram of the rolling amplitudes of the ship B-354 in typical loading conditions (corresponding to the Fig. 2)

As the metacentric height is an important parameter governing ship’s rolling amplitude the statistical study was carried out to analyze the typical values of GM for a wide group of actually operated ships. The number of 86 vessels of 4 main types was investigated and the resultant distributions of their metacentric heights are presented in Fig. 4.

Fig. 4. Distributions of actual values of metacentric height for ships in operation
In every analyzed group of vessels the mean and modal values of metacentric height are greater then 1 meter. Thus the expected rolling amplitude would reach rather the value 20 degrees or even greater.

The next step of the research focused on the rolling amplitude estimation was an expert investigation. The pool was carried out among 22 experts with many years sea practice as deck officers (the mean value of professional sea experience equals 11.6 year). According to the results of the pool the distribution of the metacentric height and the extreme rolling amplitude are obtained which presents Fig. 5.

The further discussion during an expert panel suggests that the most severe rolling amplitudes, even reaching 40 degrees in some cases, were characteristic for resonant rolling. The local peak of rolling amplitude frequency around the value 20 degrees reflects the non-resonant mode of relatively heavy rolling.

The probability of reaching high values of rolling amplitude rapidly grows in case of a resonance of ship motion sailing on rough sea. Usually two modes of resonance are classified, the simple resonance and the parametric resonance. The first one takes place when the wave encounter frequency \( \omega_E \) is similar to the natural period of ship’s rolling \( \omega_\Phi \) which is expressed by \( \omega_E \approx \omega_\Phi \). The increase in rolling amplitude can be shown in Fig. 6.

![Fig. 5. Distributions of metacentric height values and extreme rolling amplitude according to the experts’ pool](image)

![Fig. 6. Non-resonant (left) and resonant (right) modes of rolling [4]](image)
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The amplification of ship rolling motion due to equality of natural and exciting frequencies \( \omega_e \cong \omega_n \) would lead to capsizing of the ship. However actually due to non-linear character of ship righting moment the natural rolling frequency varies with the increase in rolling amplitude and thanks to this phenomenon the amplification of rolling amplitude becomes weaker \([10]\). It enables to stay out of resonant tune range for large rolling amplitudes and avoid capsizing in many cases.

The more complex and usually more dangerous mode of ship motion is the parametric resonance. The basis of this phenomenon is an intensive excitation of ship rolling motion due to another coupled motions, especially ship heaving. The aftermath of large heave is a variation of transverse righting moment which could excite rolling \([2]\). The general conditions for parametric resonance appearance are following \([4]\):

1. the ratio of the natural rolling frequency to the wave encounter frequency is roughly about \( \frac{\omega_n}{\omega_e} \cong \frac{1}{2} \);
2. the wave encounter frequency is similar to ship heave frequency \( \omega_e \cong \omega_z \);
3. the ship’s vertical center of gravity shall be significantly different then draft.

The general scenario of developing of large rolling amplitudes during ship parametric rolling is presented in Fig. 7.

![Parametric rolling scenario with large rolling amplitude generation](image)

Since the ship rolling phenomenon is very complex there is no single accurate method for rolling amplitude estimation. The authors of study \([12]\) compare 14 methods of numerical simulations of ship parametric rolling. All the benchmarked methods were all non-linear time domain seakeeping methods. The ship hydrodynamics were modeled within the potential theory for the motion of ships in waves, either with a strip method or a panel method. The simulation methods differ...
also with respect to the employed roll damping models, where linear roll damping was applied by five methods and non-linear, quadratic or cubic, applied by the other nine methods. The parameters of these models were determined on the basis of the roll decay data, whereas one method was based only on semi-empirical model [12]. The prediction capabilities were determined on the basis of the group of the best performing simulation tools. On this basis, the mean probability to successfully detect the inception of the parametric roll resonance was estimated to be 0.78, while the predictions for the amplitude of roll motion deviated on average 6.4 deg. The corresponding figures for the overall benchmarked tools were 0.62 and 10.5 deg respectively [12].

Another important approach applied for rolling amplitude estimation consists in models tests. This method is rather costly and like any others it has its own limitations, however it has numerous advantages [7]. The exemplary results of Italian model tests from 1999 are shown in Fig. 8.

