ESTIMATING FLOW CHARACTERISTICS OF DIFFERENT WEIR TYPES AND OPTIMUM DIMENSIONS OF DOWNSTREAM RECEIVING POOL

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This paper presents the results of a laboratory study on the flow characteristics of sharp-crested weirs, broad-crested weirs, and labyrinth weirs. The variation of the maximum bubble penetration depth for different weir types is investigated depending on overfall jet expansion, discharge, and drop height. Moreover, most efficient depth, length and width of the downstream receiving pool in an open channel system are studied by considering the penetration depth, overfall jet expansion, jet trajectory and the bubble zone. The results show that overfall jet expansion at the labyrinth weirs is significantly wider than the rectangular sharp-crested weirs and the trapezoidal sharp-crested weir. It is demonstrated that the labyrinth weirs have the lowest values of bubble penetration depth among the weirs tested. Furthermore, it is found that the rectangular and the trapezoidal weirs are observed to have the highest bubble penetration depth among all weirs. Consequently, empirical equations are obtained for predicting the maximum penetration depth of bubbles, trajectory of free overfall nappe, jet expansion of free overfall nappe, and the length of the bubble zone.

KEY WORDS: Weir, Penetration Depth, Jet Expansion, Jet Trajectory, Bubble, Air Entrainment, Aeration.

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Príspevok prezentuje výsledky laboratórneho výskumu charakteristík prúdenia cez ostrohranné priepady, cez priepady so širokou hranou a cez labyrintové priepady. Bola študovaná variácia maximálnej hĺbky prieniku vzduchových bublín pre rozdielne typy priepadov v závislosti od rozšírenia prúdu, prietoku a výšky priepadu. Okrem toho bola analyzovaná efektívna hĺbka, šírka a dĺžka vývaru v systéme otvorených kanálov s uvažovaním rozšírenia prúdu, prietoku a výšky priepadu a oblasti so vzduchovými bublinami. Z výsledkov vyplýva, že rozšírenie prúdu pri labyrintových priepadoch je výrazne väčšie, ako pri pravouhlých a lichobežníkových ostrohranných priepadoch. Bolo ukázané, že labyrintové priepady majú najmenšiu hĺbku prieniku vzduchových bublín spomedzi všetkých testovaných priepadov. Zistilo sa však, že pravouhlé a lichobežníkové priepady majú najväčšie hĺbky prieniku vzduchových bublín spomedzi všetkých testovaných priepadov. Tieto výsledky výskumu viedli k empirickým rovniciam, umožňujúcim výpočet maximálnej hĺbky prieniku vzduchových bublín, trajektórii prepadového lúča, rozšírenia prúdu a dĺžky prevzdušnenej oblasti.

KLÚČOVÉ SLOVÁ: priepad, hĺbka prieniku vzduchových bublín, rozšírenie prúdu, trajektória prúdu, bublina, prevzdušňovanie.

1. Introduction

The overflow section shape cut with a sharp upstream corner into a thin plate is the weir notch, sometimes called the overflow section. If the notch plate is mounted on the supporting bulkhead such that the water does not contact or cling to the downstream weir plate or supporting bulkhead, but springs clear, the weir is a sharp-crested weir

(USBR, 2001). A broad-crested weir is a flat-crested structure with a crest length large compared to the flow thickness. The ratio of crest length to upstream head over crest must be typically greater than 3 (Henderson, 1966; Chanson, 2004). The flow control section can have different shapes, such as rectangular, triangular, trapezoidal or circular. Labyrinth weirs are those for which the weir crest is not straight in planform (Wormleaton and Soufiani,

1998). The increased crest length of labyrinth weirs gives them the clear advantage of reducing upstream levels for a particular discharge over low head ranges (Wormleaton and Tsang, 2000). The flow over weirs would be classified as a free jet, as shown in Figs. 1a–c) and Figs. 2a, b).

Weirs have a large number of uses in engineering applications. Listing the more usual, these are: 1. to measure the flow of water in small stream and constructed channels; 2. to regulate the flow of water in open channel and reservoir; 3. to take the water for irrigation systems; 4. to divert flow during high flow conditions in sewer systems; 5. to increase the dissolved oxygen content of the water by mean of aeration the water; 6. to reduce tastes and odors caused by dissolved gases in the water by mean of aeration the water, such as hydrogen sulphide, which are then released; and also to oxidize and remove organic matter; 7. to decrease the carbon dioxide content of a water by mean of aeration the water and thereby reduce its corrosiveness and raise its pH value; 8. to remove certain volatile organic compounds by mean of aeration the water.

An overflow jet that plunges into a channel after passing through a gas phase entrains a substantial amount of air into the receiving pool, and forms a submerged two-phase region with a considerable interfacial area. Almost all the mass transfer substantially takes place in this two-phase region. Therefore, maximum penetration depth of the bubble swarm is one of the most important parameters that characterize the performance of weir aeration system and the design of downstream receiving pool. Moreover, estimating flow characteristics of weirs would be useful for scour depth in the downstream receiving channel because flow characteristics of the weirs are different from each other.

Apted and Novak (1973), Avery and Novak (1978), and Nakasone (1987) reported that tailwater depth was an important weir operation parameter, influencing mass transfer. Tailwater depth is also an important parameter for energy dissipation and scouring cases. Grindrod (1962) and Albrecht (1968) pointed out that the aeration efficiency would increase with increasing tailwater depth. However, there should be a limit because the penetrating air bubbles will not go to infinite depths. Actually, for each combination of discharge, drop height, and weir type, there would be an approximate maximum depth to which the bubbles would penetrate, thus limiting the aeration efficiency. Studies by Nakasone (1975, 1976, and 1987) contributed substantially to the experimental investigations of the weir aeration. Nakasone (1987) did not investigate

the case whether or not air bubbles reached the floor of the stilling basin. For tailwater depth, Nakasone suggested to adapt as $2/3$ of drop height. However, the bubble penetration depth and the overfall jet expansion would be different for each weir type and shape and it would not be true to give the bubble penetration depth as a function of only drop height. Moreover, the bubble penetration depth differs from each weir type.

