Cost -Benefit Analysis of a Single-Unit System with Preventive Maintenance and Weibull Distribution for Failure and Repair Activities

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Abstract

This paper deals with a reliability model developed for a single-unit system which goes for preventive maintenance after a pre-specific time 't' up to which no failure occurs. There is a single server who takes some time to arrive at the system for doing repair activities. The unit does not work as new after repair at complete failure and so called the degraded unit. The degraded unit is replaced by new one after its failure with some replacement time. The failure time, preventive maintenance time, replacement time and repair time of the unit are taken as Weibull distributed with common shape parameter and different scale parameters. The switching devices are perfect. The system is observed at suitable regenerative epochs to obtain various measures of system effectiveness of interest to system designers and operation managers.

Additional Key Words and Phrases: Single-Unit System, Preventive Maintenance, Degradation, Arrival Time of the Server and Cost-Benefit Analysis.

1. INTRODUCTION

In reliability theory, it has been proved that performance and reliability of repairable systems can be improved by the method of redundancy. But there are many systems in which a unit cannot be kept as spare due to its high cost. Therefore, reliability models of single-unit systems with different failure modes have also been probed by the authors including Malik and Bansal [7], Malik [8], Kharoufeh J.P. et al. [5], Malik et al. [9] and Uematsu and Nishida [11] and Pawar and Malik [10] keeping in view of their practical utility and common man's affordability under the following assumptions:

- i. The Unit has a constant failure rate.
- ii. The unit can work as new after repair and maintenance.
- iii. The unit can work forever without conducting preventive maintenance.
- iv. Inspection of the failed degraded unit will be conducted to see the feasibility of repair.
- v. Server is immediately available as and when required.

But the instantaneous hazard rates of many systems such as rotating shafts, valves and cams are of non-linear nature due to wear out under mechanical stress and so their failure and repair times follow arbitrary distributions like Weibull distribution. Gupta et al. [3] analyzed a two dissimilar unit cold standby system model by taking Weibull failure and repair distribution. Further, sometimes, unit does not work as new after repair. Since the capability of the unit after repair depends on the repair mechanism adopted and unit may have increased failure rate, if it is repaired by an ordinary server. In such a situation unit becomes degraded. Furthermore, repair of a degraded unit is not always feasible to the system may because of its excessive use and high cost of maintenance. In such cases, the failed degraded unit may be replaced by new one immediately. Kadyan and Promila [4] suggested a reliability model under the concept of degraded unit. Again, the continued operation and ageing of systems gradually reduce their performance and reliability. Therefore, PM of these systems may be conducted after a pre-specific time 't' to slow the deterioration process as well as to restore the systems in a younger age or state. The concept of preventive maintenance has been used by kumar and Malik [6] while analyzing the profit of a computer system with maximum operation time and maximum repair time.

But, in practice, the assumption (v) is not always true due to pre occupations of the repair facility and in such a situation service facility may take some time to arrive at the system. Chander [2] has suggested reliability models of a standby system with arrival time of the server.

The purpose of the present paper is to analyze a single-unit system model by incorporating Weibull distribution for failure, preventive maintenance, replacement and repair times with different scale parameters. The characteristics of Weibull distribution are to represent the three well known phases of failure/repair/replacement/maintenance rates of a unit for different values of shape parameter.

Let the random variable T denotes the time to failure of an item/device having Weibull distribution, its pdf will be:

$$f(t) = \theta p t^{p-1} \exp(-\theta t^p); \quad t \geq 0 \text{ and } \theta, p > 0 \quad (*)$$

The reliability/survival function and hazard (failure/repair) rate function for Weibull distribution are given by

$$R(t) = \exp(-\theta t^p) \quad \text{and} \quad h(t) = \theta p t^{p-1} \quad t \geq 0 \quad \text{and} \quad \theta, p > 0$$

It is important to note that p and θ are the shape and scale parameters, respectively. If we put $p = 1$ in (*), Weibull distribution reduces to Exponential distribution and if $p = 2$, it reduces to Rayleigh distribution. The hazard function of this distribution will be constant for $p = 1$, linearly increasing for $p = 2$, non-linearly increasing for $p > 2$ and uniformly decreasing for $p < 1$. Brkic [1] deals with the interval estimation of the parameters of Weibull distribution. The following measures of system effectiveness, useful to system designers and operations managers, are obtained by using regenerative point technique.

