Evaluation of Continuous Sampling Plan (CSP-5) Parameters Using GERT Technique and MATLAB

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Abstract

In this paper, we have designed the Continuous sampling plan (CSP-5) plan using GERT analysis and the software package MATLAB 2007b. The formula for finding the performance parameters such as AOQ, E(I) and the parameters i and f are found out by GERT technique and developing program for the construction of table values and plotting graph using MATLAB.

Zentralblatt MATH classification: 90B15
Keywords: GERT, W-function, Average Outgoing Quality, Process Average

1. INTRODUCTION

Acceptance sampling plans provide criteria and decision rules for determining whether to accept or reject a batch based on a sample. They are therefore widely used by manufacturers, suppliers, contractors and subcontractors, and service providers in a wide range of industries. Continuous sampling is the best plan when products are made individually in a continuous flow. It makes no sense to pick samples inside each batch. It consists of several phases:

(1) At the beginning, each piece is checked (screening).
(2) After a certain number of pieces were found satisfactory, only certain pieces are checked randomly (sampling).
(3) If a piece is defective then back to screening.

The concept of continuous sampling plan (CSP-1) was introduced by Dodge (1943) as a sampling inspection plan for a product consisting of individual units manufactured in quantity by an essentially continuous process [1]. The detailed procedure and tables for construction and selection of CSP-1 plans have been given by Stephens (1981). Ghosh (1988) and Govindaraju (1989). Whitehouse (1973; 401 - 403) and also Ohta and Kase (1984) and Chakraborty and Rathie (1989) modeled and analyzed the Dodges CSP-1 continuous sampling plan through GERT approach. Continuous sampling plans are useful when the formation of lots for sampling
inspection is impracticable and or artificial [2]. Continuous sampling plans are appropriate when there is no natural aggregation of product into lots, such as in conveyor-line production. Because most continuous-inspection procedures remove defective articles from production, nearly all continuous-inspection plans are of the AOQL type.

Dodge (1943) introduced the concept of continuous sampling and provided the mathematical rationale and rules of operation for his first continuous sampling plan designated as CSP-1. This CSP-1 plan is the most commonly used single level continuous sampling plan because of its simplicity. The other continuous sampling plans namely, CSP-2 and CSP-3 have been developed by Dodge and Torrey as the generalizations of Dodge's basic CSP-1, CSP-2, CSP-3, a multiple level CSP-F, CSP-T and CSP-5 contain some differences due to the implementation and the theoretic foundation between them [1]. Kandasamy (1993) proposed tightened forms of CSP-2 and CSP-3 and derived performances measures by following Stephens approach and also constructed tables for the selection of plans. Derman Johns and Liberman (1959) defined two modifications for CSP-1 plan termed as CSP-4 and CSP-5.

A generalized CSP-V plan is described and is referred to here as general CSP-5 plan. Its Markov-Chain formulation performance measures and procedure for the construction. A special case of the general CSP-5 plan introduced for situation where there is no advantage in reducing the sampling frequency upon demonstration of good product quality, reduced inspection can be achieved by using a smaller clearance interval. This is the main features of CSP-5 plan proposed by Asheim (1972). It is a single level continuous sampling procedure with parameters i, f and x the reduced clearance number [1]. Asheim plan required during sampling inspection i consecutive units to be conforming in order that the reduced clearance number. Gowri Shankar and Mohapatra (1994) presented Graphical Evaluation and Review technique (GERT) originally introduced for systems design and analysis to model and analyze the dynamics of the Dodge CSP-1 plan. They also constructed tables for the selection of plans based on AOQL and optimum average amount of inspection [4]. Gauri Shankar, A.K. Sahu and R. K. Srivastava (2001) enlarge the concept of conditional repetitive group sampling plan through GERT approach. The operating characteristic and average sample number functions of the generalized plan have been GERT. The plan that provides a quick discrimination of good and bad quality lots, could be used for both the process control and goods acceptance or rejection in
various stages of production in defense organizations [4].

