DESIGN OF ROBUST GUARANTEED COST PID CONTROLLER FOR NETWORKED CONTROL SYSTEMS

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The paper addresses the problem of an output feedback guaranteed cost controller design for Networked Control Systems (NCSs) with time-delay and polytopic uncertainties. By constructing a new parameter-dependent Lyapunov functional and applying the free-weighting matrices technique, the parameter-dependent, delay-dependent design method will be obtained to synthesize PID controllers achieving a guaranteed cost such that the NCSs can be stabilized for all admissible uncertainties and time-delays. Finally, numerical examples are given to illustrate the effectiveness of the proposed method.

Key words: PID controller, output feedback, Networked Control Systems (NCSs), polytopic system, parameter-dependent quadratic stability, time-delay system

1 INTRODUCTION

Feedback control systems, wherein the loops are closed through real-time networks, are called Networked Control Systems (NCSs) [11, 12, 14, 16]. Advantages of using NCSs in the control area include simplicity, cost-effectiveness, ease of system diagnosis and maintenance, increased system agility and testability. However, integration of communication real-time networks into feedback control loops inevitably leads to some problems. As a result, it leads to a network-induced delay in the networked control closed-loop system. The existence of such a kind of delay in a network-based control loop can induce instability or poor performance of control systems [8].

In the recent years, the stability analysis and controller synthesis for systems with time-delay are important in theory and practice [13]. In the time domain, there are two approaches to controller design and studying of stability of closed-loop systems: the Razumikhin theorem and the Lyapunov-Krasovskii functional (LKF) approach. It is well known that the LKF approach can provide less conservative results than the Razumikhin theorem [4, 6, 13] and references therein. The existing criteria for asymptotic stability of the time-delay system can be classified into categories: delay-independent criteria and delay-dependent ones. It is also known that the delay-dependent criteria make use of information on the length of delays, they are less conservative than the delay-independent ones, even if the time delays are very small. On the other hand, a wide class of uncertainty types studied in the system and control literature fall into the polytopic perturbations. For the time-delay system with polytopic-type uncertainties, the parameter-dependent stability condition is of lower conservativeness than the quadratic stability condition. Recently, free-weighting matrices method or slack-variable method and cross term bounding method were developed to obtain a less conservative condition [7, 10] and reference therein.

The guaranteed cost control approach has been extended to the uncertain time-delay systems, for the state feedback case, see [9, 15, 17] and for output feedback [5]. In the paper the authors consider the full order strictly proper dynamic output feedback controller. However, it seems that there is no previous result on the delay-dependent guaranteed cost control via PID output feedback.

Motivated by the above observation, in this article the parameter-dependent, delay-dependent design method will be studied to design a robust output feedback PID controller achieving a guaranteed cost such that the NCSs can be stabilized for all admissible polytopic-type uncertainties and time-delays. The sufficient condition for the existence of a guaranteed cost output feedback controller is established in terms of matrix inequalities.

This paper is organized as follows. Section 2 gives the problem formulation. Section 3 explains the main results of the paper. In section 4, numerical examples are presented to show the effectiveness of the proposed method. Notation: Throughout this paper, for real matrix M , the notation $M \geq 0$ (or $M > 0$) means that matrix M is symmetric and positive semi-definite (or positive definite); “*” denotes a block that is readily inferred by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 PRELIMINARIES AND PROBLEM FORMULATION

Consider the following linear time-delay system described

$$\begin{aligned}\dot{x}(t) &= A(\xi)x(t) + A_d(\xi)x(t - \tau) + B(\xi)u(t), \\ y(t) &= Cx(t), \\ x(t) &= \varphi(t), \quad t \in [-\tau_M, 0]\end{aligned}\tag{1}$$

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where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^l$ is the controlled output (measured output). Matrices $A(\xi), A_d(\xi), B(\xi) \in S$ belong to the convex hull, and S is a polytope with N vertices $S_1, S_2, \dots, S_N$ which can formally be defined as

$$S := \left\{ \begin{array}{l} A(\xi), A_d(\xi) \in R^{n \times n}, B(\xi) \in R^{n \times m}, \\ A(\xi) = \sum_{i=1}^N \xi_i A_i, A_d(\xi) = \sum_{i=1}^N \xi_i A_{di}, \\ B(\xi) = \sum_{i=1}^N \xi_i B_i, \sum_{i=1}^N \xi_i = 1, \xi_i \geq 0 \end{array} \right\}, \quad (2)$$

where A_i, A_{di}, B_i are constant matrices with appropriate dimensions and ξ_i is time-invariant uncertainty, τ_M is the upper bound of time delay and $\varphi(t)$ is a continuously differentiable initial function. Note that S is a convex and bounded domain. We assume that a real-time communication network is integrated into feedback control loops of system (1), and the network induced delay in NCS is given by $0 < \tau \leq \tau_M$ and $\dot{\tau} \leq \mu \leq 1$.

