Petri nets properties related to the unboundness and analyzed using coverability multigraph

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Abstract

Petri nets represent a powerful tool for modeling the discrete event systems. The Petri net markings correspond to the system states. The infinity of the marking set means that the Petri net is unbounded and this may be the sign of an incorrect system model. In that case instead of the reachability set and the reachability graph the coverability set and the coverability multigraph can be used to represent the Petri net state space. A systematic way of building the notion of the coverability set and coverability multigraph based on the notion of the ω -marking is given in the paper. Algorithm for its construction is introduced. Then the use of the coverability multigraph for the analysis of several properties of the unbounded Petri nets is described.

Additional Key Words and Phrases: Petri nets, unboundness and coverability property, ω -marking, coverability set, coverability multigraph.

1. INTRODUCTION

In the course of decades Petri nets proved to be a powerful tool for modelling and design of the discrete event systems [1], [2]. There are many modifications of the Petri nets spanning from the classic place/transition [3] nets up to the colour Petri nets [4]. Theory and practice of the Petri nets are described in many survey publications such as for example [5], [6], [7], [8], [9].

The states of the system to be modelled by a Petri net are represented by the Petri net markings. System modeling and control using Petri nets is described in detail in [8], [10]. The number of the states of the real man-made system is finite and consequently its model having the infinite number of the markings either is not correct or it is correct but the trajectory giving the infinite number of markings is finished after the finite number of steps, which is due to limited resources of the system, e. g. inputs of the system, and/or due the system control action. A thorough analysis and check of the Petri net model is necessary to verify its acceptability for

the next usage. The problems related to the unboundness of the system models based on the Petri nets will be treated in this paper.

2. PETRI NETS AND THE BOUNDNESS PROPERTY

Let us consider the standard Petri nets [5], [7], [8] called place/transition Petri nets. The following definition delimitates that Petri net class.

DEFINITION 1. The Petri net is the 5 – tuple $PN = (P, T, F, w, m_0)$ where

P is a finite non-empty set of elements called places: $P = \{p_1, p_2, \dots, p_n\}$,

T is a finite non-empty set of elements called transitions: $T = \{t_1, t_2, \dots, t_m\}$,

$$P \cap T = \emptyset,$$

F is a non-empty binary relation the so-called flow relation:

$F \subseteq (P \times T) \cup (T \times P)$, where each $p_i \in P$ is at least in one ordered pair of F , similarly for each t_j , (the relation F is finite as it follows from the definition of P and T),

w is the weight function: $F \rightarrow I^+$, where I^+ is the set of positive integers,

m_0 is the function called initial marking: $P \rightarrow I^+ \cup \{0\} = N$, the set of the natural numbers.

The Definition 1 defines the Petri net model structure. Given a transition t_k in the Petri net, the set $\bullet t = \{p_i \mid [p_i, t] \in F\}$ is the set of the pre-places. Similarly $t\bullet = \{p_i \mid [t, p_i] \in F\}$ is the set of the post-places.

Its dynamics is given by sequences of markings starting from the initial marking. The event of the marking change is bound with the transition firing. The transition firing will be described below (Definition 2). Any marking m_i in a

marking sequence analogously to the initial marking m_0 is the function $P \rightarrow N$.

The event is notated as

$$m_i \xrightarrow{t_k} m_j,$$

which means that the firing of t_k is associated with the change of m_i to m_j . The notation $m_i [t_k] m_j$.

