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Validation of transport and friction formulae for upper plane bed by experiments in rectangular pipe

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Abstract: The paper describes results of validation of authors' recently proposed formulae for sediment transport and bed friction in the upper plane bed regime using laboratory experiments in a pressurized pipe. Flows of mixture of water and fine to medium ballotini (d_{50} = 0.18 mm) were observed in a rectangular pipe (51 x 51 mm) with a deposit at the bottom of the pipe. A comparison of test results with transport-formula predictions shows a satisfactory match confirming a good prediction ability of the proposed transport formula at high bed shear. A prediction ability of the friction formulae appears to be less convincing but still reasonable. A joint use of the formulae for transport and friction predicts the delivered concentration of transported sediment within the accuracy range of ± 40 per cent for flows in which transported sediments strongly affect the bed friction, i.e. for flows with delivered concentration of sediment higher than say 3 per cent.

Keywords: Sediment transport; Sheet flow; Bed load; Suspended load.

INTRODUCTION

Intense sediment transport at high bed shear is a phenomenon associated with flows in open mobile-bed channels and pressurized industrial slurry pipes. For instance, flood flows in steep mountain streams can carry sediments at high concentration over the upper plane bed. Lately, increasing research attention has been focused on high concentrated flows in natural channels (e.g. Frey and Church, 2009; Chiari et al., 2010; Rickenmann and Koschni, 2010; El Kadi Abderrezzak and Paquier, 2011; Nitsche et al., 2011) and laboratory conduits. The well-controlled laboratory flows have been subject to both mathematical (e.g. Berzi, 2011; Capart and Fraccarollo, 2011; Talukdar et al., 2012) and physical modeling in either steep flumes (Recking, 2010; Recking et al., 2008; Capart and Fraccarollo, 2011) or pipe loops (Matoušek, 2009; Matoušek and Krupička, 2009, 2012a; Vlasák et al., 2012). Current laboratory investigations of pipeline transport of high concentrated slurry flows focuses on a complex flow behaviour associated with a very broad particle size distribution of solids fractions typically transported in industrial slurry pipelines (e.g. Graham et al., 2011; Vlasák and Chára, 2011).

Recently, a solids transport formula has been proposed for settling-slurry flows (only coarse particles, i.e. no fines leading to complex behaviour) in the upper plane bed (UPB) regime in pressurized pipes (Matoušek, 2009). The formula is based on an analytical derivation for the structure of sheet flow (details e.g. in Matoušek, 2011). Assumptions are made for shapes of profiles across a shear layer through which solids are transported, i.e. the linear shape of a concentration profile and the power-law shape for a solids-velocity profile. The analysis arrives to the transport formula of the MPM (Meyer-Peter and Műller) type

$$\Phi = \alpha \cdot \theta^{\beta} \tag{1}$$

in which Φ is the Einstein transport parameter $\left(\Phi = \frac{q_s}{\sqrt{(s-1) \cdot g \cdot d_{50}^3}} \right)$, q_s is sediment discharge per unit width of bed, s is relative density of solids, g is gravitational acceleration, d_{50} is mass-median particle diameter, θ is the Shields parameter ($\theta = \frac{u_{*b}^2}{(s-1) \cdot g \cdot d_{50}}$, u_{*b} is bed shear velocity), and α, β are coefficients. Compared to the original MPM formula, Eq. (1) neglects the threshold Shields parameter θ_C for the incipient motion of particles (θ values are much higher than θ_C in the UPB regime) and does not consider α, β as constants. The analytical solution for a sheet flow relates α to the power of the power-law velocity profile and to the coefficient of internal friction of particles, tanφ. A comparison with the database containing data for very different solids fractions (sand, bakelite, nylon) indicated that both α and β tended to vary for different fractions in the UPB regime and this could be satisfied by relating the coefficients with the particle Reynolds number Re_p. The proposed relationships read $\alpha = \frac{3.13}{\tan\varphi} + \frac{58}{\mathrm{Re}_p^{0.62}}$ and $\beta = 1.2 + \frac{1.3}{\mathrm{Re}_p^{0.39}}$, where tan$\varphi$ is coefficient of internal friction of solid particles and $\mathrm{Re}_p = \frac{w_t \cdot d_{50}}{\nu}$ (w_t is terminal settling velocity of particle, ν is kinematic viscosity of liquid). Taking a typical value of tan φ = 0.6 for sands and gravels, the Eq. (1) reads

$$\Phi = \left(5.2 + \frac{58}{\mathrm{Re}_p^{0.62}} \right) \cdot \theta^{\left(1.2 + \frac{1.3}{\mathrm{Re}_p^{0.39}} \right)} . \tag{2}$$