For system (1), we consider the following PID control algorithm

$$u(t) = K_p y(t-\tau) + K_I \int_0^t y(t-\tau) dt + K_D \frac{d}{dt} y(t-\tau). \quad (3)$$

Consider $z(t) = \int y(t-\tau) dt$, $\frac{d}{dt} y(t-\tau) = C_d \dot{x}(t-\tau)$, where C_d is an output matrix for derivative output feedback, and then by using the Newton-Leibniz formulas

$$x(t-\tau) = x(t) - \int_{t-\tau}^t \dot{x}(s) ds,$$

and

$$\dot{x}(t-\tau) = \dot{x}(t) - \int_{t-\tau}^t \ddot{x}(s) ds,$$

the PID control algorithm (3) can be written as

$$u(t) = F C_n X(t) + F_D C_D \dot{X}(t) - F_P C_P \int_{t-\tau}^t \dot{X}(s) ds - F_d C_d \int_{t-\tau}^t \ddot{X}(s) ds, \quad (4)$$

where

$$\begin{aligned} X(t) &= [x^\top(t), z^\top(t)]^\top, \\ F &= [K_P \quad K_I], F_P = [K_P \quad 0], F_D = [K_D \quad 0], \\ C_n &= \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}, C_P = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}, C_D = \begin{bmatrix} C_d & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

Consider $\dot{z}(t) = C_i x(t-\tau) = C_i x(t) - C_i \int_{t-\tau}^t \dot{x}(s) ds$, where C_i is an output matrix for integral output feedback, the system (1) can be expanded in the following form

$$\dot{X}(\xi) = A_n(\xi) X(t) + B_n(\xi) u(t) - A_{dn}(\xi) \int_{t-\tau}^t \dot{X}(s) ds, \quad (5)$$

where

$$\begin{aligned} A_n(\xi) &= \sum_{i=1}^N \xi_i A_{ni}, & A_{ni} &= \begin{bmatrix} A_i + A_{di} & 0 \\ C_i & 0 \end{bmatrix}, \\ B_n(\xi) &= \sum_{i=1}^N \xi_i B_{ni}, & B_{ni} &= \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \\ A_{dn}(\xi) &= \sum_{i=1}^N \xi_i A_{dni}, & A_{dni} &= \begin{bmatrix} A_{di} & 0 \\ C_i & 0 \end{bmatrix}. \end{aligned}$$

Applying the PID control algorithm (4) to system (5) will result in the closed-loop system

$$M_d(\xi) \dot{X}(t) + A_c(\xi) X(t) + A_{dc}(\xi) \int_{t-\tau}^t \dot{X}(s) ds + A_{dd}(\xi) \int_{t-\tau}^t \ddot{X}(s) ds = 0, \quad (6)$$

where

$$\begin{aligned} M_d(\xi) &= \sum_{i=1}^N \xi_i M_{di}, & M_{di} &= I - B_{ni} F_D C_D, \\ A_c(\xi) &= \sum_{i=1}^N \xi_i A_{ci}, & A_{ci} &= -(A_{ni} + B_{ni} F C_n), \\ A_{dc}(\xi) &= \sum_{i=1}^N \xi_i A_{dci}, & A_{dci} &= A_{dni} + B_{ni} F_P C_P, \\ A_{dd}(\xi) &= \sum_{i=1}^N \xi_i A_{ddi}, & A_{ddi} &= B_{ni} F_D C_D, \end{aligned}$$

Given positive definite symmetric matrices Q, R and S , we will consider the cost function

$$J = \int_0^\infty J(t) dt, \quad (7)$$

where

$$J(t) = X^\top(t) Q X(t) + u^\top(t) R u(t) + \dot{X}^\top(t-\tau) S \dot{X}(t-\tau).$$