1. Steady state transition probabilities and mean sojourn times.
2. Reliability of the system and Mean time to system failure (MTSF).
3. Point-wise and steady state availabilities of the system.
4. Expected busy period of the repairman in time interval $(0, t)$.
5. Net expected profit earned by the system in time interval $(0, t)$ and in steady state.

2. SYSTEM DESCRIPTION, NOTATIONS AND STATES OF THE SYSTEM

The system consists of single-unit which has two modes: Normal (N) mode and total failure (F) mode. There is a single server who takes some time to arrive at the system for doing repair and maintenance activities. The unit becomes degraded after repair. The server replaces the failed degraded unit with some replacement time. Preventive maintenance of the operative unit is carried out after a pre-specific time ' t ' up to which no failure occurs. The failure, repair, preventive maintenance, arrival time of the server, transition rate and replacement time distributions of each unit are taken to be independent having the Weibull density with common shape parameter and different scale parameters as follows:

$$h_i(t) = \alpha_i \eta t^{\eta-1} \exp(-\alpha_i t^\eta), \quad g(t) = \theta \eta t^{\eta-1} \exp(-\theta t^\eta),$$

$$f_i(t) = \beta_i \eta t^{\eta-1} \exp(-\beta_i t^\eta),$$

$k(t) = \gamma \eta t^{\eta-1} \exp(-\gamma t^\eta)$, $m_i(t) = \tau_i \eta t^{\eta-1} \exp(-\tau_i t^\eta)$ and
 $l(t) = l_1 \eta t^{\eta-1} \exp(-l_1 t^\eta)$, where $t \geq 0$ and $\eta, \alpha_i, \beta_i, \gamma, \theta, \tau_i, l_1 > 0$
 respectively for original and degraded unit respectively.

2.1. Notations

α_i / β_i ($i = 1, 2$): Scale parameter of failure/maintenance time distribution for original and degraded unit.

$\gamma / \theta / l_1$: Scale parameter of arrival time of the server/repair time of the original unit/replacement rate of the degraded unit.

τ_1 / τ_2 : Scale parameter of transition rate for original and degraded unit.

η : Shape parameter of failure/ repair/ arrival time of the server/ preventive maintenance/ transition rate/replacement time distribution for original and degraded unit.

$Q_{ij}(\cdot) / q_{ij}(\cdot)$: Cdf/pdf of transition time from state Si to state Sj.

$h_i(\cdot) / g_i(\cdot) / k(\cdot) / l(\cdot) / m_i(\cdot) / f_i(\cdot)$: Failure/repair/arrival time of server/replacement/transition rate/preventive maintenance rates of original and degraded unit having the form.

$h_i(t) = \alpha_i \eta t^{\eta-1} e^{-\alpha_i t^\eta}$, $g(t) = \theta \eta t^{\eta-1} e^{-\theta t^\eta}$, $k(t) = \gamma \eta t^{\eta-1} e^{-\gamma t^\eta}$, $l(t) = l_1 \eta t^{\eta-1} e^{-l_1 t^\eta}$, $m_i(t) = \tau_i \eta t^{\eta-1} e^{-\tau_i t^\eta}$
 and $f_i(t) = \beta_i \eta t^{\eta-1} e^{-\beta_i t^\eta}$ where $t \geq 0$ and $\eta, \alpha_i, \beta_i, \gamma, \theta, \tau_i, l_1 > 0$

p_{ij} : Direct/indirect steady state transition probabilities from state Si to state Sj.

ψ_i : Mean sojourn time in state Si

*

S : Dummy variable in Laplace Transform.

2.2. Symbols for the states of the system:

N_o / D_o : Unit is in original/degraded mode and operative.

Fur / Fwr : Unit is in failure mode and under repair/ waiting for repair.

$Pm / WPm / DPm$: Unit is under preventive maintenance / waiting for preventive maintenance/ Degraded unit under preventive maintenance.

DF_{urp} : Failed degraded unit under replacement.