Kobus and Koschitzky (1991) presented the empirical correlation for penetration depth $D_p = 0.00433 \text{Re}_j^{0.39} F_j^{0.24}$, where Re_j – jet Reynolds

number, $\text{Re}_j = \frac{q_j}{\nu}$ and F_j – the jet Froude

number, $F_j = \left[\frac{gh^3}{2q_j^2} \right]^{1/4}$. Nakasone (1987) advised

that the receiving pool depth D_p ceases to affect aeration efficiency when it exceeds two-thirds of the drop height h (i.e., $D_p \geq 2/3 h$), and Avery and Novak (1978) gave the relationship as $D_p \geq 7.5h^{0.58} F_j^{-0.53}$, where h – drop height and F_j – the jet Froude number.

Kumagai et al. (1993), Clanet and Lasheras (1997), and Ito et al. (2000) studied depth of penetration of bubbles entrained by a plunging water jet. They presented correlations for predicting the bubble penetration depth of plunging water jet as a function of the operational conditions and nozzle geometry. Baylar and Emiroglu (2002), and Emiroglu and Baylar (2003 and 2005) studied air entrainment rate of the broad crested, sharp crested weirs and labyrinth weirs and they stated that weir shape is very sensitive for flow characteristics. The behavior of water flowing over a weir varies significantly with weir shape and type. It is therefore highly desirable to investigate flow characteristics for different weir types and their shapes.

This paper presents an experimental investigation of the bubble penetration depth D_p , trajectory of free overfall nappe from the weirs, the length of the bubble zone, and the overfall jet expansion J_e of the sharp crested weirs, broad crested weirs, and labyrinth weirs. In addition, the paper studies most efficient length, width and depth of the downstream receiving pool for especially weir aerators.

2. Dimensional analysis

The functional relationship involving the maximum bubble penetration depth and influencing variables can be written as

a)

b)

c)

Fig. 1. Weirs; a) Sharp-crested weir, b) Broad-crested weir, c) Labyrinth weir.
Obr. 1. Priepady; a) ostrohranné; b) so širokou korunou; c) labyrintové.

Fig. 2. Sketch of flow patterns of free jet over weirs; a) Rectangular sharp-crested weir; b) Labyrinth weir.
Obr. 2. Náčrt voľného prúdu nad priepadom; a) pravouhlý, ostrohranný, b) labyrintový priepad.

$$D_p = f_1 \left(\frac{\Delta H, B_w, q_j, \rho, \Delta \rho, g, \mu, \sigma, P_{on}, \Delta P, z, b_r, b_\ell, b_d, b_u}{\sigma, P_{on}, \Delta P, z, b_r, b_\ell, b_d, b_u} \right), \quad (1)$$

in which D_p – maximum bubble penetration depth, ΔH – energy difference between the flow over the weir crest and the receiving pool, B_w – water surface width over the crest of the weir, q_j – discharge through the per unit water surface width at the crest of the weir ($q_j = Q/B_w$; Q – total discharge through the weir), ρ – density of fluid (water), $\Delta \rho = \rho - \rho_a$, ρ_a – density of gas (air), g – gravity acceleration, μ – viscosity of the fluid (water), σ – surface tension of the fluid (water), P_{on} – ambient pressure on the outer nappe of the jet, P_{un} – ambient pressure on the lower nappe of the jet (under the nappe), $\Delta P = (P_{on} - P_{un})$, z – depth of flow in the receiving pool, b_r, b_ℓ, b_u, b_d – the distances of the right side, left side, upstream and downstream walls of the receiving pool to the center point of the jet where it impacts the water surface in the receiving pool, respectively as shown in Fig. 3a–c).

Note that $\Delta H = h + H_w + v_o^2 / 2g$, in which h – vertical height of the weir crest with respect to the water surface level in the receiving pool (i.e., drop height), H_w – the vertical difference between the water surface in the upstream pool and the weir crest, v_o – average flow velocity in the upstream pool. It is well known that if H_w is measured at a distance sufficiently far upstream of the weir (i.e., more than $4H_w$), $v_o^2 / 2g$ is generally negligible. Therefore one can take

$$\Delta H = h + H_w. \quad (2)$$

It should be mentioned here that the air entrainment in a jet highly depends on the shear hence velocity, between the surface of the free jet (nappe) and air. Also, the fluid and flow conditions about the bubble cloud within the receiving pool are very complex. Due to these reasons, the average flow velocity may not be that good influencing factor to be chosen the dimensional analysis. The average flow velocity may cause under estimation of the quantities. This is why, the discharge per unit water surface width ($q_j = Q/B_w$) is preferred to the average flow velocity as an influencing variable in the dimensional analysis. The dimensional analysis in Eq. (1) yields,

$$\frac{D_p}{\Delta H} = f_1 \left[\frac{F_j, \text{Re}_j, W, E_u, \frac{z}{B_w}, \frac{b_r}{B_w}}{\frac{b_\ell}{B_w}, \frac{b_d}{B_w}, \frac{b_u}{B_w}, \frac{\Delta P}{P_{on}}} \right], \quad (3)$$

in which, F_j – jet Froude number = $\left[\frac{g h^3}{2 q_j^2} \right]^{1/4}$,

Re_j – jet Reynolds number = $\frac{q_j}{\nu}$ ($\nu = \mu / \rho$),

W – jet Weber number = $\frac{\rho q_j^2}{B_w \sigma}$ and

$$E_u = \text{jet Euler number} = \frac{q_j}{B_w \sqrt{\frac{\Delta \rho}{\rho}}}.$$

Fig. 3. Receiving pool; a) Plan, b) Cross-section, c) Trapezoidal weir, Cross-section.
Obr. 3. Vývar; a) náčres; b) priečny rez, c) lichobežníkový priepad, priečny rez.