Consider $\eta(t) =$

$$\left[\dot{X}^\top(t) \quad X^\top(t) \quad \int_{t-\tau}^t \dot{X}^\top(s) ds \quad \int_{t-\tau}^{t-\tau} \dot{X}^\top(s) ds \quad \int_{t-\tau}^t \ddot{X}^\top(s) ds \right]^\top$$

and by substituting $u(t)$ from (4) to $u^\top(t) R u(t)$ we obtain

$$u^\top(t) R u(t) = \eta^\top(t) K^\top R K \eta(t),$$

where $K = [F_D C_D \quad F C_n \quad -F_P C_P \quad 0 \quad -F_D C_D]$. Using $\eta(t)$, $J(t)$ can be rewritten as follows

$$J(t) = \eta^\top(t) M_Q(\xi) \eta(t), \quad (8)$$

where

$$M_Q(\xi) = \begin{bmatrix} C_D^\top F_D^\top R F_D C_D + S & C_D^\top F_D^\top R F C_n & * & * & * \\ * & C_n^\top F^\top R F C_n + Q & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ -C_D^\top F_D^\top R F_P C_P & 0 & -C_D^\top F_D^\top R F_D C_D - S \\ -C_n^\top F^\top R F_P C_P & 0 & -C^\top F^\top R F_D C_D \\ C_P^\top F_P^\top R F_P C_P & 0 & C_P^\top F_P^\top R F_D C_D \\ * & 0 & 0 \\ * & * & C_D^\top F_D^\top R F_D C_D + S \end{bmatrix}$$

Associated with the cost, the guaranteed cost controller is defined as follows:

DEFINITION 1. Consider the uncertain system (1). If there exists a controller of form (3) and a positive scalar J_0 such that for all uncertainties (2) the closed-loop system (6) is asymptotically stable and closed-loop value of the cost function (7) satisfies $J \leq J_0$ then J_0 is said to be a guaranteed cost and the controller (2) is said to be a guaranteed cost controller.

Finally we introduce the well known results from LQ theory.

LEMMA 1. Consider the continuous-time delay system (5) with control algorithm (3). The control algorithm (3) is the guaranteed cost control for system (5) if and only if there exists LKF $V(\xi, t)$ such that the following condition holds

$$\frac{d}{dt}V(\xi, t) + J(t) \leq 0. \quad (9)$$

The objective of this paper is to develop a procedure to design a robust PID controller of form (4) which ensures parameter-dependent the closed-loop system stability and guaranteed cost.

3 MAIN RESULTS

The following theorem provides robust parameter-dependent quadratic stability and robust performance results for the closed-loop system (6).

THEOREM 1. Consider the uncertain linear time-delay system (1) with network-induced delay τ satisfying $0 \leq \tau \leq \tau_M$, $\dot{\tau} \leq \mu \leq 1$ and the cost function (7). If there exists a PID controller of form (3), scalar J_0 , and matrices $P_i > 0$, $G_i > 0$, $G_{1i} > 0$, $G_{2i} > 0$, $G_{3i} > 0$ ($i = 1, \dots, N$), N_1, N_2, N_3, N_4 , and N_5 that satisfy the following matrix inequality

$$W_i = \begin{bmatrix} w_i^{11} & w_i^{12} & w_i^{13} & w_i^{14} & w_i^{15} \\ * & w_i^{22} & w_i^{23} & w_i^{24} & w_i^{15} \\ * & * & w_i^{33} & w_i^{34} & w_i^{35} \\ * & * & * & w_i^{44} & w_i^{45} \\ * & * & * & * & w_i^{55} \end{bmatrix} \leq 0, \quad (10)$$

where

$$\begin{aligned} w_i^{11} &= N_1 M_{di} + M_{di}^\top N_1^\top + \tau_M G_{1i} + \mu G_{3i} + \\ &\quad C_D^\top F_D^\top R F_D C_D + S, \\ w_i^{12} &= N_1 A_{ci} + M_{di}^\top N_2^\top + P_i + C_D^\top F_D^\top R F C_n, \\ w_i^{13} &= N_1 A_{dci} + M_{di}^\top N_3^\top - C_D^\top F_D^\top R F_P C_P, \\ w_i^{14} &= M_{di}^\top N_4^\top, \\ w_i^{15} &= N_1 A_{ddi} + M_{di}^\top N_5^\top + (1 - \mu) \mu G_{3i} \\ &\quad - C_D^\top F_D^\top R F_D C_D - S, \\ w_i^{22} &= N_2 A_{ci} + A_{ci}^\top N_2^\top + \mu G_i + C_n^\top F^\top R F + Q, \end{aligned}$$