Using these symbols, the system can be in any one of the following states.

$$S_0 = N_o \quad S_1 = WPm \quad S_2 = Pm \quad S_3 = Fwr \quad S_4 = Fu \quad S_5 = D_o \quad S_6 = DFur$$

$$S_7 = DPm$$

We observe that the states S_0 and S_5 are up states and S_2, S_3, S_4, S_6 and S_7 are failed states. It is also revealed that all the states are regenerative states.

3. TRANSITION PROBABILITIES AND SOJOURN TIMES

The transition probability matrix (t.p.m.) of the embedded Markov Chain is

$$P = \begin{pmatrix} p_{00} & p_{01} & p_{02} & p_{03} & p_{04} & p_{05} & p_{06} & p_{07} \\ p_{10} & p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} & p_{17} \\ p_{20} & p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} & p_{27} \\ p_{30} & p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} & p_{37} \\ p_{40} & p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} & p_{47} \\ p_{50} & p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} & p_{57} \\ p_{60} & p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} & p_{67} \\ p_{70} & p_{71} & p_{72} & p_{73} & p_{74} & p_{75} & p_{76} & p_{77} \end{pmatrix}$$

With non-zero elements:

$$\begin{aligned} p_{12} = 1, p_{20} = 1, p_{34} = 1, p_{45} = 1, p_{60} = 1, p_{75} = 1, p_{01} = \frac{\tau_1}{\tau_1 + \alpha_1}, p_{03} = \frac{\alpha_1}{\tau_1 + \alpha_1}, \\ p_{57} = \frac{\tau_2}{\tau_2 + \alpha_2}, p_{56} = \frac{\alpha_2}{\tau_2 + \alpha_2} \end{aligned} \quad (1)$$

and the other elements of the transition probability matrix will be zero.

As an illustration, to obtain p_{45} , the probability that the system transits from state S4 to state S5 during time interval $(0, \infty)$ we observe as follows:

$$p_{45} = \int [\text{Probability that repair of the failed unit in state S4 completes during time } (v, v+dv)] dv \\ = \int \theta \eta v^{\eta-1} e^{-\theta v^{\eta}} dv = 1$$

It can be easily verified that $p_{01} + p_{03} = 1, p_{56} + p_{57} = 1$

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transitioning into any other state. If random variable U_i denotes the sojourn time in state S_i then $\psi_i = \int P[U_i > t] dt$

Therefore, its values for various regenerative states are as follows:

$$\psi_0 = \frac{\Gamma(1+\frac{1}{\eta})}{\frac{1}{(\alpha_1 + \tau_1)^{\eta}}}, \psi_1 = \psi_3 = \frac{\Gamma(1+\frac{1}{\eta})}{\frac{1}{(\gamma_1)^{\eta}}}, \psi_2 = \frac{\Gamma(1+\frac{1}{\eta})}{\frac{1}{(\beta_1)^{\eta}}}, \psi_4 = \frac{\Gamma(1+\frac{1}{\eta})}{\frac{1}{(\theta)^{\eta}}}, \psi_5 = \frac{\Gamma(1+\frac{1}{\eta})}{\frac{1}{(\alpha_2 + \tau_2)^{\eta}}}, \psi_6 = \frac{\Gamma(1+\frac{1}{\eta})}{\frac{1}{(l_1)^{\eta}}} \\ \text{and } \psi_7 = \frac{\Gamma(1+\frac{1}{\eta})}{\frac{1}{(\beta_2)^{\eta}}} \quad (2)$$

As an illustration, to obtain ψ_0 , we observe as follows:

$$\psi_0 = \int [\text{Probability that the operating unit in state } S_0 \text{ neither fail nor undergoes for PM up to time } t] dt \\ = \int e^{-(\alpha_1 + \tau_1)t^{\eta}} dt = \frac{\Gamma(1+\frac{1}{\eta})}{\frac{1}{(\alpha_1 + \tau_1)^{\eta}}}$$

4. ANALYSIS OF RESULTS

4.1. Reliability of the system and MTSF

Let the random variable T_i denotes the time to system failure i.e., time taken by the system to reach into any of the failed state, when the system initially starts from regenerative state S_i , then the reliability of the system is given by $R_i(t) = P(T_i > t)$