Markings can be represented by the vectors with components obtained through the marking function so that

$$\mathbf{m}_i = \begin{pmatrix} m_i(p_1) \\ m_i(p_2) \\ \vdots \\ m_i(p_n) \end{pmatrix}.$$

The next vectors enabling an effective work with Petri nets are those defined for $\forall t \in T$

$$\mathbf{t}^+ = \begin{pmatrix} w([t, p_1]) \text{ if } [t, p_1] \in F, \text{ otherwise } 0 \\ w([t, p_2]) \text{ if } [t, p_2] \in F, \text{ otherwise } 0 \\ \vdots \\ w([t, p_n]) \text{ if } [t, p_n] \in F, \text{ otherwise } 0 \end{pmatrix},$$

$$\mathbf{t}^- = \begin{pmatrix} w([p_1, t]) \text{ if } [p_1, t] \in F, \text{ otherwise } 0 \\ w([p_2, t]) \text{ if } [p_2, t] \in F, \text{ otherwise } 0 \\ \vdots \\ w([p_n, t]) \text{ if } [p_n, t] \in F, \text{ otherwise } 0 \end{pmatrix}$$

DEFINITION 2. A transition t_k on the given Petri net is fireable at the marking $\mathbf{m}_i$ iff

$$\mathbf{t}_k^- \leq \mathbf{m}_i$$

and when the transition is fired it holds

$$\mathbf{m}_i \xrightarrow{t_k} \mathbf{m}_j \Rightarrow \mathbf{m}_j = \mathbf{m}_i + \mathbf{t}_k^+ - \mathbf{t}_k^-. \quad (1)$$

The equation (1) determines the transition firing rules (when the transition t_k is fireable): for each pre-place p_r of the transition t_k from the marking of p_r the value of the weight for ordered pair $[p_r, t_k]$ is subtracted and for each post-place p_s the value of the weight for $[t_k, p_s]$ is added to the marking of p_s . If the function value $m_i(p_r)$ is interpreted as the number of the tokens residing in the place p_r by the marking m_i then the firing means taking the number of tokens from each $p_r \in \bullet t_k$ given by the weight of the edge going from p_r to t_k and adding to each $p_s \in t_k \bullet$ the number of tokens according to the weight of the edge going into p_s . The rules just described are additive ones. There exist other kind of the firing rules [9].

DEFINITION 3. Let a Petri net PN be given. A marking m' , for which a finite firing sequence $\tilde{\mu}$ exists such that

$$\tilde{\mu} = t_{j_1} t_{j_2} \dots \quad \text{and} \quad m_0 \xrightarrow{t_{j_1}} m_{i_1} \xrightarrow{t_{j_2}} m_{i_2} \dots = m', \quad \text{written also as} \\ m_0 [\tilde{\mu}] m',$$

is the reachable marking in the Petri net PN . The set $R_{PN}(m_0)$ of all such markings is called the reachability set in the given Petri net PN . Each firing sequence starts always at the initial marking m_0 .

As we will see later the firing sequence can be infinite. The above introduced firing rules are the basic and mostly treated ones. It is sufficient and good substantiated to presume in the most cases of modeling that only one transition fires (that is only one system event occurs) in one time point.

Petri nets are a kind of the mathematical graphs as it can be seen from the following definition.

DEFINITION 4. The labeled directed simple mathematical graph is a 6-tuple

$$G = (A, R, f_1, f_2, S_1, S_2)$$

where

A is a finite set of the nodes,

R is a relation on A , which can be empty,

f_1 is a partial function $A \rightarrow S_1$ defined if the set S_1 is defined,

f_2 is a partial function $R \rightarrow S_2$ defined if the set S_2 is defined,

the sets S_1 and S_2 can be empty.

The set $P \cup T$ of a Petri net corresponds to the set A and the relation F to R in the Definition 4. The Petri net as the graph is a specific one, namely it is a bipartite labeled directed simple mathematical graph because the set A is split into two sets, i. e. $A = P \cup T$ and $R = F \subseteq (P \cup T) \cup (T \cup P)$. Places and transitions of a Petri net are the graph nodes and the elements of F , i. e. $[p_i, t_k] \in F$ and $[t_r, p_s] \in F$, are directed edges of the graph. The functions w and m_0 correspond to f_1 and f_2 , respectively. For an actual marking m_i there is the actual graph with the different function f_1 .