In this study, since the jet nappe is freely ventilated and open into atmosphere and no clinging of the nappe onto the weirs, $\Delta P/P_{on}$ has no physical importance and keeping in mind the scale effects, the effect of Weber and Euler number are negligible in air entrainment problems (Kells and Smith, 1991). Thus, under these reasons Eq. (1) becomes,

$$\frac{D_p}{\Delta H} = f_1 \left[F_j, Re_j, \frac{z}{B_w}, \frac{b_r}{B_w}, \frac{b_l}{B_w}, \frac{b_d}{B_w}, \frac{b_u}{B_w} \right], \quad (4)$$

$$\frac{D_p}{\Delta H} = f_1 [F_j, Re_j]. \quad (5)$$

If one conducts the similar dimensional analysis for L_b – length of the bubble zone; b_u – horizontal distance of the jet impact point at the water surface in the receiving pool and to the upstream back wall, one gets

$$\frac{L_b}{\Delta H} = f_2 \left[F_j, Re_j, \frac{z}{B_w}, \frac{b_r}{B_w}, \frac{b_l}{B_w}, \frac{b_d}{B_w}, \frac{b_u}{B_w} \right], \quad (6)$$

$$\frac{L_b}{\Delta H} = f_2 [F_j, Re_j], \quad (7)$$

$$\frac{b_u}{\Delta H} = f_3 [F_j, Re_j], \quad (8)$$

$$\frac{J_e}{\Delta H} = f_4 [F_j, Re_j]. \quad (9)$$

Note that since D_p , L_b , b_u and J_e are just before the impact of the jet at the water surface they are independent of z, b_r, b_l, b_d and b_u as shown in Eqs. (5), (7), (8) and (9).

3. Air Entrainment mechanisms-classification and break-up length of the jets

Tsang (1987) described four basic air entrainment mechanisms with reference to a free falling jet from a weir. *Tsang's* four air entrainment mechanisms are smooth, rough, oscillating, and disintegrated. For small drop heights, a water jet with a relatively smooth surface issues from the weir. The major source of air supply is visualized as a thin layer surrounding the jet and carried into the water upon impact, and therefore the air entrainment capacity is limited. The water surface in the receiving pool is relatively undisturbed. As the drop height increases, the surface of the jet becomes roughened. The air supply can be considered as coming largely from small air pockets entrapped between the jet surface roughness and the receiving water. At impact, the jet produces ripples on the pool surface. Compared with the smooth jet under similar conditions, this mechanism results in shallower bubble penetration but increased entrainment rate, because the bubbles are more densely packed in the biphasic zone. As the drop height is increased further, the jet begins to oscillate during the fall. The primary air source originates from large air pockets entrapped between the undulating jet and the pool surface. The pool surface is considerably agitated, and air may also be entrained by surface roller action and splashing. Large air pockets are transported from the surface into the water depth and broken down due to turbulence. With an even larger drop height, the jet breaks up into discrete droplets. The pool surface is intensely agitated, and air is entrained by the action of surface rollers and by the engulfing of air pockets as jet fragments hit the pool surface. The bubbles are generally only transported to relatively shallow depths. Disintegrated jets have the advantage over solid jets of greater surface area; however, air entrainment rate Q_A and bubble penetration are significantly reduced because of energy loss to the surrounding atmosphere during the fall (*Wormleaton and Tsang, 2000*).

The break-up length L_{br} at which the jet begins to disintegrate can be determined using *Horeni's* Equation for rectangular jets $L_{br} = 31.19 q_w^{0.319}$, where L_{br} is the break-up length [cm] and q_w is the specific flow over the weir in $\text{cm}^3 \text{s}^{-1} \text{cm}^{-1}$. On the other hand, the equation of the break-up length for rectangular jet, established by *Castillo* (2007) is

$$\frac{L_{br}}{B_i F_i^2} = \frac{0.85}{(1.07 T_u F_i^2)^{0.82}}, \quad (10)$$

where B_i and F_i are the jet thickness and Froude number at issuance conditions, respectively. T_u is the initial turbulence intensity when the flow passes on spillway ($0 \leq T_u \leq 3\%$).

4. Experimental equipment and procedure

Fig. 4 shows the laboratory arrangement for the current study. The experiments have been conducted in a rectangular flume with glass sides and well painted steel bed. The flume is 3.40 m in length, 0.6 m in width, and 0.6 m in depth and the bed has an average Manning's roughness coefficient $\bar{n}$ of 0.0099. A digital point gauge has been mounted on rails along the channel allowed the upstream water surface profile, then the nappe profile, and H_w have been measured. Furthermore, a digital point gauge has been mounted on the received pool to measure h . The water in the experimental channel has been re-circulated by a pump. The water jet from the test weir has plunged into a downstream water pool, whose height is adjusted using a release valve. The plan-view dimensions of the downstream water pool made of glass-wall are 1.75 m in length, 1.50 m in width, and 2 m in height.

The experiments reported here are carried out with rectangular sharp-crested weirs ($B_w = 0.10, 0.20$ and 0.30 m), trapezoidal sharp-crested weir (Cipolletti weir with $1H/4V$, $b' = 0.20$ m and, $L_w = 0.40$ m), triangular sharp-crested weirs ($\alpha = 30^\circ, 60^\circ, 90^\circ$ and 120°), triangular labyrinth weirs ($\theta = 45^\circ, 90^\circ$ and 135°) all having the same total sill length of 0.30 m, rectangular broad-crested weirs ($B_w = 0.10, 0.20$ and 0.30 m and, $L_w = 0.40$ m), trapezoidal broad-crested weir ($1H/4V$, $b' = 0.20$ and, $L_w = 0.40$ m), triangular broad-crested weirs ($\alpha = 30^\circ, 60^\circ, 90^\circ$ and 120° and, $L_w = 0.40$ m), as can be seen in Figs. 1a–c). The difference between crest and top in all weirs s is 0.25 m and crest height p is 0.35 m. Each weir configuration has been tested under flow rates Q varying from 1.0 to 5.0 L s^{-1} in 1 L s^{-1} steps. The drop height h , defined as the difference between the water levels upstream and downstream of the weir, is varied between 0.20 to 1.00 m in 0.20 m steps.