1.1. Graphical Evaluation & Review Technique (GERB)

Graphical Evaluation and Review Technique commonly known as GERT, is a network analysis technique used in project management that allows probabilistic treatment of both network logic and estimation of activity duration. The technique was first discovered in 1966 by Dr. Alan B. Pritsker of Purdue University. Compared to other techniques, GERT is only rarely used in complex systems. The GERT approach addresses the majority of the limitations associated with PERT/CPM technique. GERT allows loops between tasks. The fundamental drawback associated with the GERT technique is the complex programme required to model the GERT system. Development in GERT includes Q-GERTS allowing the user to consider queuing within the system. This has been applied to modeling of sampling plans and promises to be value in encouraging statistical quality control [6]. In this paper we analyze the dynamics of the Dodge-Romig sampling inspection plan by GERT and propose a new sampling plan which secures the advantages of both consumer and producer. It is a technique for the analysis of a class of networks, which have the following characteristics: (1) a probability that a branch of the network is indeed part of a realization of the network; and (2) an elapsed time or time interval associated with the branch if the branch is part of the realization of the network. Such networks will be referred to as stochastic networks and consist of a set of branches and nodes. A realization of a network is a particular set of branches and nodes, which describe the network for one experiment. If the time associated with a branch is a random variable, then a realization also implies that a fixed time has been selected for each branch. GERT will derive both probability that a node is realized and the conditional moment generating function (M.G.F) of the elapsed time required to traverse between any two nodes [4,6].

1.2. Steps in Applying GERT

The foregoing material described the qualitative aspects of GERT. Basically, the steps employed in applying GERT are:

(1) Convert a qualitative description of a system or problem to a model in network form;
(2) Collect the necessary data to describe the branches of the network;
(3) Obtain an equivalent one-branch function between two nodes of the network;
(4) Convert the equivalent function into the following two performance measures of
the network;
   (a) The probability that a specific node is realized and (b) The M.G.F of the time
   associated with an equivalent network;
(5) Make inferences concerning the system under study from the information obtained
in 4 above.

1.3. Brief Review of GERT

GERT was initiated by Pritsker and Happ (1966), Pritsker and Whitehouse
(1966) and Whitehouse and Pritsker (1969) as a procedure for the analysis of
stochastic networks having the following features: [6]
(1) Each network consists of logical nodes (or events) and directed branches (or
activities).
(2) A branch has a probability that the activity associated with it will be performed.
(3) Other parameters describe the activities represented by the branches.
Gowri Shankar and Mohapatra (1994) presented Graphical Evaluation and Review
technique (GERT) originally introduced for systems design and analysis to model
and analyze the dynamics of the Dodge CSP-1 plan. They also constructed tables for
the selection of plans based on AOQL and optimum average amount of inspection.
Subrata Chakraborthy (1989) presented a paper for the analysis of inspection error
concept of conditional repetitive group sampling plan through GERT approach. The
operating characteristic and average sample number functions of the generalized plan
have been GERT. The plan that provides a quick discrimination of good and bad
quality lots, could be used for both the process control and goods acceptance or
rejection in various stages of production in defense organizations.

The basic setback pertaining to the GERT technique is the complex program
needed to model the GERT system. A.A.B. Pritsker (1966) presented a Memorandum,
one of series done by the RAND Corporation on the Apollo Checkout Study for Head
quarters, National Aeronautics and Space Administration under Contract Nasr-21(08),
evolved from a study of the terminal countdown of an Apollo space system. The
result of the research on this problem GERT, a procedure for the analysis of stochastic
networks. This procedure makes it possible to analyze complex systems and problems
in a less inductive manner and hence should stimulate efforts in the network analysis
1.4. W-Function of the Plan

The above states enable us to construct GERT network representation of the inspection system as shown in Fig. 1. The network diagram consists of logical nodes and directed branches. The nodes are each state of the plan and a branch has probability that the activity associated with it will be performed. The sample size $n$ associated with a branch is characterized by the moment generating function (mgf) of the form $M_n(\theta) = \sum_n \exp(n\theta)f(n)$, where $f(n)$ denotes the density function of $n$ and $\theta$ is any real variable. The probability $\phi$ that the branch is realised is multiplied by the mgf to yield the W-function such that $W(\theta) = \phi M_n(\theta)$. The W-function is used to obtain the information on the relationship which exists between the nodes [4].