3. UNBOUNDED PETRI NETS AND COVERABILITY

The reachability set $R_{PN}(m_0)$ of the given Petri net PN can be infinite or in other words the Petri net is unbounded as it is illustrated by the following example.

EXAMPLE 1. Let a Petri net be given graphically in Fig. 1.

Fig. 1. An unbounded Petri net.

By convention if the weight is not introduced it is 1. Using definition we have the Petri net:

$$P = \{p_1, p_2, p_3\}, \quad T = \{t_1, t_2\}, \quad F = \{[p_1, t_1], [p_2, t_1], [p_3, t_2], [t_1, p_3], [t_2, p_1], [t_2, p_2]\},$$

$$w([p_1, t_1]) = 2, \quad w([p_2, t_1]) = 3, \text{ etc.}, \quad m_0(p_1) = 2, \quad m_0(p_2) = 3, \quad m_0(p_3) = 0.$$

The firing sequence

$$\mathbf{m}_0 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \xrightarrow{t_1} \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \xrightarrow{t_2} \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \xrightarrow{t_1} \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \xrightarrow{t_2} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \xrightarrow{t_1} \dots$$

shows that the number of markings is infinite because the firing sequence continues without limit. The number of tokens in the place p_3 grows to the infinity.

The described Petri net property is formally expressed by the following definition.

DEFINITION 5. Given a Petri net $PN = (P, T, F, w, m_0)$. A place $p_i \in P$ is bounded iff

$$m(p_i) \leq K, \quad K \in \mathbb{N},$$

holds true for all reachable marking $m \in R_{PN}(m_0)$. A Petri net PN is bounded iff each place $p_i \in P$ of PN is bounded. If a Petri net is not bounded it is unbounded.

THEOREM 1. A Petri net is bounded iff its reachability set is finite.

PROOF.

$\Leftarrow$: Suppose the reachability set is finite. The initial marking always maps the places into the finite natural numbers. Therefore the firing rules (Definition 2) yield again the finite marking numbers as sums of the finite integers (see the equation 1 and the construction of $\mathbf{t}^+$ and $\mathbf{t}^-$). Conclusion: the Petri net is bounded.

$\Rightarrow$: Conversely, if PN is bounded markings of the reachability set are always achieved by the finite number of firings so that the reachability set is finite.

If the Petri net is bounded the corresponding reachability set and its creation can be presented by the reachability graph.

DEFINITION 6. The reachability graph is the labeled directed simple mathematical graph where following the Definition 4 $A = R_{PN}(m_0)$, R is the relation $R \subseteq R_{PN}(m_0) \times R_{PN}(m_0)$ such that for each $[m_i, m_j] \in R$: $\exists t_k \in T$, for which is $m_i \xrightarrow{t_k} m_j$. The function $f_2 : R \rightarrow T$ so that $f_2([m_i, m_j]) = t_j$ and the function $f_1 : A \rightarrow \emptyset$.

COROLLARY 1. The reachability graph of a Petri net can be constructed iff the Petri net is bounded.

The coverability is a notion, which can help to analyze and evaluate the properties of the unbounded Petri nets using the coverability graph instead of the non-constructable reachability graph.

DEFINITION 7. Given a Petri net $PN = (P, T, F, W, M_0)$. A marking $\mathbf{m}'$ covers another marking $\mathbf{m}$ iff

$$\mathbf{m}' \geq \mathbf{m}.$$

It follows from the definition that a marking $\mathbf{m}$ in the Petri net is covered by itself that is $\mathbf{m} \geq \mathbf{m}$.

The marking covering or the coverability property in the unbounded Petri nets needs to enhance the marking definition in the following way.

DEFINITION 8. The ω -marking in the Petri net is the function

$$m_\omega : P \rightarrow \mathbf{I}^+ \cup \{0\} \cup \{\omega\}$$

where a. $\mathbf{I}^+$ is the set of positive integers

b. $\omega \succ n$ for any $n \in \mathbf{I}^+$

c. $\omega \pm n = \omega$ for any $n \in \mathbf{I}^+$

d. $\omega \geq \omega$

e. $\exists p \in P$ such that $m_\omega(p) = \omega$.