$$\begin{aligned} w_i^{23} &= N_2 A_{dci} + A_{ci}^\top N_3^\top + (1 - \mu) G_i + G_{2i} \\ &\quad - C_n^\top F^\top R F_P C_P, \\ w_i^{24} &= A_{ci}^\top N_4^\top + G_{2i}, \\ w_i^{25} &= N_2 A_{ddi} + A_{ci}^\top N_5^\top - C_n^\top N^\top R F_D C_D, \\ w_i^{33} &= N_3 A_{dci} + A_{dci}^\top N_3^\top - (1 - \mu) G_i - \frac{1}{\tau_M} G_{1i} - G_{2i} + \\ &\quad C_P^\top F_P^\top R F_P C_P, \\ w_i^{34} &= A_{dci}^\top N_4^\top - G_{2i}, \\ w_i^{35} &= N_3 A_{ddi} + A_{dci}^\top N_5^\top + C_P^\top F_P^\top R F_D C_D, \\ w_i^{44} &= -G_{2i} - \frac{1}{\tau_M} G_{1i}, \\ w_i^{45} &= N_4 A_{ddi}, \\ w_i^{55} &= N_5 A_{ddi} + A_{ddi}^\top N_5^\top - (1 - \mu) G_{3i} + \\ &\quad C_D^\top F_D^\top R F_D C_D + S. \end{aligned}$$

then the uncertain system (1) with controller (3) is parameter-dependent quadratically stable and the cost function (7) satisfies the following bound

$$J \leq J_0 = \sqrt{\lambda_{MP}^2 + \lambda_{MG}^2 + \lambda_{MG1}^2 + \lambda_{MG2}^2 + \lambda_{MG3}^2} * J_M \quad (11)$$

where

$$\begin{aligned} \lambda_{MP} &= \max_{i=1 \dots N} (\text{MaxEigenvalue}(P_i)), \\ \lambda_{G_i} &= \max_{i=1 \dots N} (\text{MaxEigenvalue}(G_i)), \\ \lambda_{MG1} &= \max_{i=1 \dots N} (\text{MaxEigenvalue}(G_{1i})), \\ \lambda_{MG2} &= \max_{i=1 \dots N} (\text{MaxEigenvalue}(G_{2i})), \\ \lambda_{MG3} &= \max_{i=1 \dots N} (\text{MaxEigenvalue}(G_{3i})), \end{aligned}$$

$$\begin{aligned} J_M &= \left(\|x_0\|^4 + \left(\int_{-\tau}^0 \|\varphi(s)\|^2 ds \right)^2 + \left(\int_{-\tau}^0 d\theta \int_{\theta}^0 \|\dot{\varphi}(s)\|^2 ds \right)^2 \right. \\ &\quad \left. + \left(\int_{-\tau_M}^{-\tau} \|\varphi(s)\|^2 ds \right)^2 + \left(\int_{-\tau}^0 \|\dot{\varphi}(s)\|^2 ds \right)^2 \right)^{1/2}. \end{aligned}$$

Proof. Consider the Lyapunov-Krasovskii functional as follows

$$\begin{aligned} V(\xi, t) &= \sum_{i=1}^5 V_i(\xi, t), \\ V_1(\xi, t) &= X^\top(t) P(\xi) X(t), \\ V_2(\xi, t) &= \int_{t-\tau}^t X^\top(s) G(\xi) X(s) ds, \\ V_3(\xi, t) &= \int_{-\tau}^0 d\theta \int_{t+\theta}^t \dot{X}^\top(s) G_1(\xi) \dot{X}(s) ds, \\ V_4(\xi, t) &= \int_{t-\tau_M}^t X^\top(s) G_2(\xi) X(s) ds, \end{aligned} \quad (12)$$

$$V_5(\xi, t) = \int_{t-\tau}^t \dot{X}^\top(s) G_3(\xi) \dot{X}(s) ds.$$

Differentiating $V(\xi, t)$ with respect to t and using the Newton-Leibniz formula $x(t-\tau) = x(t) - \int_{t-\tau}^t \dot{x}(s) ds$ we obtain