To determine $R_i(t)$, we regard the failed states S_i and S_3 of the system as absorbing. By probabilistic arguments, we have

$$R_0(t) = Z_0(t) \text{ where } Z_0(t) = e^{-(\alpha_1 + \tau_1)t^\eta} \quad (3)$$

Taking the Laplace Transforms of relations (3) and simplifying for $R_0^*(s)$, omitting the argument 's' for brevity, we get

$$R_0^*(s) = Z_0^*(s) \quad (4)$$

Taking the Inverse Laplace Transforms of (4), we can get the reliability of the system when it starts from state S_0 . The MTSF when system initially starts from S_0 is given by,

$$E(T_0) = \int_0^\infty R_0(t) dt = \lim_{s \rightarrow 0} R_0^*(s) = \psi_0 \quad (5)$$

4.2. Availability analysis

Let $A_i(t)$ be the probability that the system is operative at epoch t when initially, it starts from regenerative state S_i . By simple probabilistic arguments, we have the following recursive relations in $A_i(t)$:

$$\begin{aligned}
A_0(t) &= Z_0(t) + q_{03}(t) \odot A_3(t) + q_{01}(t) \odot A_1(t) \\
A_1(t) &= q_{12}(t) \odot A_2(t) \\
A_2(t) &= q_{20}(t) \odot A_0(t) \\
A_3(t) &= q_{34}(t) \odot A_4(t) \\
A_4(t) &= q_{45}(t) \odot A_5(t) \\
A_5(t) &= Z_5(t) + q_{56}(t) \odot A_6(t) + q_{57}(t) \odot A_7(t) \\
A_6(t) &= q_{60}(t) \odot A_0(t) \\
A_7(t) &= q_{75}(t) \odot A_5(t)
\end{aligned} \tag{6}$$

where $Z_0(t) = e^{-(\alpha_1 + \tau_1)t^\eta}$ and $Z_5(t) = e^{-(\alpha_2 + \tau_2)t^\eta}$

Taking the Laplace Transforms of relations (6) and simplifying for $A_0^*(s)$, omitting the argument 's' for brevity, we get

$$A_0^*(s) = \frac{Z_0^*[1 - q_{57}^*q_{75}^*] + Z_5^*[q_{03}^*q_{34}^*q_{45}^*]}{[1 - q_{57}^*q_{75}^*] - [1 - q_{57}^*q_{75}^*]q_{20}^*q_{12}^*q_{01}^* - q_{03}^*q_{34}^*q_{45}^*q_{60}^*q_{56}^*} \tag{7}$$

Taking the Inverse Laplace Transform of (7), we can get availability of the system for known values of the parameters.

In the long run, the steady state probability that the system will be operative, is given by,

$$A_0 = \lim_{t \rightarrow \infty} A_0(t) = \lim_{s \rightarrow 0} sA_0^*(s) = \frac{\psi_0[1 - p_{57}p_{75}] + \psi_5[p_{03}p_{34}p_{45}]}{[\psi_0 + p_{01}\psi_1 + p_{01}\psi_2 + p_{03}\{\psi_3 + \psi_4\}][1 - p_{57}] + [\psi_5 + p_{57}\psi_7 + p_{56}\psi_6]p_{03}} \tag{8}$$

The expected up time of the system during time interval (0,t) is

$$\mu_{up}(t) = \int_0^t A_0(u) du \text{ so that } \mu_{up}^*(s) = \frac{A_0^*(s)}{s} \tag{9}$$