The relationship has been verified and validated for $5 \leq \mathrm{Re}_p \leq 280$. The coefficients are particularly sensitive to the particle Reynolds number for sediment fractions of Re_p smaller than say 30. At Re_p bigger than say 150, the effect of Re_p becomes small.

A recent comparison of formula predictions with laboratory flume data by Smart (1984) and Capart-Fraccarollo (2011) has revealed (Matoušek and Krupička, 2012b) that the formula can be applied to sheet flows in open channels as well.

Besides developing the formula for solids transport, the authors derived friction coefficient correlations for the UPB regime (Matoušek and Krupička, 2012a). A logarithmic formula for a hydraulically rough boundary was taken as a basis for the analysis of the bed friction coefficient λ_b. The formula recognizes k_s/R_b as a dimensionless length scale for bed roughness (k_s is equivalent roughness height, R_b is hydraulic radius of discharge area associated with bed),

$$\sqrt{\frac{8}{\lambda_b}} = 2.5\cdot\ln\left(\frac{B_s\cdot R_b}{k_s}\right), \tag{3}$$

where the typical values of the formula constant are $B_s = 14.8$ for pressurized-pipe flows and $B_s = 11.1$ for open-channel flows. For granular beds, the roughness height k_s is usually related to the bed particle size through k_s/d_{50} = const. In the UPB regime, however, the determination of k_s/d_{50}, or λ_b, is more complex and requires additional parameters in the relationship (see e.g. Matoušek and Krupička, 2009). Based on the authors' slurry-pipe database, the empirical formula for the equivalent roughness in the log-law (Eq. (3)) is suggested as

$$\frac{k_s}{d_{50}} = 0.0037\cdot\frac{W_{s*}^{1.71}}{i_E^{1.94}}\cdot\left(\frac{d_{50}}{R_b}\right)^{0.56}\cdot\theta^{2.89} \tag{4}$$

in which i_E is the slope of the energy grade line, and the dimensionless settling velocity $W_{s*} = \left[\frac{(s-1)^2}{\mathrm{g}\cdot\nu}\right]^{1/3}\cdot w_t$. It can be assumed, however, that a velocity profile deviates from a logarithmic shape in a concentrated bed-load flow above an eroded bed. Hence, a power-law type of a relationship between λ_b and pertinent dimensionless groups might be more appropriate and a calibration of such a formula using slurry-pipe data suggests

$$\lambda_b = 0.0122\cdot\frac{W_{s*}^{0.77}}{i_E^{0.99}}\cdot\left(\frac{d_{50}}{R_b}\right)^{0.80}\cdot\theta^{1.43}. \tag{5}$$

An aim of this work is to validate the formulae using new experimental data collected for the solids fraction and conduit geometry that have not been available in the database yet.