Penetration depth, D_p , of the bubbles produced by the jet, which is defined as the vertical distance from the water surface to the lower end of the submerged biphasic region in the water. Penetration

depth is measured by a movable scale from the receiving pool wall. This scale is capable of reading to the nearest 1.0 mm. All measurements are repeated three times and then averaged, especially for penetration depth measurements. In all of the experiments, the tailwater depth is selected greater than the maximum bubble penetration depth so that bubbles cannot reach the floor of the downstream water pool. The expansion measured at impact

point on the liquid pool of free overfall jet is named as jet expansion, J_e . And, it is measured by a scale which is capable of reading to the nearest 0.5 mm. The length, L_b , shown in Fig. 3a) is named as the length of the bubble zone. Also, it is measured by a scale which is capable of reading to the nearest 1.0 mm. A total of 675 tests are performed in this study.

Fig. 4. Experimental apparatus.
Obr. 4. Aparatūra.

5. Evaluation of experimental results

For different weir types, the bubble penetration depth D_p , the overfall jet expansion J_e , the length of the bubble zone L_b , and the horizontal distance to overfall jet impact point from the weir b_u are determined for a range of discharges and drop heights. The following sections present and discuss the experimental results.

To examine the effect of weir types on the overfall jet expansion, J_e values are plotted against drop height. The experimental data are presented in Fig. 5. The overfall jet expansion of the weirs is highly different from each other. The experimental data indicate that the overfall jet expansion increases with an increasing water discharge. As shown in Fig. 5, the labyrinth weirs have the highest values of overfall jet expansion among weirs tested. At labyrinth weirs, the overfall jet expansion increases as the included angle in the triangular labyrinth weirs decreases. Overfall jet expansion at the labyrinth weirs is significantly more than the trapezoidal sharp-crested weir, the rectangular sharp-

crested weirs, and circular sharp-crested weir. This is due to the fact that the labyrinth weirs have more crest length than the other weirs tested. The overfall jet expansion of the labyrinth weirs show an increasing tendency when drop height increases. But, the tendency of overfall jet expansion for the trapezoidal sharp-crested weir, the rectangular sharp-crested weirs slightly show a decrease as drop height increases, as illustrated in Fig. 5. The results indicate that the trapezoidal sharp-crested weir and the rectangular sharp-crested weirs have the lowest overfall jet expansion values. At the rectangular sharp-crested weirs, the overfall jet expansion increases with an increasing crest width B_w . It can be seen from the Fig. 5 that overfall jet expansion for the rectangular broad-crested weirs is more than the rectangular sharp-crested weirs. For triangular sharp-crested weirs, the overfall jet expansion increases as the angle of triangular weirs α decreases. Fig. 5 shows that overfall jet expansion for the triangular broad-crested weirs is significantly less than the triangular sharp-crested weirs.

Fig. 5. Variation in overfall jet expansion of different weir types with drop height for $Q = 5 \text{ L s}^{-1}$.
 Obr. 5. Variácie rozšírenia prepádového prúdu rozdielnych typov priepadov s výškou $Q = 5 \text{ L s}^{-1}$.

Fig. 6. Variation in penetration depth of different weir types with drop height for $Q = 5 \text{ L s}^{-1}$.
 Obr. 6. Variácie hĺbky prieniku vzduchových bublín rozdielnych typov priepadov s výškou $Q = 5 \text{ L s}^{-1}$.

The residence time of entrained air bubbles in a water body directly affects the oxygen mass transfer. This residence time is related to the bubble flow path and hence D_p into the receiving water. It is observed from experimental data that bubble penetration depth is closely related to the jet shape and expansion that is unique to each weir type.

The bubble penetration depth of the weirs is also highly different from each other. It would not be true to give the bubble penetration depth as a function of only drop height. Fig. 6 shows variation in D_p of nineteen different weir types with drop height while the change in discharge is constant. Experiments with all nineteen weirs indicate that the weir type and shape, the water discharge, and the drop height are important factors influencing D_p . It is found from these results that D_p tends to increase with water discharge Q . It is clear from the results in Fig. 6 that D_p generally decreases with increasing drop height. The experimental results show that D_p decreases with increasing overfall jet expansion J_e . The decrease in D_p with increasing in J_e is might be due mainly to the increased buoyant forces of the entrained bubbles as a result of the increase in the air entrainment rate, which may have caused by the increase of the jet surface roughness.

As is seen from Fig. 6, the labyrinth weirs have the lowest values of bubble penetration depth among the weirs tested. It can be seen from the experimental results that D_p values for triangular labyrinth weir with included angle of 45° are observed the lowest at all discharge, from 1 to 5 L s⁻¹. The rectangular and trapezoidal sharp-crested weirs are observed to have the highest bubble penetration depth among all weirs tested (Fig. 6). Especially, the trapezoidal sharp-crested weir has the highest bubble penetration depth, D_p , values. Moreover, at both triangular sharp-crested weirs and triangular broad-crested weirs, it is demonstrated that D_p increased as the angle of α in triangular weirs increased.

It may be concluded from Fig. 5 and Fig. 6 that D_p decreases while the overfall jet expansion increases. Thus, the scour depth in the downstream channel would decrease with increasing the overfall jet expansion because the bubble penetration depth would decrease. Moreover, overfall jet's momentum is sensitive for aeration. It can be seen from these two figures that if drop height is highly increased, then the overfall jet is disintegrated and the bubble penetration depth is much decreased. At this state, the aeration performance of the weirs would be extremely little.

Fig. 7 shows the relationship between D_p and h with *Kobus* and *Koschitzky*'s results; *Avery* and *Novak*'s results and present study data when the weir type is rectangular broad crested weir. Good agreement between the measured values and the values computed from the *Kobus* and *Koschitzky*'s, and *Avery* and *Novak*'s equations are obtained.

Scimemi (1937) expressed the shape of the nappe in co-ordinates x and y , measured from an origin at the highest point, for a unit value of H as $y = Kx^n H^{1-n}$, where $K = 0.5$ and $n = 1.85$. These values are taken as $K = 0.47$ and $n = 1.80$ for Creager profile. Thus, the lower jet trajectory can define with $y = 0.50 x^{1.85} H^{-0.85}$ and $y = 0.47 x^{1.80} H^{-0.80}$ (*Scimemi*, 1930; *Scimemi*, 1937; *Creager* et al., 1945). Where x, y coordinates and H is equal to $H_w + v_o^2/2g$. Nowadays most crests have an ogee shape, (e.g. *Scimemi* profile, *Creager* profile).