4.3. Busy period analysis

Let $B_i(t)$ be the probability that the server will be busy at epoch t when initially, its operation starts from regenerative state S_i . By simple probabilistic arguments, we have the following

recursive relations in $B_i(t)$:

$$\begin{aligned}
B_0(t) &= q_{03}(t) \odot B_3(t) + q_{01}(t) \odot B_1(t) \\
B_1(t) &= q_{12}(t) \odot B_2(t) \\
B_2(t) &= Z_2(t) + q_{20}(t) \odot B_0(t) \\
B_3(t) &= q_{34}(t) \odot B_4(t) \\
B_4(t) &= Z_4(t) + q_{45}(t) \odot B_5(t) \\
B_5(t) &= q_{56}(t) \odot B_6(t) + q_{57}(t) \odot B_7(t) \\
B_6(t) &= Z_6(t) + q_{60}(t) \odot B_0(t) \\
B_7(t) &= Z_7(t) + q_{75}(t) \odot B_5(t)
\end{aligned} \tag{10}$$

where $Z_2(t) = e^{-\beta_1 t^\eta}$, $Z_4(t) = e^{-\theta t^\eta}$, $Z_6(t) = e^{-l_1 t^\eta}$ and $Z_7(t) = e^{-\beta_2 t^\eta}$

Taking the Laplace Transforms of relations (10) and simplifying for $B_0^*(s)$, omitting the argument 's' for brevity, we get

$$B_0^*(s) = \frac{Z_2^* q_{01}^* q_{12}^* [1 - q_{57}^* q_{75}^*] + q_{03}^* q_{34}^* \{Z_4^* [1 - q_{57}^* q_{75}^*] + q_{45}^* [Z_6^* q_{56}^* + Z_7^* q_{57}^*]\}}{[1 - q_{57}^* q_{75}^*] - [1 - q_{57}^* q_{75}^*] q_{20}^* q_{12}^* q_{01}^* - q_{03}^* q_{34}^* q_{45}^* q_{60}^* q_{56}^*} \tag{11}$$

Taking the Inverse Laplace Transform of (11) we can get the probability that the server will be busy at a particular epoch for known values of the parameters.

In the long run, the steady state probability that the server will be busy in the repair of a failed unit, is given by,

$$B_0 = \lim_{t \rightarrow \infty} B_0(t) = \lim_{s \rightarrow 0} s B_0^*(s) = \frac{\psi_0 p_{01} p_{12} [1 - p_{57} p_{75}] + p_{03} p_{34} \{\psi_4 [1 - p_{57} p_{75}] + p_{45} [\psi_6 p_{56} + p_{57} \psi_7]\}}{[\psi_0 + p_{01} \psi_1 + p_{01} \psi_2 + p_{03} \{\psi_3 + \psi_4\}] [1 - p_{57}] + [\psi_5 + p_{57} \psi_7 + p_{56} \psi_6] p_{03}} \tag{12}$$

The expected busy period of the server during time interval (0,t) is

$$\mu_b(t) = \int_0^t B_0(u) du \text{ so that } \mu_b^*(s) = \frac{B_0^*(s)}{s} \tag{13}$$

5. COST BENEFIT ANALYSIS

The net expected profit incurred by the system during time interval (0, t) is given by

$$C(t) = K_0\mu_{up}(t) - K_1\mu_b(t) \quad (14)$$

where K_0 is the revenue per unit up time by the system and K_1 is the amount paid to the server per unit time when the system is under repair. Here, we assume that the repair cost of original and degraded unit are same in par unit of time. The net expected profit incurred by the system per unit of time in steady state is given by

$$C = K_0A_0 - K_1B_0 \quad (15)$$

where A_0 and B_0 has been defined already in previous sections.

6. CASE STUDIES WITH DISCUSSIONS

- i. When shape parameter $\eta = 0.5$ then failure/preventive maintenance/arrival time of the server/replacement/transition rate/ repair time distributions reduce to:

$$h_i(t) = \frac{\alpha_i}{2\sqrt{t}} e^{-\alpha_i\sqrt{t}}, g(t) = \frac{\theta}{2\sqrt{t}} e^{-\theta\sqrt{t}}, k(t) = \frac{\gamma}{2\sqrt{t}} e^{-\gamma\sqrt{t}}, l(t) = \frac{l_1}{2\sqrt{t}} e^{-l_1\sqrt{t}}, m_i(t) = \frac{\tau_i}{2\sqrt{t}} e^{-\tau_i\sqrt{t}}$$

and $f_i(t) = \frac{\beta_i}{2\sqrt{t}} e^{-\beta_i\sqrt{t}}$; where $t \geq 0$ and $\eta, \alpha_i, \beta_i, \gamma, \theta, \tau_i, l_1 > 0$ (16)