$$\begin{aligned} \dot{V}_1(\xi, t) &= z X^\top(t) P(\xi) \dot{X}(t), \\ \dot{V}_2(\xi, t) &\leq \eta_1^\top(t) \begin{bmatrix} \mu G(\xi) & (1-\mu)G(\xi) \\ * & -(1-\mu)G(\xi) \end{bmatrix} \eta_1(t), \\ \eta_1^\top(t) &= \begin{bmatrix} X^\top(t) & \int_{t-\tau}^t \dot{X}^\top(s) ds \end{bmatrix}, \\ \dot{V}_3 &\leq \tau_M \dot{X}^\top(t) G_1(\xi) \dot{X}(t) \\ &\quad - \frac{1}{\tau_M} \int_{t-\tau}^t \dot{X}^\top(s) ds G_1(\xi) \int_{t-\tau}^t \dot{X}(s) ds, \\ \dot{V}_4(\xi, t) &\leq \eta_2^\top(t) \begin{bmatrix} 0 & G_2(\xi) & G_2(xi) \\ * & -G_2(xi) & -G_2(xi) \\ * & * & -G_2(xi) \end{bmatrix} \eta_2(t), \\ \eta_2^\top(t) &= \begin{bmatrix} X^\top(t) & \int_{t-\tau}^t \dot{X}^\top(s) ds & \int_{t-\tau_M}^{t-\tau} \dot{X}^\top(s) ds \end{bmatrix}, \\ \dot{V}_5(\xi, t) &\leq \eta_3^\top(t) \begin{bmatrix} \mu G_3(\xi) & (1-\mu)G_3(\xi) \\ * & -(1-\mu)G_3(\xi) \end{bmatrix} \eta_3(t), \\ \eta_3^\top &= \begin{bmatrix} X^\top & \int_{t-\tau} \dot{X}^\top(s) ds \end{bmatrix}. \end{aligned}$$

Applying the free-weighting matrices technique, equation (8) is represented in the following equivalent form

$$\alpha(t) = 2\eta^\top(t) [N_1^\top \quad N_2^\top \quad N_3^\top \quad N_4^\top \quad N_5^\top]^\top [M_d(\xi) \quad A_c(\xi) \quad A_{dc}(\xi) \quad 0 \quad A_{dd}(\xi)] \eta(t) = 0.$$

After manipulation of the above equation, we obtain

$$\alpha(t) = \eta^\top(t) M_\alpha(\xi) \eta(t) = 0 \quad (13)$$

where

$$M_\alpha(\xi) = \begin{bmatrix} N_1 M_d(\xi) + M_d^\top(\xi) N_1^\top & N_1 A_c(\xi) + M_d^\top(\xi) N_2^\top & N_1 A_{dc}(\xi) + M_d^\top(\xi) N_3^\top & M_d^\top(\xi) N_4^\top & N_1 A_{dd}(\xi) + M_d^\top(\xi) N_5^\top \\ * & N_2 A_c(\xi) + A_c^\top(\xi) N_2^\top & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ N_2 A_{dc}(\xi) + A_{dc}^\top(\xi) N_3^\top & A_{dc}^\top(\xi) N_4^\top & N_2 A_{dd}(\xi) + A_{dc}^\top(\xi) N_5^\top & * & * \\ N_3 A_{dc}(\xi) + A_{dc}^\top(\xi) N_3^\top & A_{dc}(\xi) N_4^\top & N_3 A_{dd}(\xi) + A_{dc}^\top(\xi) N_5^\top & * & * \\ * & 0 & N_4 A_{dd}(\xi) & * & * \\ * & * & N_5 A_{dd}(\xi) + A_{dd}^\top(\xi) N_5^\top & * & * \end{bmatrix}.$$