Fig. 7. Comparison of the D_p values of rectangular broad crested weir with the equations of *Kobus* and *Koschitzky* (1991), and *Avery* and *Novak* (1978).

Obr. 7. Porovnanie hodnôt D_p pravouhlého priepadu so širokou hranou s výsledkami podľa rovníc *Kobusa* a *Koschitzkého* (1991), a *Avery*, *Novaka* (1978).

Fig. 8 shows the jet trajectories for different weir types tested. Equations for the jet trajectory of weirs other than rectangular sharp-crested weir do not exist in literature as far as I know. But, free jet trajectory equations for an open channel, a flip bucket, an orifice through a dam and an overtopping dam crest exist in literature (*Davis* et al., 1999; *Dey*, 2002; *Wahl* et al., 2008). Therefore, present study data are compared with *Scimemi* and *Creager* profiles (Fig. 9). Good agreement between the present study b_u values and the values computed from the *Scimemi* and *Creager* equations is obtained.

The nature of the nappe affects significantly the aeration mechanism due to bubble behavior. There-

fore, the jet break-up was considered in this experimental study. The break-up length L_{br} was calculated using Horeni's and Castillo's equations and, the L_{br} values for the experimental data were obtained between 120 cm to 250 cm. Castillo stated that the Horeni's formula gives correct values for flows smaller than $q_w = 0.25 \text{ m}^2 \text{ s}^{-1}$. The jet break-up was not observed in the study because maximum drop height was taken as 100 cm. This situation was confirmed with Horeni's and Castillo's equations.

Empirical correlations predicting the overfall jet expansion J_e , the bubble penetration depth D_p ,

Fig. 8. Jet trajectories for the different weir types.
Obr. 8. Trajektórie prúdu rozdielnych typov priepadov.

Table 1. Values of constants (D_p) in Eq. (10).

Ta b u ľ k a 1. Hodnoty konštant (D_p) v rov. (10).

Weir Type	A	B	a_1	a_2	a_3	ε [%]
Sharp-crested triangular ¹⁾	$0.421^{\cos \alpha}$	2.353	0.009	0.032	-48.10	5.2
Sharp-crested rectangular ²⁾	4.369	68.90	-0.264	-0.583	1.523	6.5
Sharp-crested trapezoidal ³⁾	7.706	15.805	-0.163	-0.392	1.629	4.6
Broad-crested triangular ⁴⁾	$F_j^{-0.612} 0.706^{\cos \alpha}$	0.72	0.100	0.108	-5.046	6.7
Broad-crested rectangular ⁵⁾	$\ln(F_j^{1.977})$	14.649	-0.140	-0.311	5.933	5.5
Broad-crested trapezoidal ⁶⁾	0.859	$\exp(7.151)$	-0.406	-1.000	1.871	5.5
Triangular labyrinth ⁷⁾	$0.724^{\cos \theta}$	0.945	0.064	0.241	-5.717	8.1

¹⁾ostrohranný trojuholníkový priepad, ²⁾ostrohranný pravouhlý priepad, ³⁾ostrohranný lichobežníkový priepad, ⁴⁾trojuholníkový priepad so širokou hranou, ⁵⁾pravouhlý priepad so širokou hranou, ⁶⁾lichobežníkový priepad so širokou hranou, ⁷⁾trojuholníkový labyrintový priepad.

the distance between the channel intake and the overfall jet impact point b_u , and the length of the bubble zone L_b are developed for the weirs. The resulting correlations are given in Eqs. (11)–(14) and the values of constants in these correlations are shown in Tabs. 1–4.

$$\frac{D_p}{\Delta H} = A \left[\ln \left(B \cdot \text{Re}_j^{a_1} \cdot F_j^{a_2} \right) \right]^{a_3}, \quad (11)$$

$$\frac{L_b}{\Delta H} = A \left[\ln \left(B \cdot \text{Re}_j^{a_1} \cdot F_j^{a_2} \right) \right]^{a_3}, \quad (12)$$

$$\frac{J_e}{\Delta H} = A \left[\ln \left(B \cdot \text{Re}_j^{a_1} \cdot F_j^{a_2} \right) \right]^{a_3}, \quad (13)$$

$$\frac{b_u}{\Delta H} = A \left[\ln \left(B \cdot \text{Re}_j^{a_1} \cdot F_j^{a_2} \right) \right]^{a_3}, \quad (14)$$

where J_e – the overfall jet expansion [m], D_p – the penetration depth [m], b_u – the distance between the channel intake and the overfall jet impact point [m], L_b – the length of the bubble zone [m], H – the drop height [m], Q – the water discharge [$\text{m}^3 \text{ s}^{-1}$], B_w – the crest width in rectangular sharp and broad-crested weirs [m], α – the angle in triangular sharp and broad-crested weirs [°], θ – the included angle in triangular labyrinth weirs [°].

T a b l e 2. Values of constants (L_b) in Eq. (11).
T a b u l k a 2. Hodnoty konštant (L_b) v rov. (11).

Weir Type	A	B	a_1	a_2	a_3	ε [%]
Sharp-crested triangular ¹⁾	$Re_j^{0.394} 0.414^{\cos\alpha}$	1.979	-0.035	-0.097	2.233	4.7
Sharp-crested rectangular ²⁾	$Re_j^{0.304}$	1.580	-0.030	-0.061	1.000	9.6
Sharp-crested trapezoidal ³⁾	$\ln(Re_j^{0.165})$	1.084	0.059	0.243	-6.248	12.3
Broad-crested triangular ⁴⁾	$0.449^{\cos\alpha}$	4.201	-0.001	-0.258	5.719	4.8
Broad-crested rectangular ⁵⁾	3.215	7.02	-0.075	-0.216	5.866	6.9
Broad-crested trapezoidal ⁶⁾	$\ln(Re_j^{0.028})$	1.001	0.046	0.193	-6.067	8.1
Triangular labyrinth ⁷⁾	$1.079^{\cos\alpha}$	1.809	-0.033	0.466	-2.580	5.8

¹⁾ostrohranný trojuholníkový priepad, ²⁾ostrohranný pravouhlý priepad, ³⁾ostrohranný lichobežníkový priepad, ⁴⁾trojuholníkový priepad so širokou hranou, ⁵⁾pravouhlý priepad so širokou hranou, ⁶⁾lichobežníkový priepad so širokou hranou, ⁷⁾trojuholníkový labyrintový priepad.