![Fig. 8. Non-resonant and resonant rolling (jump from rolling amplitude at ab. 20 degrees up to 40 degrees due to bifurcations) - results of Italian model test from 1999 [7]](image)

In the light of statistical research, experts’ panel results and the literature review, two typical values of ship rolling amplitudes emerge. The marginal extreme rolling amplitude seems to be about 40 degrees and the typical heavy rolling amplitude equals about 20 degrees. The very precise prediction of rolling amplitude is not achievable nowadays even for a specified ship [12]. Moreover, such prediction is unfeasible for ships analyzed generally as a group. Thus, the assumptions about two characteristic values of ship heavy rolling amplitudes equal 20 degrees and 40 degrees seems to be justified for the purpose of further investigation related to liquid sloshing phenomenon.
4. Computation of Heeling Moment due to Liquid Sloshing

The heeling moment due to liquid sloshing in a partly filled tank was computed with the use of CFD technique with the use of software FlowVision. The simulations were carried out in 3D mode. The hexahedral computational mesh was used with two coupled reference frames, the stationary and a moving ones. The Sub-Grid Geometry Resolution (SGGR) was applied where the triangulated surfaces naturally cut Cartesian cells and reconstructing the free surface [5]. The method consists in natural splitting of the boundary cells by the triangulated boundaries. The equations of a given mathematical model are approximated on the polyhedrons without simplifications. The approach enables accurate calculations in a complex domain on a reasonably coarse mesh [5].

The number of the obtained child cells depends on the geometry peculiarities. The child cells are arbitrary polyhedrons. The equations of a given mathematical model are approximated on the polyhedrons without simplifications. The approach enables accurate calculations in a complex domain on a reasonably coarse mesh [5]. The FlowVision code is based on the finite volume method (FVM) and uses the VOF method for free surface problems [5]. High accuracy of computation is achieved by solving the governing equations in the 'free surface' cells (the cells partly filled with liquid) [5]. The RANS (Reynolds-averaged Navier–Stokes) equation is implemented and the simulation of turbulent flows is based on the eddy viscosity concept. The semi-empirical k-ε model turbulence model was applied.

The result of the simulation comprises the general flow pattern, the velocity and pressure fields and the user-predefined heeling moment due to liquid sloshing being the most important for the conducted study.

The heeling moment due to liquid sloshing in tanks, obtained in the course of numerical simulations, was decomposed into two components. The first one comprises the moment due to dynamic action of solid-like liquid (i.e. ‘frozen’) at an angle of heel equal 0 degrees. The second component of the dynamic heeling moment due to liquid sloshing covers only the moment resulting from letting free the liquid to slosh inside the tank. The core idea of this approach may be expressed by the formula:

\[ M_{Total\_dyn} = M_{FL\_dyn} + M_{ff} \]  

(5)

where:

- \( M_{Total\_dyn} \) – total dynamic moment due to liquid sloshing in a tank;
- \( M_{FL\_dyn} \) – dynamic heeling moment due to the weight of solid-like liquid in a tank;
- \( M_{ff} \) – free floating component of the dynamic moment due to liquid sloshing.
According to the formula (5) the core component of the heeling moment due to sloshing is a difference between the total dynamic moment due to liquid sloshing and the dynamic moment due to solid-like weight in a tanks.

Although the heeling moment was computed in the course of time-domain calculations, the considered free floating component of the moment was plotted versus an angle of ship’s heel. Then thanks to the application of the decomposition of the heeling moment (formula 5), the resultant hysteresis loop of the free floating component may be simplified by the use of a previously worked out linearization procedure [11]. The linearization formula can be shown in following notation [11]:

\[
\int_{0}^{\phi} M_{\theta}(\phi) \cdot d\phi - \int_{0}^{\phi} M_{\theta}(\phi) \cdot d\phi + \int_{0}^{\phi} M_{\theta}(\phi) \cdot d\phi = ... \\
M_{\theta} = 4 \int_{0}^{\phi} M_{\theta, LA} \cdot d\phi = \phi_{4} \cdot M_{\theta, LA}
\]

where:
- \( M_{\theta} \) – free floating component of the dynamic moment due to liquid sloshing;
- \( \phi \) – angle of ship’s heel;
- \( \phi_{4} \) – ship’s rolling amplitude;
- \( M_{\theta, LA} \) – linear approximation of the free floating component of the heeling moment for a given ship’s rolling amplitude;
- \( M_{\theta, LA} \) – the value of the linear moment \( M_{\theta, LA} \) for an angle of heel equal rolling amplitude \( \phi_{4} \).