1.5. MATLAB R2007b

Cleve Moler, the chairman of the computer science department at the University of New Mexico, started developing MATLAB in the late 1970s. He designed it to give his students access to LINPACK and EISPACK without them having to learn Fortran. It soon spread to other universities and found a strong audience within the applied mathematics community. Jack Little, an engineer, was exposed to it during a visit Moler made to Stanford University in 1983. Recognizing its commercial potential, he joined with Moler and Steve Bangert. They rewrote MATLAB in C and founded Math Works in 1984 to continue its development. These rewritten libraries were known as JACKPAC. In 2000, MATLAB was rewritten to use a newer set of libraries for matrix manipulation, LAPACK. MATLAB was first adopted by researchers and practitioners in control engineering, Little’s specialty, but quickly spread to many other domains. It is now also used in education, in particular the teaching of linear algebra, numerical analysis, and is popular amongst scientists involved in image processing [9].

MATLAB (matrix laboratory) is a multi-paradigm numerical computing environment and fourth-generation programming language. A proprietary programming language developed by Math Works, MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages, including C, C++, Java, Fortran and Python. Although MATLAB is intended primarily for numerical computing, an optional toolbox uses the MuPAD symbolic engine, allowing access to symbolic computing abilities [10].

An additional package,
Simulink, adds graphical multi-domain simulation and model-based design for dynamic and embedded systems [10].

1.6. GERT analysis of the CSP-2 plan

The possible states of the CSP-2 inspection system are as follows:

- $S_0$ : Initial state of the plan.
- $S_1(k)$ : State in which $k(= 1, 2, ..., i)$ preceding units are found clear of defects during 100% inspection.
- $S_p$ : Initial state of partial inspection.
- $S_2$ : State in which a unit is not inspected (i.e. passed) during sampling inspection.
- $S_{PA}$ : State in which a unit is found defective and no other defectives are found in the successive $k$ units during sampling inspection.
- $S_{PR}$ : State in which more than one unit is found defective during partial inspection.
- $S_A$ : State in which current unit is accepted.
- $S_R$ : State in which current unit is rejected.

These states enable us to construct network diagram of the plan and is shown in Fig.1 [5]. Suppose that the process is in statistical control, then the probability of any incoming unit being defective is $p$ and the probability of any unit being non-defective is $q = 1 - p$.

![Network diagram of CSP-2 plan](image-url)

Fig. 1. Network diagram of CSP-2 plan
The W-functions from the initial node \( S_0 \) to the terminal nodes \( S_A \) and \( S_R \) are respectively found as
\[
W_A(\theta) = \frac{(f + (1-f))qe^\theta + (1-f)(1 - (qe^\theta)^i)}{1 - (1-f)pe^\theta[(1 - (qe^\theta)^i) / ((1-qe^\theta))]
\]
\[
W_R(\theta) = \frac{(f + (1-f))pe^\theta}{(1 - (1-f)pe^\theta)[(1 - (qe^\theta)^i) / ((1-qe^\theta))]}\]

Therefore
\[
P_A = [W_A(\theta)]_{\theta=0} = \frac{q^i(2-q^i)}{f(1 - q^i)(1 - q^i) + (2 - q^i)q^i}
\]
\[
P_R = [W_R(\theta)]_{\theta=0} = \frac{f(1 - q^i)(1 - q^i)}{f(1 - q^i)(1 - q^i) + (2 - q^i)q^i}
\]

where \( P_A \) and \( P_R \) stands for probability of acceptance and rejection (of a unit) by CSP-2 plan respectively. Since \( P_A \) fraction of accepted units are defective with probability \( p \) and \((1 - P_A)\) fraction are non-defective with probability \( q = 1 - p \).

2. OPERATING PROCEDURE OF CONTINUOUS SAMPLING PLAN (CSP-5)

The operating procedure of CSP-5 plan are described as follows:
i). At the outset, inspect consecutive units in the order of production continue this 100\% inspection until 'i’ successive units are found conforming when 'i’ successive units are found conforming, discontinue 100\% inspection and begin sampling inspection.

ii). During sampling inspection inspect units at rate 'f’. Continue sampling inspection until a non-conforming revert to 100\% inspection as per paragraph (1) above. If a non-conforming unit is found after 'k' conforming units have been found, revert to 100\% inspection with reduced clearance number 'x'.

iii). During 100\% inspection with clearance number 'x’, inspect consecutive units in the order of production and continue inspection until a nonconforming units is found or 'x’ conforming units are found, whichever occurs earlier. If a non conforming unit is found, revert to 100\% inspection with larger clearance number 'i'. If 'x’ conforming units are found, discontinue 100\% inspection and begin sampling inspection.

iv). Correct or replace all conforming units found with conforming units.