T a b l e 3. Values of constants (J_e) in Eq. (12).
T a b u l k a 3. Hodnoty konštant (J_e) v rov. (12).

Weir Type	A	B	a_1	a_2	a_3	ε [%]
Sharp-crested triangular ¹⁾	$0.848^{\cos\alpha}$	2.223	0.039	-0.418	1.000	5.5
Sharp-crested rectangular ²⁾	16.608	2.101	0.028	0.086	-28.998	10.8
Sharp-crested trapezoidal ³⁾	$\ln(Re_j^{0.115})$	1.596	0.043	0.107	-19.246	4.3
Broad-crested triangular ⁴⁾	$0.394^{\cos\alpha}$	3.383	-0.004	-0.307	3.049	6.7
Broad-crested rectangular ⁵⁾	$(\ln(Re_j / F_j))^{-22.097}$	18.427	0.585	-1.148	23.904	11.6
Broad-crested trapezoidal ⁶⁾	0.719	16.94	-0.118	-0.480	2.486	2.2
Triangular labyrinth ⁷⁾	$1.445^{\cos\alpha}$	1.789	0.018	0.133	-8.208	2.3

¹⁾ostrohranný trojuholníkový priepad, ²⁾ostrohranný pravouhlý priepad, ³⁾ostrohranný lichobežníkový priepad, ⁴⁾trojuholníkový priepad so širokou hranou, ⁵⁾pravouhlý priepad so širokou hranou, ⁶⁾lichobežníkový priepad so širokou hranou, ⁷⁾trojuholníkový labyrintový priepad.

The calculated penetration depth D_{po} is compared with the observed penetration depth D_{pc} to yield the average percent error ε as

$$\varepsilon = \frac{100}{N} \sum_{i=1}^N \left| \frac{D_{po} - D_{pc}}{D_{po}} \right|, \quad (15)$$

in which N is the number of data points. The average percent error ε is a function of the constant in the penetration depth equation. Eq. (15) can be also written for L_b , J_e and b_u . The average percent error values belong to Eqs. (11)–(14) are given in Tabs. (1–4). Moreover, the measured bubble penetration depths are compared with those predicted with Eq.

(11). Good agreement between the measured bubble penetration depth and the values computed from the predictive equations is obtained. The values of L_b measured are also compared with those predicted with Eq. (12). Good agreement between the measured L_b and the values computed from the predictive equations is obtained. The measured J_e are compared with those predicted with Eq. (13). Good agreement between the measured J_e and the values computed from the predictive equations is obtained. The values of b_u measured are compared with those predicted with Eq. (14). Excellent agreement between the measured b_u and the values computed from the predictive equations is obtained. Further

Table 4. Values of constants (b_u) in Eq. (13).Tabuľka 4. Hodnoty konštant (J_e) v rov. (13).

Weir Type	A	B	a_1	a_2	a_3	ε [%]
Sharp-crested triangular ¹⁾	$0.995^{\cos \alpha}$	5.371	-0.044	0.164	-2.177	1.4
Sharp-crested rectangular ²⁾	1.42	3.961	-0.03	0.12	-4.600	1.3
Sharp-crested trapezoidal ³⁾	1.924	3.642	-0.018	0.093	-5.787	1.3
Broad-crested triangular ⁴⁾	$1.054^{\cos \alpha}$	2.288	-0.006	-0.190	0.867	2.1
Broad-crested rectangular ⁵⁾	0.667	4.004	-0.051	0.146	-2.652	1.2
Broad-crested trapezoidal ⁶⁾	0.125	5.781	0.082	-0.343	2.112	1.4
Triangular labyrinth ⁷⁾	$1.007^{\cos \alpha}$	0.930	-0.042	1.388	-1.035	2.3

¹⁾ostrohranný trojuholníkový priepad, ²⁾ostrohranný pravouhlý priepad, ³⁾ostrohranný lichobežníkový priepad, ⁴⁾trojuholníkový priepad so širokou hranou, ⁵⁾pravouhlý priepad so širokou hranou, ⁶⁾lichobežníkový priepad so širokou hranou, ⁷⁾trojuholníkový labyrintový priepad.

confidence in the correlations is seen in Figs. 10, 11, 12, and 13.

The following equations can be written for the determination of the minimum width, length and depth of the downstream receiving pool.

$$W_d \geq J_e, \quad (16)$$

$$L_d \geq b_u + L_b, \quad (17)$$

$$z_d \geq D_p, \quad (18)$$

Fig. 9. Comparison of the b_u values of rectangular broad crested weir with the equations of Scimemi and Creager.

Obr. 9. Porovnanie hodnôt b_u pravouhlého priepadu so širokou korunou a výsledkami rovníc Scimemi a Creagera.

where W_d – minimum width of downstream receiving pool [m], L_d – minimum length of downstream receiving pool [m], and z_d – minimum tailwater depth [m].

In this study, tested discharges are smaller than that of some prototype applications. But, tested drop heights are similar to a lot of prototype scale. Scaling of bubble penetration depth data to prototype size is virtually impossible, largely due to the relative invariance of bubble size. Clearly, tests at higher discharges should be carried out to see if this trend extrapolates.

Fig. 10. Comparison of observed D_p values with those calculated from Eq. (11).

Obr. 10. Porovnanie pozorovaných hodnôt D_p s vypočítanými podľa rov. (11).

Fig. 11. Comparison of observed L_b values with those calculated from Eq. (12).
Obr. 11. Porovnanie pozorovaných hodnôt L_b s vypočítanými podľa rov. (12).

Fig. 12. Comparison of observed J_e values with those calculated from Eq. (13).
Obr. 12. Porovnanie pozorovaných hodnôt J_e s vypočítanými podľa rov. (13).