Because of $\alpha(t) = 0$, thus

$$\begin{aligned} \dot{V}(\xi, t) &= \sum_{i=1}^5 \dot{V}_i(\xi, t) + \alpha(t) \leq \eta^\top(t) [M_\alpha(\xi) \\ &\quad + M_V(\xi)] \eta(t), \quad (14) \end{aligned}$$

where

$$M_V(\xi) = \begin{bmatrix} \tau_M G_1(\xi) + \mu G_3(\xi) & P(\xi) & * & * & * & * \\ * & \mu G(\xi) & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ 0 & 0 & (1-\mu)G_3(\xi) & 0 & 0 & 0 \\ (1-\mu)G(\xi) & G_2(\xi) & 0 & 0 & 0 & 0 \\ -(1-\mu)G(\xi) - G_2(\xi) & -G_2(\xi) & 0 & 0 & 0 & 0 \\ -\frac{1}{\tau_M} G_1(\xi) & * & * & * & * & * \\ * & -G_2(\xi) - \frac{1}{\tau_M} G_1(\xi) & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}.$$

Due to Lemma 1, the closed-loop system (6) is robustly asymptotically stable and give an upper bound (a guaranteed cost) for the cost function (7) if

$$\dot{V}(\xi, t) + J(t) \leq \eta^\top(t) W(\xi) \eta(t) \leq 0 \iff W(\xi) \leq 0, \quad (15)$$

where $W(\xi) = \sum_{i=1}^N \xi_i W_i = M_\alpha(\xi) + M_V(\xi) + M_Q(\xi)$. If for each $W_i \leq 0$, $I = 1, \dots, N$, then $W(\xi) = \sum_{i=1}^N \xi_i W_i \leq 0$. Therefore, $\dot{V}(\xi, t) \leq -J(t) \leq 0$ ($J(t) > 0$), or $J(t) \leq -\dot{V}(\xi, t)$. By integrating $J(t) \leq -\dot{V}(\xi, t)$ we obtain

$$\begin{aligned} J &\leq - \int_0^\infty \dot{V}(\xi, t) dt = V_0 = X_0^\top P(\xi) X_0 + \\ &\quad \int_{-\tau}^0 X^\top(s) G(\xi) X(s) ds + \int_{-\tau}^0 d\theta \int_\theta^0 \dot{X}^\top(s) G_1(\xi) \dot{X}(s) ds \\ &\quad + \int_{-\tau_M}^0 X^\top G_2(\xi) X(s) ds + \int_{-\tau}^0 \dot{X}^\top(s) G_3(\xi) \dot{X}(s) ds \end{aligned}$$

Because of $X(t) = [\varphi^\top(t) \quad 0]^\top$, $\forall t \in [-\tau_M, 0]$

$$\begin{aligned} V_0 &\leq \lambda_{MP} \|x_0\|^2 + \lambda_{MG} \int_{-\tau}^0 \|\varphi(s)\|^2 ds \\ &\quad + \lambda_{MG1} \int_{-\tau}^0 d\theta \int_\theta^0 \|\dot{\varphi}(s)\|^2 ds + \lambda_{MG2} \int_{-\tau_M}^0 \|\varphi(s)\|^2 ds \\ &\quad + \lambda_{MG3} \int_{-\tau}^0 \|\dot{\varphi}(s)\|^2 ds. \end{aligned}$$

It is known that for two arbitrary vectors $\mathbf{X}, \mathbf{Y}$, the following inequality hold

$$|\mathbf{X}\mathbf{Y}| \leq \|\mathbf{X}\| \|\mathbf{Y}\|. \quad (16)$$

Consider $\vec{X} = [\lambda_{MP} \quad \lambda_{MG} \quad \lambda_{MG1} \quad \lambda_{MG2} \quad \lambda_{MG3}]^\top$,

$$\begin{aligned} \mathbf{Y} &= \left[\|x_0\|^2 \quad \int_{-\tau}^0 \|\varphi(s)\|^2 ds \quad \int_{-\tau}^0 d\theta \int_\theta^0 \|\dot{\varphi}(s)\|^2 ds \quad \int_{-\tau_M}^0 \|\varphi(s)\|^2 ds \quad \int_{-\tau}^0 \|\dot{\varphi}(s)\|^2 ds \right]^\top. \end{aligned}$$

Applying inequality (16) to the above equation the upper bound cost function (7) J_0 is obtained as (11).

The theorem 1 is proved.

4 EXAMPLES

In this section we present the results of numerical calculations of two examples to design a robust output feedback PID controller with guaranteed cost for NCSs with time-delay. The design procedure is based on BMI inequalities (10).