The flowchart of the plan is given in Fig. 2.
3. GERT ANALYSIS OF CSP-5 PLAN

The network diagram of CSP-5 plan is shown in Fig. 3. The W-functions from the initial node $S_0$ to the terminal nodes $S_A$ and $R$ are respectively found as

$$W_A(\theta) = \frac{(qe^\theta)^i}{(qe^\theta)^i + f(1 - (qe^\theta)^i) + f(qe^\theta)^i(qe^\theta)^i - (qe^\theta)^i}$$  \hspace{1cm} (1)

and

$$W_R(\theta) = \frac{(pfgqe^\theta)^i}{(qe^\theta)^{i+1} + f(qe^\theta)^i(1 - (qe^\theta)^i) + f(qe^\theta)^i(qe^\theta)^i - (qe^\theta)^i}$$  \hspace{1cm} (2)

The probability of acceptance and rejection of a unit by GERT approach can be calculated in terms of w-function. Since $P_A$ fractions of accepted units are defective with probability $p$ and $P_R$ fraction are non defective with probability $1 - p$ or $q$. 

Fig. 2. Flow chart of CSP-5 plan
\[ P_A = \frac{W_A(\theta)}{\theta=0} \]

\[ W_A(\theta) = \frac{(qe^\theta)^i}{(qe^\theta)^i + f(1 - (qe^\theta)^i) + f(qe^\theta)^i(qe^\theta)^i - (qe^\theta)^x} \]

\[ W_A(\theta)/\theta=0 = \frac{(qe^\theta)^i}{(qe^\theta)^i + f(1 - (qe^\theta)^i) + f(qe^\theta)^i(qe^\theta)^i - (qe^\theta)^x/\theta=0} \]

\[ = \frac{q^i}{q^i + f(1 - q^i) + f q^i(q^i - q^*)} \]

\[ P_A = \frac{q^i}{q^i + f(1 - q^i) + f q^i(q^i - q^*)} \]

\[ P_R = \frac{W_R(\theta)}{\theta=0} \]

\[ W_R(\theta) = \frac{(pf q e^\theta)^i}{(pf q e^\theta)^i + f(qe^\theta)^i(1 - (qe^\theta)^i) + f(qe^\theta)^i(qe^\theta)^i - (qe^\theta)^x} \]

\[ W_R(\theta)/\theta=0 = \frac{(pf q e^\theta)^i}{(pf q e^\theta)^i + f(qe^\theta)^i(1 - (qe^\theta)^i) + f(qe^\theta)^i(qe^\theta)^i - (qe^\theta)^x/\theta=0} \]

\[ = \frac{pf q^i}{q^i + f(1 - q^i) + f q^i(q^i - q^*)} \]

\[ P_R = \frac{pf q^i}{q^i + f(1 - q^i) + f q^i(q^i - q^*)} \]
Where \( P_{A} \) and \( P_{R} \) are the probability of acceptance and rejection of a unit by CSP-5 plan. The average outgoing quality of a lot using GERT method can be calculated in terms of \( P_{A} \) and \( P_{R} \) is

\[
AOQ = \frac{PP_{A} - qP_{R}}{P_{A} + P_{R}} = \frac{pq - pqf q^{-1}}{q' + f(1-q') + f q'(q' - q^1)}
\]

(5)

Therefore the average outgoing quality level (AOQL),

\[
PL = \frac{p_{1}q_{1} - q_{1}p_{1}f q_{1}^{-1}}{q_{1}’ + f(1-q_{1}’) + f q_{1}’(q_{1}’ - q_{1}^1)}
\]

(6)

where \( p_{1} \) is the value of \( p \) for which AOQL is reached and \( q_{1} = 1 - p_{1} \). To determine \( f \), we differentiate (1) with respect to \( p \) and equate to zero and after substituting \( p_{1} \) for \( p \).