6. Conclusions

The present experimental investigation estimates the free jet trajectory, the overfall jet expansion, and bubble penetration depth for different weir types and so dimensions of receiving pool. The correlations are developed for predicting the jet

trajectory, overfall jet expansion, the maximum penetration depth, and the length of the bubble zone for different weir types. Downstream receiving pool using equations presented in this study have properly been designed as the optimum case. The concluding remarks obtained are as follows.

Fig. 13. Comparison of observed b_u values with those calculated from Eq. (14).
Obr. 13. Porovnanie pozorovaných hodnôt b_u s vypočítanými podľa rov. (14).

1. The values of overfall jet expansion J_e , the bubble penetration depth D_p , jet trajectory, and the length of the bubble zone L_b are different for each weir type and shape.
2. The overfall jet expansion J_e increases with an increasing water discharge. The labyrinth weirs have the highest values of J_e . At the labyrinth weirs, J_e increases as the included angle in the triangular labyrinth weirs decreases. The overfall jet expansion J_e at the labyrinth weirs is significantly wider than the trapezoidal sharp-crested weir and the rectangular sharp-crested weirs. J_e of the labyrinth weirs has shown an increasing tendency when drop height increased. The results indicate that the trapezoidal sharp-crested weir and the rectangular sharp-crested weirs have the lowest J_e . At the rectangular sharp-crested weirs, J_e increases with an increasing crest width B_w . Furthermore, J_e for the rectangular broad-crested weirs is wider than the rectangular sharp-crested weirs. For triangular sharp-crested weirs, J_e increases as the angle of triangular weirs α decreases. The overfall jet expansion J_e for the tri-

angular broad-crested weirs is significantly smaller than the triangular sharp-crested weirs.

3. D_p is closely related to the jet shape and expansion that is different for each weir type. The bubble penetration depth D_p increases with increasing the water discharge. At the same time, the bubble penetration depth D_p decreases with increasing drop height. The bubble penetration depth D_p decreased with increasing J_e . The labyrinth weirs have the lowest values of D_p among the weirs tested. The bubble penetration depth D_p values for triangular labyrinth weir with included angle of 45° are observed as the lowest one. The rectangular and trapezoidal sharp-crested weirs are observed to have the highest D_p among all weirs tested. Especially, the trapezoidal sharp-crested weir has the highest D_p values. At both triangular sharp-crested weirs and triangular broad-crested weirs, D_p increases as the angle of α in triangular weirs increases.
4. The labyrinth weirs and triangular sharp-crested weirs with small angle are not appropriate due to enormous overfall jet expansion, if the downstream receiving pool width is very small.

List of symbols

B_w – water surface width over the crest of the weir [m],
 b_d – distance of the downstream wall of the receiving pool to the center point of the jet where it impacts the water surface in the receiving pool [m],
 b_ℓ – distance of the left side wall of the receiving pool to the center point of the jet where it impacts the water surface in the receiving pool [m],
 b_L – half-crest length in triangular labyrinth weir [m],
 b_r – distance of the right side wall of the receiving pool to the center point of the jet where it impacts the water surface in the receiving pool [m],
 b_u – distance of the upstream wall of the receiving pool to the center point of the jet where it impacts the water surface in the receiving pool [m],
 d – breadth in triangular labyrinth weir [m],
 D_p – bubble penetration depth [m],
 D_{pc} – calculated penetration depth [m],
 D_{po} – observed penetration depth [m],
 E_u – jet Euler number,
 F_i – Froude number at the issuance,
 F_j – jet Froude number,
 g – acceleration due to gravity [m s^{-2}],
 h – drop height [m],
 H_w – the vertical difference between the water surface in the upstream pool and the weir crest [m],
 J_e – overfall jet expansion [m],
 L – the experimental channel width [m],
 L_b – length of the bubble zone [m],
 L_d – minimum length of the downstream pool [m],
 L_{br} – jet break-up length [m],
 L_W – length in triangular broad-crested weir [m],
 P_{on} – ambient pressure on the outer nappe of the jet [N m^{-2}],

P_{un} – ambient pressure on the lower nappe of the jet [N m^{-2}],
 Re_j – jet Reynolds number,
 s – difference between crest and top in all weirs [m],
 T_u – initial turbulence intensity,
 Q – water discharge [$\text{m}^3 \text{s}^{-3}$],
 Q_A – air entrainment rate [$\text{m}^3 \text{s}^{-3}$],
 q_j – discharge through the per unit water surface width at the crest of the weir [$\text{m}^2 \text{s}^{-1}$],
 v_o – average flow velocity in the upstream pool [m s^{-1}],
 W – jet Weber number,
 w – width in triangular labyrinth weir [m],
 W_d – minimum width of the downstream receiving pool [m],
 z – tailwater depth [m],
 z_d – minimum depth of the downstream pool [m],
 α – angle in triangular sharp and broad-crested weir [$^\circ$],
 ε – average percent error,
 θ – included angle in triangular labyrinth weir [$^\circ$],
 μ – viscosity of the fluid (water) [$\text{kg s}^{-1} \text{m}^{-1}$],
 ρ – density of fluid (water) [kg m^{-3}],
 ρ_a – density of gas (air) [kg m^{-3}],
 σ – surface tension of the fluid (water) [N m^{-1}].