So that the results given by (16) will be as follows:

$$\psi_0 = \frac{2}{(\alpha_1 + \tau_1)^2}, \psi_1 = \psi_3 = \frac{2}{\gamma_1^2}, \psi_2 = \frac{2}{\beta_1^2}, \psi_4 = \frac{2}{\theta^2}, \psi_5 = \frac{2}{(\alpha_2 + \tau_2)^2}, \psi_6 = \frac{2}{l_1^2} \text{ and } \psi_7 = \frac{2}{\beta_2^2} \quad (17)$$

- ii. When shape parameter $\eta = 1.0$ then failure/preventive maintenance/arrival time of the server/replacement/transition rate/ repair time distributions reduce to exponentials, then

$$h_i(t) = \alpha_i e^{-\alpha_i t}, g(t) = \theta e^{-\theta t}, k(t) = \gamma e^{-\gamma t}, l(t) = l_1 e^{-l_1 t}, m_i(t) = \tau_i e^{-\tau_i t} \text{ and } f_i(t) = \beta_i e^{-\beta_i t};$$

where $t \geq 0$ and $\eta, \alpha_i, \beta_i, \gamma, \theta, \tau_i, l_1 > 0$

(18)

So that the results given by (18) will be as follows:

$$\psi_0 = \frac{1}{(\alpha_1 + \tau_1)}, \psi_1 = \psi_3 = \frac{1}{\gamma_1}, \psi_2 = \frac{1}{\beta_1}, \psi_4 = \frac{1}{\theta}, \psi_5 = \frac{1}{(\alpha_2 + \tau_2)}, \psi_6 = \frac{1}{l_1} \text{ and } \psi_7 = \frac{1}{\beta_2}$$
(19)

iii. When shape parameter $\eta = 2.0$ then failure/preventive maintenance/arrival time of the server/replacement/transition rate/ repair time distributions reduce to Rayleigh having the pdf-

$$h_i(t) = 2\alpha_i t e^{-\alpha_i t^2}, g(t) = 2\theta t e^{-\theta t^2}, k(t) = 2\gamma t e^{-\gamma t^2}, l(t) = 2l_1 t e^{-l_1 t^2}, m_i(t) = 2\tau_i t e^{-\tau_i t^2} \text{ and } f_i(t) = 2\beta_i t e^{-\beta_i t^2}$$

where $t \geq 0$ and $\eta, \alpha_i, \beta_i, \gamma, \theta, \tau_i, l_1 > 0$

(20)

So that the results given by (20) will be as follows:

$$\psi_0 = \frac{1}{2} \sqrt{\frac{\pi}{(\alpha_1 + \tau_1)}}, \psi_1 = \psi_3 = \frac{1}{2} \sqrt{\frac{\pi}{\gamma_1}}, \psi_2 = \frac{1}{2} \sqrt{\frac{\pi}{\beta_1}}, \psi_4 = \frac{1}{2} \sqrt{\frac{\pi}{\theta}}, \psi_5 = \frac{1}{2} \sqrt{\frac{\pi}{(\alpha_2 + \tau_2)}}, \psi_6 = \frac{1}{2} \sqrt{\frac{\pi}{l_1}} \text{ and}$$

$$\psi_7 = \frac{1}{2} \sqrt{\frac{\pi}{\beta_2}}$$
(21)

By using the MATLAB software, we have obtained the values of reliability characteristics (MTSF, availability and net expected steady state profit) of the system for above three cases for different values of the parameters in Tables 1, 3 and 5. In table 1, MTSF of above three cases are shown with respect to α_1 for two different values of τ_1 (0.5, 0.6). The values of other parameters are kept fixed as $(\alpha_{2=0.8}, \tau_2 = 1.2, l_1 = 1.4, \beta_1 = 1.7, \beta_2 = 1.6, \theta = 0.5 \text{ and } \gamma = 5)$.

Similarly, in table 3 the values for availability are shown for the same values of all the parameters as in the case of MTSF.