The symbol ω represents the infinity in the domain of the natural numbers $\mathbf{N} = \mathbf{I}^+ \cup \{0\}$. The item e. in the Definition 8 means that there is at least one place in the given Petri net, which is mapped to ω .

An ω -marking m_ω can be represented analogously to a Petri net marking (ordinary) with help of the vector

$$\mathbf{m}_\omega = \begin{pmatrix} m_\omega(p_1) \\ \cdot \\ \cdot \\ \cdot \\ m_\omega(p_n) \end{pmatrix}$$

Let the place p_i of the Petri net is unbounded. The mapping of ω to the place p_i of the Petri net means that the value of the marking for that place is greater than any arbitrary positive integer $n \in \mathbf{N}$. So that an ω -marking can be specified for unbounded Petri nets.

Now the analogous definition to the Definition 7 considering the ω -marking can be submitted, namely:

DEFINITION 9. Given a Petri net. An ω -marking $\mathbf{m}_\omega$ is covered by an ω -marking $\mathbf{m}'_\omega$ iff

$$\mathbf{m}'_\omega \geq \mathbf{m}_\omega \quad (2)$$

The expression (2) means that the relation $\geq$ should be valid for each corresponding entries of the vectors. Moreover as it was defined earlier $\omega \geq n$ for $\forall n \in \mathbf{N}$ and $\omega \geq \omega$. Note that the marking (as it was ordinary defined - Definition 1 and later) and ω -marking (Definition 8) are distinguished.

The strict covering is introduced in what follows. It concerns the marking or the ω -marking (the latter case is introduced in brackets).

DEFINITION 10. Given a Petri net $PN = (P, T, F, W, M_0)$. A marking $\mathbf{m}$ (ω -marking $\mathbf{m}_\omega$) is strictly covered by another marking $\mathbf{m}'$ (ω -marking $\mathbf{m}'_\omega$) iff $\mathbf{m}' > \mathbf{m}$ ($\mathbf{m}'_\omega > \mathbf{m}_\omega$).

The inequality sign in the definition means that $\mathbf{m}'(p_i) > \mathbf{m}(p_i)$ ($\mathbf{m}'_\omega(p_i) > \mathbf{m}_\omega(p_i)$) holds true at least for one place $p_i \in P$. Obviously, when a place p_r has value $m_\omega(p_r) = \omega$, it does not contribute to the strict coverability because by definition $\omega \geq \omega$.

THEOREM 2. Given a Petri net PN and two firing sequences $\tilde{\lambda}, \tilde{\nu}$, where $\tilde{\nu}$ is a non-empty firing sequence while $\tilde{\lambda}$ can be the empty one (that is $\mathbf{m}_0 [\tilde{\lambda}] \mathbf{m}_0$ if $\tilde{\lambda}$ is empty). If

$$\mathbf{m}_0 [\tilde{\lambda}] \mathbf{m} [\tilde{\nu}] \mathbf{m}' \quad \text{and} \quad \mathbf{m}' > \mathbf{m}$$

then the places $p_i, p_j, \dots \in P$, for which $\mathbf{m}'(p_i) > \mathbf{m}(p_i), \mathbf{m}'(p_j) > \mathbf{m}(p_j), \dots$ are unbounded, and consequently the whole Petri net is unbounded.

PROOF. Inequality $\mathbf{m}' > \mathbf{m}$ means that at least for one entry of those vectors is

$$m'(i) > m(i), i \in 1, 2, \dots, |P| \quad (3)$$

while for the other entries, for which it is not valid, the equality holds. The vector entries considered in (3) correspond to the place p_i .