EXPERIMENTAL WORK

In 2010–2011, validation tests were carried out with stratified flows of an aqueous mixture of medium to fine ballotini (Re_p smaller than 5) in a pressurized conduit of a rectangular cross section (H x B = 51.2 x 50.8 mm). The pipe is a part of the experimental test loop for slurry flows (see Fig. 1) which was recently set up in the Water Engineering Laboratory of the Czech Technical University in Prague. As Fig. 1 shows, the loop is composed of horizontal pipes and a vertical invert U-tube. Except for the rectangular conduit made of plexiglass and implemented to the horizontal part of the loop for this particular experiment, the rest of the loop is composed of circular pipes made of PP (polypropylene). The internal diameter of the circular pipes is 51.2 mm. The total length of the loop is approximately 22.5 m. Three measuring sections for the differential pressure (DP) are located in straight pipes. The horizontal measuring section (DP3) is 2 m long and the length of the straight pipe in front of the section is 3.7 m, i.e. 74 times pipe height. The vertical sections (DP1 and DP2) are 1.3 m long and the straight-pipe lengths in front of the sections are 54 times internal diameter in case of DP1 and 45 times internal diameter in case of DP2. DP1 and DP2 serve primarily for determining the density of flowing mixture of water and sediment (this density is further interpreted as the delivered volumetric concentration of particles in the mixture), DP3 is used for measuring the hydraulic gradient in the horizontal flow and for additional flow analyses. Differential pressure transmitter Siemens Sitrans P DSIII is used to record differential pressures over DP3, transmitters Fisher Rosemount DP1151 are used in vertical sections. A magnetic flow meter (Krohne Optiflux 5000) is mounted to the vertical pipe section to measure the cross-sectional averaged velocity (and thus the volumetric flow rate) of the flowing mixture. A centrifugal pump (Ebara, input power 2.2 kW) is run by the variable-frequency drive which makes a continuous regulation of pump speed (from zero to maximum) possible. The system is employed to control the flow rate in the test loop. An electronic data acquisition system gathers signals of all measured parameters except the thickness of deposit in the horizontal pipe section. The thickness is determined visually in the transparent plexiglass pipe.

The tested granular fraction was an industrial ballotini (fraction B134 produced by the company Preciosa) that is relatively narrow graded (d_{15} = 0.16 mm, d_{50} = 0.18 mm d_{85} = 0.24 mm) and has the density similar to natural sands and gravels (2450 kg m^{-3}). Our tests determine the sediment properties mentioned above and the settling velocity of the sediment, $w_t \approx 18$ mm s^{-1}. Hence, a typical value of the ballotini Reynolds number $\mathrm{Re}_p \approx 3.2$.

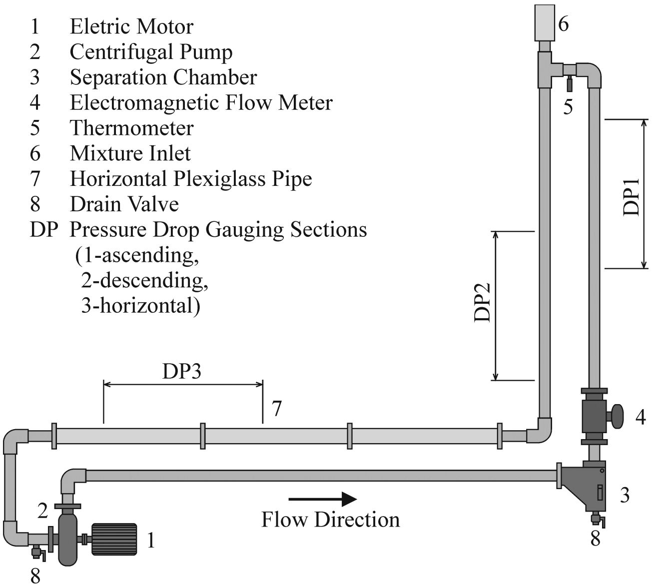

Fig. 1. Experimental test loop in Water Engineering Laboratory of Czech Technical University in Prague.

VALIDATION OF TRANSPORT FORMULA

Plotting experimental data to the Φ-θ plot exhibits a tight correlation (squares in Fig. 2) between the dimensionless numbers. Thus a transport formula of the MPM type is appropriate for this ballotini mixture. However, considering $\beta = 1.5$ as suggested in the original MPM formula (and derived analytically for the bed-load transport layer, see e.g. Matoušek, 2011) leads to no success in approximating the data for any suggested value of the constant α in the MPM formula (Eq. (1)). Values of the constants as used in Eq. (2) are considerably higher than in the original MPM, namely $\alpha \approx 32$; $\beta \approx 2.0$. Eq. (2) predicts Φ values that are in a very good agreement with the measured values, particularly for θ higher than 1.5 (Fig. 2). At lower values of θ, the transport formula (Eq. (2)) tends to overestimate values of the sediment flow rate q_s with slightly more than 50 per cent (Fig. 3).