Table 1 Values of MTSF for different values of τ_1 and η with respect to α_1

α_1	$\eta = 0.5$		$\eta = 1.0$		$\eta = 2.0$	
	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$
0.10	41.3223	25.5102	4.5455	3.5714	1.8894	1.6748
0.11	37.8072	23.7812	4.3478	3.4483	1.8479	1.6457
0.12	34.7222	22.2222	4.1667	3.3333	1.8090	1.6180
0.13	32.0000	20.8117	4.0000	3.2258	1.7725	1.5917
0.14	29.5858	19.5313	3.8462	3.1250	1.7380	1.5666
0.15	27.4348	18.3655	3.7037	3.0303	1.7055	1.5427
0.16	25.5102	17.3010	3.5714	2.9412	1.6748	1.5199
0.17	23.7812	16.3265	3.4483	2.8571	1.6457	1.4980
0.18	22.2222	15.4321	3.3333	2.7778	1.6180	1.4770
0.19	20.8117	14.6092	3.2258	2.7027	1.5917	1.4569
0.20	19.5313	13.8504	3.1250	2.6316	1.5666	1.4376

Similarly, in table 5 the values of profit function are shown for the same values of all the parameters as in the case of MTSF and availability. The values of K_0 and K_1 for profit function are assumed as ($K_0 = 1200$, $K_1 = 600$).

From table 1, we observed that the MTSF decreases more rapidly as the shape parameter η increases. Also with the increase of the value α_1 , MTSF decreases uniformly irrespective of the values of η and τ_1 . Moreover, as the value of τ_1 increases from 0.12 to 0.18, we observe that the MTSF decreases.

Table 2 Values of MTSF for different values of τ_1 and η with respect to α_1

α_1	$\eta = 0.9$		$\eta = 1.0$		$\eta = 1.1$	
	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$
0.10	5.6589	4.3287	4.5455	3.5714	3.8220	3.0695
0.11	5.3862	4.1632	4.3478	3.4483	3.6706	2.9732
0.12	5.1374	4.0093	4.1667	3.3333	3.5313	2.8829
0.13	4.9096	3.8659	4.0000	3.2258	3.4026	2.7982
0.14	4.7003	3.7319	3.8462	3.1250	3.2834	2.7186
0.15	4.5072	3.6064	3.7037	3.0303	3.1727	2.6436
0.16	4.3287	3.4888	3.5714	2.9412	3.0695	2.5729
0.17	4.1632	3.3782	3.4483	2.8571	2.9732	2.5059
0.18	4.0093	3.2741	3.3333	2.7778	2.8829	2.4426
0.19	3.8659	3.1759	3.2258	2.7027	2.7982	2.3825
0.20	3.7319	3.0832	3.1250	2.6316	2.7186	2.3254

Table 3 Values of Availability for different values of τ_1 and η with respect to α_1

α_1	$\eta = 0.5$		$\eta = 1.0$		$\eta = 2.0$	
	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$
0.10	0.8917	0.8617	0.7010	0.6810	0.5469	0.5373
0.11	0.8784	0.8472	0.6870	0.6679	0.5401	0.5313
0.12	0.8649	0.8325	0.6738	0.6556	0.5339	0.5257
0.13	0.8511	0.8177	0.6613	0.6440	0.5283	0.5205
0.14	0.8371	0.8029	0.6496	0.6330	0.5230	0.5158
0.15	0.8230	0.7881	0.6385	0.6227	0.5182	0.5113
0.16	0.8088	0.7734	0.6280	0.6128	0.5138	0.5072
0.17	0.7945	0.7587	0.6180	0.6035	0.5096	0.5033
0.18	0.7802	0.7441	0.6086	0.5946	0.5057	0.4997
0.19	0.7659	0.7297	0.5996	0.5862	0.5021	0.4963
0.20	0.7517	0.7154	0.5910	0.5781	0.4987	0.4931

Further, from tables 3 and 5, we observe that availability and profit function of the system model are also follows the same trend as MTSF. Also with the increase of the value α_1 , availability and profit decreases uniformly irrespective of the values of η and τ_1 . Moreover, as the value of τ_1 increases from 0.12 to 0.18, we observe that the availability and profit decreases. Here one thing is important that if we increase shape parameter η and decrease τ_1 then system is more profitable in comparison to the situation when η and τ_1 both increases.