\[
\frac{d}{dp}(AOQ) = \frac{[q' + f(1-q') + f q'(q' - q^1)]dP_{A}}{q' + f(1-q') + f q'(q' - q^1)} - \frac{[p(1-f)q']dP_{R}}{q' + f(1-q') + f q'(q' - q^1)}
\]

\[
\frac{d}{dp}(AOQ) = \frac{[q' + f(1-q') + f q'(q' - q^1)][(1-f)(-ip_{1}q_{1}^{-1} + q_{1}) - ip_{1}q_{1}^{-1} + f(ip_{1}q_{1}^{-1} + q_{1}(-ip_{1}q_{1}^{-1} + q_{1}^{-1}) - ip_{1}q_{1}^{-1} - q_{1}^{-1})]}{q' + f(1-q') + f q'(q' - q^1)}
\]

(7)

Replacing \( p \) by \( p_{1} \) and \( q \) by \( q_{1} \) in equation (7), we get

\[
0 = \frac{[(1-f)(-ip_{1}q_{1}^{-1} + q_{1})] - p_{L} \times \frac{[q_{1}^{-1} + f(ip_{1}q_{1}^{-1} +q_{1}(-ip_{1}q_{1}^{-1} + q_{1}^{-1}) - ip_{1}q_{1}^{-1} - q_{1}^{-1})]}{q_{1}^{-1} + f(1-q') + f q_{1}'(q_{1}' - q_{1}^1)}}{q_{1}^{-1} + f(1-q') + f q_{1}'(q_{1}' - q_{1}^1)}
\]

(8)

\[
\frac{d}{dp}(AOQ) = 0 \Rightarrow (1-f)(-ip_{1}q_{1}^{-1} + q_{1}) - p_{L}[iq_{1}^{-1} + f(ip_{1}q_{1}^{-1} + q_{1}(-ip_{1}q_{1}^{-1} + q_{1}^{-1}) - ip_{1}q_{1}^{-1} - q_{1}^{-1})] = 0
\]

\[
f = \frac{[A + p_{L}B]}{[A + p_{L}C]} \text{, where } A = \{ (-ip_{1}q_{1}^{-1} + q_{1}^{-1}) \}, \ B = ip_{1}q_{1}^{-1}
\]

and \( C = [iq_{1}^{-1} + q_{1}^{-1}(-ip_{1}q_{1}^{-1} + q_{1}^{-1}) - ip_{1}q_{1}^{-1} - q_{1}^{-1}]. \)

(9)

corresponding to AOQL, \( p_{L} \) and process average \( p_{1} \) we can find \( f \) and \( i \) using (8) and (9).
4. MATLAB PROGRAM FOR DESIGNING CONTINUOUS SAMPLING (CSP-5) PLAN

clc;
clear all;
p1 = 0.3; q = 0; x = 0;
for p=0.1:0.1:1
if p < p1
display(‘invalid’);
else
i = (1 - p)/(p - p1);
i = ceil(i);
q = 1 - p;
for x = 1:1:i - 1;
A = -p*i*q*i*(i - 1) + q*i*(i);
B = i*(q) * (i - 1);
C = i*(q)*i*(i - 1) + (q) * (i - 1) + x*(q) * (i - 1) - i*(q)*i*(i - 1) + ((q) * (i - 1) - (q) * (i - x));
f = A + p1 * (B) / A + p1 * (C);
P_a(x) = q * (i) / (q) * (i) + f * (1 - q * (i)) + f * (q) * (i) * q * (i) - q * (i);
AOQ(x) = p * (1 - f) * (q) * (i) / q * (i) + f * (1 - q * (i)) + f * (q) * (i) * q * (i) * (i - q) /
E(x) = f + 1 + (q) * (i) * ((q) * (i) - (q) * (x)) / ((q) * (i) + f * (1 - (q) * (i)) + f * (q) * (i) * ((q) * (i) - q * (x)));
end;
end;

5. SIMULATION OF DATA FROM THE MATLAB PROGRAMMING

The following table gives the values of $i, f, x, E(I)$ and AOQ for given values of $p_L$ and process average $p_1$
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