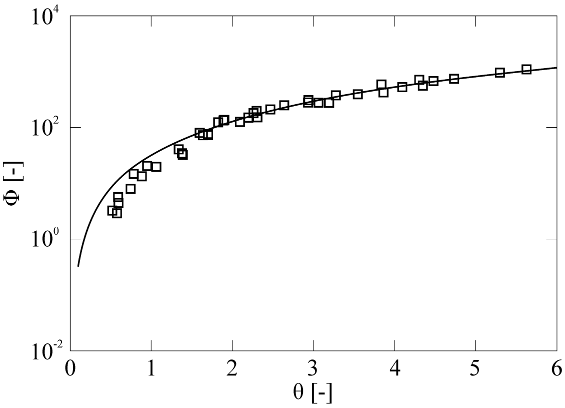

Fig. 2. Dimensionless sediment flow rate: comparison of measurements (see Table 1) and predictions using Eq. (2). Legend: □ measurements, —— predictions using Eq. (2).

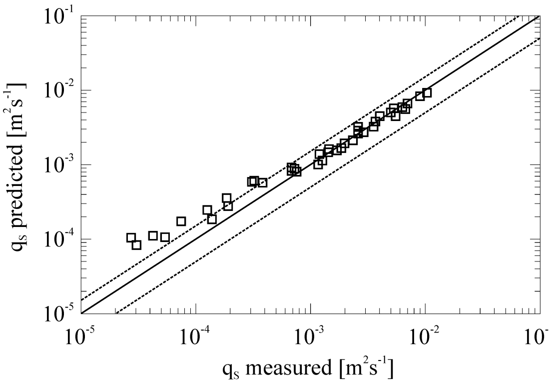

Fig. 3. Sediment flow rate per unit width of bed: comparison of measurements and predictions using Eq. (2). Legend: □ measurements, —— line of perfect match, ······ 50 per cent deviation from perfect match.

The values of α and β as used in Eq. (2) ($\alpha \approx 32$; $\beta \approx 2.0$) agree very well with values derived directly from the experimental data for $\theta > 1.5$ ($\alpha = 29$; $\beta = 2.1$). Values derived from all data (i.e. including those for $\theta < 1.5$) are very different ($\alpha =$ $=19$; $\beta = 2.5$). This indicates rather different transport behaviour in two UPB sub-regimes separated by the threshold value of θ equal to roughly 1.5. This threshold value of θ seems to correspond with the threshold at which the bed friction coefficient starts to increase due to the presence of transported sediment. Fig. 4 shows that λ_b, tends to increase if the delivered concentration C_{vd} of sediment exceeds of about 3 per cent (Fig. 4). This value of C_{vd} is reached in the ballotini mixture at θ of about 1.5 (see Table 1). This θ value also corresponds with u_{*b}/w_t of about 3.5 (Table 1) which is a value sufficient for producing a considerable proportion of turbulent suspension in pressurized flows of settling mixtures (e.g. Matoušek, 2007; Wilson, 2005). Particle suspension by turbulent eddies of flowing carrier may be a reason for the required high value of β in the transport formula for the tested ballotini flow.

VALIDATION OF FRICTION FORMULAE

Fig. 4 shows that the friction coefficient of the mobile bed composed of the ballotini particles is very sensitive to the delivered concentration of sediment in the flow above the bed. If the concentration remains small (smaller than say 3 per cent by volume), then values of the measured λ_b correspond well with values predicted using the law of the rough wall (Eq. (3)) for a fixed bed with the equivalent roughness equal to two particle diameters. However, if the concentration exceeds 3 per cent the coefficient starts to increase and the λ_b value almost triples when the concentration exceeds 20 per cent.

The effect of the delivered concentration on the bed friction coefficient is incorporated in the proposed friction formulae Eqs. (3) and (4) or Eq. (5). A comparison of formulae predictions with the experimental data shows that both proposed formulae tend to underestimate a value of the bed friction coefficient and both formulae give very similar results (see Fig. 5). At the lowest values of θ, however, a combination of Eqs. (3) and (4) predicts better than Eq. (5).