The linear function of heeling moment can be determined by the fixing of two in-line points having the coordinates (\( \phi, M \)). One of them is the point (0,0) and the second one the point (\( \phi_{4}, M_{\theta, LA} \)). Therefore the complete description of the linear approximation of the moment \( M_{\theta} \) may be done by one scalar only. Such a scalar is the value \( M_{\theta, LA} \) of the linear free floating component due to sloshing for the angle of heel equal \( \phi_{4} \) and obtained according to the given formula (6) [11].

The value \( M_{\theta, LA} \) of the heeling moment’s linear free floating component depend not only on tank dimensions and location but obviously one the weight of liquid inside. Thus the wider and higher tanks are considered the greater value of moment \( M_{\theta, LA} \) is noticed. This effect can be avoided and instead of presentation of the value \( M_{\theta, LA} \), the ratio \( M_{\theta, LA}/q \) could be plotted, where \( q \) is just the weight of liquid in each tank calculated as mass times gravity acceleration. The quotients moment over liquid weight which can be actually called moments’ arms.

In the course of the study two typical ship dimensions were taken into consideration. In case of quasi static approach to the free surface effect the location of a partly filled tank does not play any role. Reversely, the dynamic approach is
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related to ship rolling and the location of considered tank is crucial. The particulars applied in the research are given in the table 1.

<table>
<thead>
<tr>
<th>Ship particulars [m]</th>
<th>ship 1</th>
<th>ship 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>breadth B</td>
<td>32</td>
<td>20</td>
</tr>
<tr>
<td>height H</td>
<td>20</td>
<td>12,5</td>
</tr>
<tr>
<td>elevation of rolling axis KR</td>
<td>9,00</td>
<td>5,62</td>
</tr>
</tbody>
</table>

Regardless the ships dimensions an exemplary tank is taken into account. It is located in ships symmetry plane at different elevations (Fig. 9). Its breadth equals 10m and height 1,5m. The filling level equals 50%. The ratio \( M_{FfA}/q \) was computed for two rolling amplitudes 20 and 40 degrees. The results are shown in Fig. 9.

Fig. 9. Linear free floating arm \( M_{FfA}/q \) for different location of the exemplary partly filled tank – computations performed for ship’s rolling amplitudes equal 20 and 40 degrees

The graph presented in Fig. 9 reveals an increasing value of the ratio \( M_{FfA}/q \) with the rise in rolling amplitude. Such a result suggests that the rolling amplitude assumed for the purpose of liquid sloshing computation shall be fairly adjusted to avoid the underestimation of final heeling moment values.

5. Conclusion

The research described in the paper is focused on ship’s rolling amplitude as an important factor influencing dynamic effects of liquid sloshing in partly filled tanks. Beyond the literature review providing numerous suggestions regarding expected rolling amplitudes of ships, the statistical study and the expert panel was carried out.
As the result of investigation, two significant values for rolling amplitudes were obtained. The amplitude about 20 degrees reflects typical heavy rolling of ship without suffering parametric rolling. The amplitude equal 40 degrees is an extreme amplitude noticed in marginal situations onboard ships experiencing parametric rolling. Beyond the 40 degrees margin the stability accident is likely to be noticed. Both obtained rolling amplitudes are the basis for further research with regard to dynamic effects of liquid sloshing onboard ships.

The exemplary computations for a partly filled tank located at different elevations over ship’s base line, reveals the characteristic of the ratio $\frac{M_{\text{ffA}}}{q}$ increasing with the rise in rolling amplitude. Such result is important from the stability assessment point of view. The stability standards are to ensure the relevant level of ship safety with regard to her stability. Thus, any underestimation of stability decaying factors cannot be accepted, therefore the high values of ships’ rolling amplitudes obtained in the course of the research are justified.

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6. References,

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