Let the first transition in the non-empty firing sequence $\tilde{v}$ be t_k . If t_k is fireable by $\mathbf{m}$ even more it is fireable by the marking $\mathbf{m}'$. The structure and weights of the Petri net PN are fixed. Some tokens are removed by firing t_k from p_i and some are added in the place p_i according to the firing rules. If there are more tokens added to p_i than removed from it then (3) holds. Consider that the firing sequence $\tilde{v}$ is again applied starting from marking $\mathbf{m}'$ and $\mathbf{m}'[\tilde{v} > \mathbf{m}''$. The inequality $\mathbf{m}'' > \mathbf{m}'$ holds due to the above-mentioned reason. Continuing the reasoning in the described way it can be concluded that the number of tokens in place p_i grows to infinity. A similar situation can occur for more than one place. In such a case the number of tokens in such places increase beyond any integer limit.

If the places $p_i, p_j, \dots$ from the Theorem 2 are mapped to ω in the marking $\mathbf{m}'$ then the ω -marking $\mathbf{m}'_\omega$ is obtained with $\mathbf{m}'_\omega(p_i), \mathbf{m}'_\omega(p_j), \dots$. When mapping to ω is done immediately after occurring a strict inequality in a firing sequence according the Theorem 2 and this is done for all possible firing sequences, a set of ω -markings for the given Petri net is obtained. Now there is a right point to define the coverability set.

DEFINITION 11. For a given Petri net the coverability set $C_{PN}(\mathbf{m}_0)$ is the set, which contains reachable markings of the Petri net and/or ω -markings so that each reachable markings is in $C_{PN}(\mathbf{m}_0)$ or it is covered by an ω -marking from $C_{PN}(\mathbf{m}_0)$.

The Theorem 2 can be used for the construction of the coverability multigraph. Before proceeding further the directed labeled multigraph is going to be defined.

DEFINITION 12. The directed labeled mathematical multigraph MG is given by the ordered 7-tuple

$$MG = (A, H, \mu, S_1, S_2, f_1, f_2)$$

where

A	is a finite non-empty set of the multigraph nodes
H	is the set of directed arcs
$\mu : H \rightarrow A \times A$	is the incidence function mapping the arcs into
the	ordered 2-tuples (ordered pairs) of the nodes
and	determining the arc direction, as well
S_1, S_2	are finite sets, where either can be empty
$f_1 : H \rightarrow S_1$	is the function labeling the arcs
$f_2 : A \rightarrow S_2$	is the function labeling the nodes.

The Definition 12 opens the way to the definition of the coverability multigraph in what follows.

DEFINITION 13. For the Petri net PN the coverability multigraph is a directed labeled mathematical multigraph defined by the expression

$$CMG_{PN} = (C_{PN}(\mathbf{m}_0), H, \mu, T, \emptyset, f_1, f_2)$$

where

$C_{PN}(\mathbf{m}_0)$	is a coverability set of the Petri net PN
$H = \{h_1, h_2, \dots, h_r\}$	is a finite set of the arcs
$\mu : H \rightarrow C_{PN}(\mathbf{m}_0) \times C_{PN}(\mathbf{m}_0)$	is the incidence function
T	is the set of the PN transitions

$\emptyset$	the second set according to the Definition 12 of the mathematical multigraph is empty
$f_1 : H \rightarrow T$	is the function, which specifies label of each arc such that firing of the transition labeling an arc changes the ω -markings, which are incident with that arc, formally if $\mu(h_i) = [\mathbf{n}_{j\omega}, \mathbf{n}_{k\omega}], h_i \in H, \mathbf{n}_{j\omega} \in C_{PN}(\mathbf{m}_0),$ $\mathbf{n}_{k\omega} \in C_{PN}(\mathbf{m}_0)$ and $t_u \in T$ is fireable by $\mathbf{n}_{j\omega}$, that is $\mathbf{n}_{j\omega} [t_m > \mathbf{n}_{k\omega}, \text{ then}$ $f_1(h_i) = t_m$
f_2	is the function f_2 from the Definition 12 mapping to $\emptyset$.