Table 4 Values of Availability for different values of τ_1 and η with respect to α_1

α_1	$\eta = 0.9$		$\eta = 1.0$		$\eta = 1.1$	
	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$
0.10	0.7314	0.7089	0.7010	0.6810	0.6747	0.6567
0.11	0.7165	0.6950	0.6870	0.6679	0.6616	0.6446
0.12	0.7024	0.6817	0.6738	0.6556	0.6493	0.6332
0.13	0.6891	0.6691	0.6613	0.6440	0.6378	0.6226
0.14	0.6763	0.6572	0.6496	0.6330	0.6270	0.6125
0.15	0.6642	0.6458	0.6385	0.6227	0.6168	0.6031
0.16	0.6527	0.6349	0.6280	0.6128	0.6073	0.5942
0.17	0.6417	0.6245	0.6180	0.6035	0.5983	0.5857
0.18	0.6312	0.6147	0.6086	0.5946	0.5897	0.5778
0.19	0.6212	0.6052	0.5996	0.5862	0.5817	0.5702
0.20	0.6116	0.5962	0.5910	0.5781	0.5740	0.5630

Table 5 Values of Profit for different values of τ_1 and η with respect to α_1

α_1	$\eta = 0.5$		$\eta = 1.0$		$\eta = 2.0$	
	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$
0.10	990.7216	924.6307	604.2070	538.0979	310.6588	257.4500
0.11	966.5537	898.0719	581.4166	517.9208	304.5149	253.6795
0.12	941.8709	871.3215	559.9886	498.9173	298.8430	250.1478
0.13	916.7710	844.4643	539.8044	480.9880	293.5810	246.8277
0.14	891.3473	817.5793	520.7588	464.0443	288.6777	243.6963
0.15	865.6881	790.7393	502.7581	448.0072	284.0909	240.7339
0.16	839.8768	764.0108	485.7186	432.8057	279.7853	237.9241
0.17	813.9909	737.4542	469.5653	418.3762	275.7307	235.2525
0.18	788.1023	711.1243	454.2308	404.6614	271.9018	232.7067
0.19	762.2775	685.0696	439.6544	391.6094	268.2765	230.2758
0.20	736.5768	659.3336	425.7813	379.1733	264.8360	227.9503

Table 6 Values of Profit for different values of τ_1 and η with respect to α_1

α_1	$\eta = 0.9$		$\eta = 1.0$		$\eta = 1.1$	
	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$	$\tau_1 = 0.12$	$\tau_1 = 0.18$
0.10	664.9017	596.5038	604.2070	538.0979	552.0869	488.2269
0.11	640.0125	573.9096	581.4166	517.9208	531.5169	470.4229
0.12	616.4046	552.4823	559.9886	498.9173	512.3054	453.7411
0.13	593.9899	532.1394	539.8044	480.9880	494.3167	438.0751
0.14	572.6876	512.8055	520.7588	464.0443	477.4331	423.3319
0.15	552.4232	494.4116	502.7581	448.0072	461.5518	409.4297
0.16	533.1282	476.8946	485.7186	432.8057	446.5829	396.2962
0.17	514.7393	460.1965	469.5653	418.3762	432.4470	383.8674
0.18	497.1985	444.2644	454.2308	404.6614	419.0740	372.0862
0.19	480.4520	429.0493	439.6544	391.6094	406.4014	360.9018
0.20	464.4504	414.5062	425.7813	379.1733	394.3738	350.2685

CONCLUSIONS

For a more concrete study of the single-unit system under preventive maintenance and Weibull distributed failure and repair activities, we obtain numerical results for MTSF, availability and profit function in tables 2,4, and 6 respectively, for different values of τ_1 (0.12, 0.18) and η (0.9, 1.0, 1.1) against the failure parameter α_1 , while the others parameters are kept fixed as: $\alpha_{2=0.8}, \tau_2 = 1.2, l_1 = 1.4$,

$\beta_1 = 1.7, \beta_2 = 1.6, \theta = 0.5, \gamma = 5, K_0 = 1200, K_1 = 600$. From Tables 2, 4 and 6 respectively, we observe that MTSF, availability and profit function decreases with the increase in η, α_1 and τ_1 .

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