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URČOVANIE PRÚDOVÝCH CHARAKTERISTÍK ROZDIELNYCH TYPOV PRIEPADOV A OPTIMÁLNY NÁVRH VÝVARU

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Práca obsahuje výsledky experimentálneho výskumu určovania voľnej dráhy (trajektórie) prúdu, expanzie prúdu pri prepade a hĺbku prieniku vzduchových bublín pre rozdielne typy priepadov. Boli navrhnuté korelačné závislosti pre výpočet trajektórie prúdu, jeho expanzie počas prepadu, maximálna hĺbka prieniku a dĺžka oblasti obsahujúcej vzduchové bubliny pre rozdielne typy priepadov. Rovnice, uvedené v tejto práci umožňujú optimálny návrh vývaru. Získané poznatky môžu byť sformulované takto:

1. Hodnoty rozšírenia prúdu nad priepadom J_e , hĺbka prieniku bublín D_p , trajektória prúdu a dĺžka oblasti obsahujúcej bubliny L_b sú pre rôzne typy priepadov rozdielne.
2. Rozšírenie prúdu nad priepadom J_e sa zväčšuje so zvyšujúcim sa prietokom vody. Labyrintové priepady majú navyššiu hodnotu J_e . Pre labyrintové priepady J_e

sa zvyšuje so znižovaním sa uhla trojuholníkového labyrintového priepadu. Expanzia prúdu vody nad priepadom J_e v labyrintových priepadoch je širšia ako v lichobežníkových ostrohranných priepadoch a v pravouhlých ostrohranných priepadoch. J_e labyrintových priepadov sa zvyšuje so zvyšujúcou sa výškou priepadu. Výsledky naznačujú, že lichobežníkový ostrohranný priepad a pravouhlý ostrohranný priepad majú najnižšie J_e . Pri pravouhlom ostrohrannom pravouhlom priepade J_e sa zvyšuje so zvyšujúcou sa šírkou hrany B_w . Navyše, J_e pre pravouhlý priepad so širokou priepadovou hranou je širšia, ako pre pravouhlý, ostrohranný priepad. Pre trojuholníkový, ostrohranný priepad J_e sa zvyšuje so znižujúcim sa uhlom priepadu α . Celkové rozšírenie prúdu J_e pre trojuholníkový priepad so širokou hranou je výrazne menšie, ako pre trojuholníkový ostrohranný priepad.

3. D_p je funkciou tvaru prúdu a jeho expanzie a je rozdielny pre rôzne typy priepadu. Hĺbka prieniku vzduchových bublín D_p sa zvyšuje so zvyšujúcim sa prietokom. Súčasne sa hĺbka prieniku bublín vzduchu D_p znižuje so zvyšujúcou sa výškou priepadu. Hĺbka prieniku bublín vzduchu D_p sa znižuje so zvyšujúcim sa J_e . Najnižšiu hodnotu D_p medzi testovanými priepadmi majú priepady labyrintové. Hĺbka prieniku bublín vzduchu D_p pre trojuholníkový priepad s uhlom 45° bola najnižšia. Hĺbka prieniku bublín vzduchu D_p medzi testovanými priepadmi bola najvyššia pre pravouhlé a trojuholníkové ostrohranné priepady. Lichobežníkové ostrohranné priepady mali najvyššie hodnoty D_p . Pre oba trojuholníkové ostrohranné priepady a pre trojuholníkový priepad so širokou hranou sa D_p zvyšovalo so zvyšujúcim sa uhlom α trojuholníkového priepadu.
4. Labyrintové a trojuholníkové ostrohranné priepady s malým uhlom nie je vhodné používať tam, kde je malý vývar, pretože majú veľkú expanziu prúdu.

Zoznam symbolov

- B_w – šírka hladiny vody nad hranou priepadu [m],
 b_d – vzdialenosť medzi dolnou stenou vývaru k bodu v strede prúdu dopadajúceho na hladinu vývaru [m],
 b_e – vzdialenosť ľavej bočnej steny vývaru k bodu v strede prúdu dopadajúceho na hladinu vývaru [m],
 b_L – polovica dĺžky priepadovej hrany trojuholníkového labyrintového priepadu [m],
 b_r – vzdialenosť pravostrannej steny vývaru od bodu v strede prúdu dopadajúceho na hladinu vývaru [m],
 b_u – vzdialenosť protiprúdnej steny vývaru od bodu v strede prúdu dopadajúceho na hladinu vývaru [m],
 d – šírka trojuholníkového labyrintového priepadu [m],
 D_p – hĺbka prieniku bublín [m],
 D_{pc} – vypočítaná hĺbka prieniku [m],
 D_{po} – pozorovaná hĺbka prieniku [m],
 E_u – Eulerovo číslo prúdu,
 F_i – Froudovo číslo vo výtoky,
 F_j – Froudovo číslo prúdu,
 g – gravitačné zrýchlenie [m s^{-2}],
 h – výška priepadu [m],

H_w – vertikálny rozdiel medzi hladinou vody v hornej a dolnej zdrži priepadu [m],	q_j – prietok na jednotkovú dĺžku povrchu vody v korune priepadu [$\text{m}^2 \text{s}^{-1}$],
J_e – rozšírenie prepadového prúdu [m],	v_o – priemerná rýchlosť prúdenia v hornej zdrži [m s^{-1}],
L – šírka experimentálneho kanála [m],	W – Weberovo číslo prúdu,
L_b – dĺžka prevzdušnenej oblasti (s bublinkami) [m],	w – šírka trojuholníkového labyrintového priepadu [m],
L_d – minimálna dĺžka vývaru [m],	W_d – minimálna šírka vývaru [m],
L_{br} – dĺžka prepadového prúdu [m],	z – hĺbka vody v odtokovom kanáli [m],
L_W – dĺžka trojuholníkového priepadu so širokou hranou [m],	z_d – minimálna hĺbka vývaru [m],
P_{on} – tlak vzduchu na vonkajšej strane prepadového prúdu [N m^{-2}],	α – uhol trojuholníkového ostrohranného priepadu a priepadu so širokou priepadovou hranou [$^\circ$],
P_{un} – tlak vzduchu na vnútornej (dolnej) strane prepadového prúdu [N m^{-2}],	ε – priemerná chyba v percentách,
Re_j – Reynoldsovo číslo prúdu,	θ – uhol v trojuholníkovom labyrintovom priepade [$^\circ$],
s – rozdiel medzi priepadovou hranou a vrcholom všetkých priepadov [m],	μ – viskozita tekutiny (vody) [$\text{kg s}^{-1} \text{m}^{-1}$],
T_u – počiatočná intenzita turbulencie,	ρ – hustota tekutiny (vody) [kg m^{-3}],
Q – prietok vody [$\text{m}^3 \text{s}^{-3}$],	ρ_a – hustota plynu (vzduchu) [kg m^{-3}],
Q_d – rýchlosť unášania vzduchu [$\text{m}^3 \text{s}^{-3}$],	σ – povrchové napätie tekutiny (vody) [N m^{-1}].