The Theorem 2 can be used for the construction of the coverability multigraph for the given Petri net. It proceeds as follows. Starting from the initial marking $\mathbf{m}_0$ all possible firing sequences are generated. A fireable transition generates a marking or ω -marking, which according to the Theorem 2 is checked up whether strictly covers some of the preceding markings or ω -markings generated by the firing sequences in the multigraph. In the places, which strictly cover the corresponding places the symbol ω is placed. The markings or ω -markings thus generated constitute the coverability multigraph nodes. The multigraph construction algorithm named ALGCOV is depicted with help of structogramme in Fig. 2. There the marking denoted with asterisk $*$ are either marking or ω -marking.

Fig. 2. The algorithm ALGCOV expressed by the structogramme.

It is evident from the coverability property and from the construction of the coverability multigraph that the coverability set $C_{PN}(\mathbf{m}_0)$ is finite and multigraph is considered because more than one oriented edges can connect a pair of the multigraph nodes.

EXAMPLE 2. A coverability multigraph is constructed with ALGCOV (Fig. 3) for the Petri net from the Example 1.

Fig. 3. A coverability multigraph for the Petri net from the Example 1.

The algorithm ALGCOV gives the unique coverability multigraph. But there is an ambiguity in that it can be constructed different coverability multigraphs fulfilling the definition: they cover all markings generated in the given Petri net and they show the possible firing sequences. See an extreme case of that illustrated in the Example 3.

EXAMPLE 3. Consider again the Petri net from the Example 1. The coverability multigraph in Fig. 4 differs from that in Fig. 3.

Fig. 4. A different coverability multigraph.

It can be seen from the introduced examples that all firing sequences are not specified by the coverability multigraph. They have to be found together with the original Petri net. It means that in a sense in the coverability multigraph some information is lost. An approach to the solution of the problem can be found in [11].

4. ANALYSIS OF THE PETRI NET PROPERTIES BASED ON THE COVERABILITY MULTIGRAPH.

Several properties of the Petri nets can be analyzed using the coverability multigraphs. Let some of them be treated in the sequel.

a. Boundedness.

For a given Petri net the question of the boundedness can be answered via the coverability multigraph constructed with ALGCOV, which can be programmed. The Petri net is bounded if there is not ω -marking in the graph. On the contrary the Petri net is unbounded.

b. Reachability.

For a given Petri net a question can be posed: is the given marking reachable in the Petri net? To find the answer the coverability multigraph is created with the

algorithm ALGCOV. One searches in the multigraph if the analyzed marking exist in it or it is covered by an ω -marking generated. In the latter case one has to search firing sequences if there is at least one leading to the marking under question.

c. Existence of the dead-lock marking.

The dead-lock marking means the dead-lock in Petri net dynamics and correspondingly in the system modeled by the Petri net. A dead-lock exist if in the coverability multigraph is a node, from which no edge is going out.

d. Reversibility.

Reversibility of the Petri net is such a property that it is possible from each reachable marking to reach the initial marking. Using the coverability multigraph we have that the Petri net is reversible if the coverability multigraph is strongly connected (strong connectivity means that each pair of the multigraph nodes are connected with a path in both directions).

e. Liveness.

Liveness is the Petri net property concerning the fireability of the transitions and the existence of the relevant firing sequences. There are more levels of the liveness. The liveness on the level L4 is defined so that [7] starting from any reachable marking each transition is fireable at least once. It can be proved that if the Petri net is L1 live (L1 means that each transition starting from the initial marking is fireable at least once) and the coverability multigraph is strongly connected the Petri net is L4 live.

CONCLUSION

The aim of this paper was to build systematically the notion of the coverability multigraph and to show its relation to several properties of the Petri nets. The coverability multigraph can be used for the analysis of some Petri net properties. In the paper was presented a way how to proceed in the analysis of the Petri net properties. Further analysis methods based on the coverability multigraph can be developed.

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