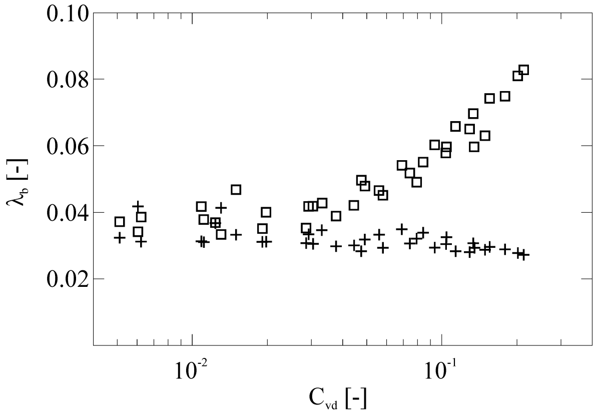

Fig. 4. Effect of delivered concentration of transported sediment on bed friction coefficient: comparison of measurements (see Table 1) and prediction using Eq. (3) with $k_s = 2 \cdot d_{50}$. Legend: □ measurements, + predictions using Eq. (3) with $k_s = 2\ d_{50}$.

DISCUSSION: COMBINED USE OF TRANSPORT AND FRICTION FORMULAE

The transport formula of the MPM type (Eq. (1)) can be used as a basis for a formulation of an explicit relationship between the bed friction coefficient, λ_b, and the delivered concentration, C_{vd}, i.e. the relationship shown in Fig. 4. In the UPB regime, it can be assumed that the liquid viscous contribution to the total bed, friction is small compared to the sediment contribution and hence the bed friction coefficient λ_b equals to the friction coefficient due to sediment transport. Assuming this and using $C_{vd} = q_s/q_T$, $q_T = v \cdot h$ (q_T is discharge of mixture per unit width of bed,

Table 1. Experimental data for flow of ballotini mixture in upper plane bed regime.

Series	Run	v	i_E	C_{vd}	h	R_b	θ	Φ	λ_b	u_{*b}/w_t
		[m s^{-1}]	[–]	[–]	[m]	[m]	[–]	[–]	[–]	[–]
2010-12-07 b	11	1.167	0.0409	0.048	0.047	0.021	3.19	273.8	0.050	5.1
	12	1.118	0.0350	0.038	0.044	0.018	2.30	196.1	0.039	4.4
	13	1.066	0.0320	0.029	0.042	0.016	1.89	133.5	0.035	4.0
	14	0.983	0.0281	0.019	0.040	0.015	1.60	79.6	0.035	3.6
	15	0.866	0.0232	0.011	0.040	0.016	1.34	40.4	0.038	3.3
	16	0.764	0.0188	0.006	0.039	0.015	1.06	19.6	0.038	3.0
	17	0.645	0.0137	0.003	0.039	0.015	0.75	7.9	0.038	2.5
2010-12-09 a	13	1.173	0.0422	0.058	0.043	0.019	2.94	305.7	0.045	4.9
	14	1.114	0.0387	0.045	0.040	0.017	2.47	209.0	0.042	4.5
	15	1.028	0.0345	0.031	0.038	0.016	2.09	125.9	0.042	4.2
	16	0.950	0.0300	0.020	0.036	0.015	1.71	72.2	0.040	3.8
	17	0.837	0.0246	0.011	0.036	0.015	1.38	34.3	0.042	3.4
	18	0.710	0.0175	0.005	0.035	0.014	0.89	13.3	0.037	2.7
	19	0.571	0.0118	0.001	0.034	0.013	0.58	2.9	0.038	2.2
2011-01-10 a	14	1.207	0.0556	0.129	0.041	0.022	4.48	669.8	0.065	6.1
	15	1.147	0.0532	0.114	0.039	0.021	4.10	529.8	0.066	5.8
	16	1.116	0.0516	0.094	0.035	0.019	3.55	388.5	0.060	5.4
	17	1.095	0.0493	0.075	0.032	0.016	2.93	274.7	0.052	4.9
	18	1.009	0.0436	0.049	0.029	0.014	2.31	152.9	0.048	4.4
	19	0.929	0.0373	0.029	0.027	0.012	1.70	76.3	0.042	3.8
	20	0.793	0.0301	0.015	0.026	0.012	1.39	32.4	0.047	3.4
	21	0.570	0.0157	0.003	0.024	0.010	0.60	4.5	0.039	2.2
2011-01-13 a	12	1.199	0.0636	0.213	0.041	0.024	5.63	1094.7	0.083	6.8
	13	1.177	0.0642	0.202	0.038	0.022	5.30	949.1	0.081	6.6
	14	1.156	0.0647	0.179	0.034	0.020	4.73	741.6	0.075	6.3
	15	1.113	0.0651	0.156	0.031	0.018	4.35	559.8	0.074	6.0
	16	1.082	0.0648	0.134	0.028	0.016	3.86	422.9	0.070	5.7
	17	1.042	0.0618	0.105	0.024	0.013	3.07	276.3	0.060	5.0
	18	0.932	0.0521	0.084	0.022	0.012	2.26	179.6	0.055	4.3
	19	0.844	0.0460	0.069	0.020	0.011	1.83	123.0	0.054	3.9
	20	0.707	0.0324	0.013	0.015	0.007	0.79	14.6	0.033	2.6
	21	0.606	0.0251	0.006	0.015	0.006	0.59	5.7	0.034	2.2
2011-07-26 a	08	1.201	0.0578	0.149	0.038	0.020	4.31	708.4	0.063	6.0
	07	1.166	0.0559	0.135	0.035	0.018	3.84	581.3	0.060	5.6
	06	1.093	0.0535	0.104	0.031	0.016	3.28	374.0	0.058	5.2
	05	1.067	0.0512	0.080	0.028	0.014	2.64	247.6	0.049	4.7
	04	1.000	0.0474	0.056	0.025	0.012	2.20	149.7	0.046	4.3
	03	0.900	0.0403	0.033	0.023	0.011	1.64	72.3	0.043	3.7
	02	0.738	0.0279	0.012	0.021	0.009	0.95	20.3	0.037	2.8
	01	0.571	0.0171	0.003	0.020	0.008	0.52	3.2	0.034	2.1