Example 1 has been borrowed from [2] to demonstrate the use of algorithm (10) for the problem of robustly stabilizing, with a guaranteed cost, vertical take-off and landing of a helicopter. The system is controlled through NCS with a time-varying time-delay $0 < \tau \leq \tau_M = 200$ ms, $\dot{\tau} \leq \mu \leq 0.99$. Let uncertain matrices A , B , C , A_d be defined as

$$A = \begin{bmatrix} -0.036 & 0.0270 & 0.0188 & -0.4555 \\ 0.0482 & -1.010 & 0.0024 & -4.0208 \\ 0.1002 & q_1(t) & -0.707 & q_2(t) \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 4.422 & 0.1761 \\ q_3(t) & -7.59222 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix}, C^\top = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, A_d = 0$$

with parameters bounds for all time $-0.6319 \leq q_1(t) \leq 1.3691$, $1.22 \leq q_2(t) \leq 1.420$, $2.7446 \leq q_3(t) \leq 4.3446$. The above model has been recalculated to the form (1). The respective eight vertices are calculated. Note that matrix A is unstable with $\max \text{real eigenvalue}(A) = 1.2675$. The results of calculation for the case $r = 1$, $q = 0.1$, $s = 0.001$, $r_0 = 10$ are as follows

$$F = [K_P \ K_I] = \begin{bmatrix} -0.2788 & 0.0927 \\ 0.5857 & 0.4086 \end{bmatrix}, K_D \cong 0.$$

The $\max \text{real eigenvalue}(\text{Close-loop}) = -0.072209$ and guaranteed cost $J_0 = 17.9128 J_M$, where $\lambda_{MP} = \lambda_{MG} = \lambda_{MG2} = 8.9794$, $\lambda_{MG1} = 8.8841$, $\lambda_{MG3} = 0.2319$.

EXAMPLE 2. We consider the linear model of two co-operating DC motors. The problem is to design two PI controllers for a laboratory MIMO system which guarantee robust stability with a guaranteed cost. The system is controlled through a NCS with time-varying time-delay $0 < \tau \leq \tau_M = 100$ ms, $\dot{\tau} \leq \mu = 0.2$. The system model is given with a time invariant matrix affine type uncertain structure, where

$$A_0 = \begin{bmatrix} 0 & -0.2148 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1.014 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2605 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.9107 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1639 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.8137 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2279 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -0.8251 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} .3148 & 0 \\ .0478 & 0 \\ 0 & -0.1028 \\ 0 & -0.0091 \\ -0.0841 & 0 \\ -0.0287 & 0 \\ 0 & .3676 \\ 0 & .2448 \end{bmatrix}, B_1 = \begin{bmatrix} .0625 & 0 \\ .0798 & 0 \\ 0 & -0.0462 \\ 0 & -0.0449 \\ .0016 & 0 \\ .0072 & 0 \\ 0 & .077 \\ 0 & .003 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & -.025 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.1395 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.0938 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.2911 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0188 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0208 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.1173 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & .0125 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .0594 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0116 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0308 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.0188 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.0156 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .0208 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} .0094 & 0 \\ .0151 & 0 \\ 0 & .0019 \\ 0 & -.003 \\ -.0121 & 0 \\ -.03 & 0 \\ 0 & -.064 \\ 0 & .0189 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}, A_d = 0.$$

The above model has been recalculated to the form (1). The respective four vertices are calculated. The results of calculation for the case $r = 1$, $q = 0.1$, $s = 0.001$, $r_0 = 20$ are as follows

$$F = [K_P \ K_I] = \begin{bmatrix} -1.8916 & 0.1128 & 0.2026 & -0.6964 \\ -0.4310 & -1.8916 & 0.8782 & -1.4985 \end{bmatrix}.$$

The $\max \text{real eigenvalue}(\text{Close-loop}) = -0.17726$. And guaranteed cost $J_0 = 37.7659 J_M$ where $\lambda_{MP} = 18.916$, $\lambda_{MG} = 15.0011$, $\lambda_{MG2} = 18.916$, $\lambda_{MG1} = 15.5723$, $\lambda_{MG3} = 15.5918$.

5 CONCLUSION

The guaranteed cost control problem is studied in this paper for a class of linear time-delay uncertain polytopic systems and a given quadratic cost function with three terms (QRS). On the basis of the Lyapunov-Krasovskii functional, new sufficient parameter-dependent quadratic stability conditions are given for the output feedback PID controller proposed design procedure in terms of bilinear matrix inequality.

The examples show the effectiveness of the proposed method.

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