v is average velocity of mixture flow, h is flow depth), and $\sqrt{\frac{8}{\lambda_b}} = \frac{v}{u_{*b}}$, the transport formula (Eq. (1)) can be re-written as

$$\sqrt{\frac{8}{\lambda_b}} = \frac{d_{50}}{h}\cdot\frac{1}{C_{vd}}\cdot\alpha\cdot\theta^{\beta-0.5} \tag{6a}$$

which can be seen as an additional bed-friction formula for the UPB regime. The formula shows that the relationship between λ_b and C_{vd} is affected by Shields parameter, flow depth and sediment properties.

A comparison of the predictions using Eq. (6a) with the ballotini experimental data in Fig. 5 shows a very good match for high values of λ_b, i.e. for $C_{vd} > 0.03$. At lower C_{vd}, however, Eq. (6a) fails. This is because λ_b predicted by Eq. (6a) represents the bed friction due to sediment transport only. The experimental data say that if $C_{vd} < 0.03$ then the effect of C_{vd} on λ_b is negligible and hence Eq. (6a) cannot be valid ($\lambda_b = 0$ for $C_{vd} = 0$ in Eq. (6a)). Instead, λ_b from Eq. (3) with $k_s = 2\cdot d_{50}$ is appropriate for $C_{vd} < 0.03$. Note that the other proposed friction formulae (Eqs. (3) with (4) or Eq. (5)) do not recognize any C_{vd} threshold. They are suggested to be valid within the entire range of measured C_{vd}. Therefore, it can be assumed that due to ignoring the two different experimentally observed regimes for λ_b in Fig. 4 the prediction ability of the proposed friction formulae may be weak at the lowest C_{vd} values. And indeed, the formulae exhibit the worst predictions at the lowest C_{vd} as Fig. 5 shows.

If rewritten to the form expressing C_{vd}, Eq. (6a) reads

$$C_{vd} = \frac{d_{50}}{h}\cdot\sqrt{\frac{\lambda_b}{8}}\cdot\alpha\cdot\theta^{\beta-0.5}. \tag{6b}$$

Combined with the proposed friction formula (i.e. Eq. (3) or (5)), Eq. (6b) can be used for a direct prediction of the delivered concentration of transported sediment in mixture flow. Assuming flow in a wide open channel (longitudinal slopes of the bed, i_b, and of the water surface, i_w, equal to i_E, and $R_b = h$), the concentration of transported sediment of known size and specific gravity can be determined directly from the channel slope and the flow depth. Note that $\lambda_b = fn\left(h, i_b, d_{50}, s, \nu\right)$ in Eqs (3), (4), and (5) and also $C_{vd} = fn\left(h, i_b, d_{50}, s, \nu\right)$ in Eq. (6b), because $\theta = \dfrac{h \cdot i_b}{\left(s-1\right) \cdot d_{50}}$. Results in Fig. 6 demonstrate a very reasonable match between the measured values of C_{vd} and the values predicted using a combination of Eq. (6b) with the coefficients α, β as used in Eq. (2) and one of the proposed friction formulae provided that C_{vd} exceeds 0.01. Both friction formulae (Eq. (5) and Eq. (3) with k_s from Eq. (4)) perform almost equally well in predicting C_{vd}.

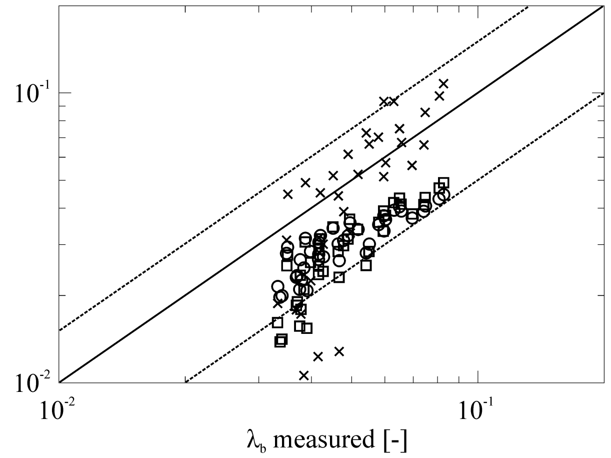

Fig. 5. Bed friction coefficient: comparison of measurements (Table 1) and predictions using various friction formulae. Legend: ○ prediction using Eqs (3) and (4), □ prediction using Eq. (5), × prediction using Eqs (2) and (6), —— line of perfect match, ┈┈ 50 per cent deviation from perfect match.

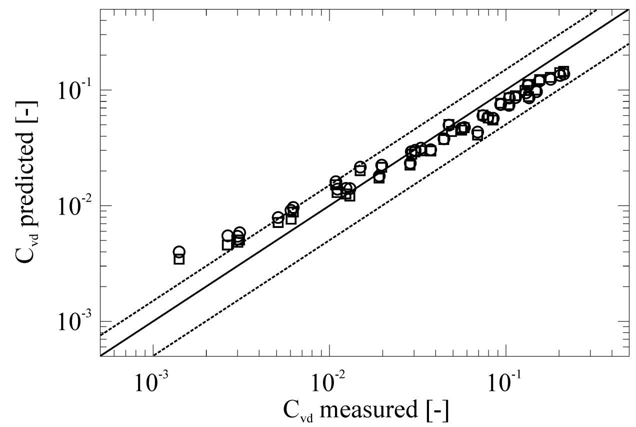

Fig. 6. Delivered concentration of transported sediment: comparison of measurements (Table 1) and predictions using combination of transport- and friction formulae. Legend: ○ prediction using Eqs (2), (3), (4), and (6b), □ prediction using Eqs (2), (5), and (6b), —— line of perfect match, ┈┈ 50 per cent deviation from perfect match.

CONCLUSIONS

Results of laboratory tests carried out for conditions not yet available in the database of settling-slurry flows above deposits confirmed a good ability of the by the authors earlier proposed transport formula to predict sediment flow rates in the upper plane bed regime, particularly at high values of Shields parameter for which high concentration of transported sediment is typical. Mixture flows of medium to fine ballotini (the median particle size of 0.18 mm and particle Reynolds number of about 3.2) exhibited a considerable increase in bed friction if the delivered concentration of transported particles exceeds say 3 per cent by volume.

The proposed friction formulae seem to grasp this increase although predictions of the bed friction coefficient are less accurate than the predictions of the sediment flow rate. It is shown that a combination of the transport and friction formulae can be used for predicting the delivered concentration of transported sediment of certain size and specific gravity from values of water depth and channel longitudinal slope.

Further investigation is required on flows near the threshold for the upper-plane regime (Shields parameter below one), where the prediction capability of the formulae seems to be the weakest.

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