

PSEUDO-ORTHOGONALIZATION OF MEMORY PATTERNS FOR COMPLEX-VALUED AND QUATERNIONIC ASSOCIATIVE MEMORIES

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Abstract

Hebbian learning rule is well known as a memory storing scheme for associative memory models. This scheme is simple and fast, however, its performance gets decreased when memory patterns are not orthogonal each other. Pseudo-orthogonalization is a decorrelating method for memory patterns which uses XNOR masking between the memory patterns and randomly generated patterns. By a combination of this method and Hebbian learning rule, storage capacity of associative memory concerning non-orthogonal patterns is improved without high computational cost. The memory patterns can also be retrieved based on a simulated annealing method by using an external stimulus pattern. By utilizing complex numbers and quaternions, we can extend the pseudo-orthogonalization for complex-valued and quaternionic Hopfield neural networks. In this paper, the extended pseudo-orthogonalization methods for associative memories based on complex numbers and quaternions are examined from the viewpoint of correlations in memory patterns. We show that the method has stable recall performance on highly correlated memory patterns compared to the conventional real-valued method.

Keywords: Hopfield neural network, pseudo-orthogonalization, complex numbers, quaternions

1 Introduction

Hebbian learning rule is a well-known scheme for embedding patterns onto associative memories, such as Hopfield neural networks [1]. This scheme is simple and straightforward, however, it has a crucial issue for the embedding patterns; the patterns should be orthogonal to each other. On embedding correlated patterns by this scheme, the storage performance of the network is significantly decreased. Thus, many researches for storing correlated patterns direct to orthogonalization of these patterns,

such as pseudo-inverse matrix method [2] and iterative learning scheme [3]. Though these methods enable all the correlated patterns to be stable local minima in the network, their computational costs grow with respect to the network size and the number of patterns to be embedded.

A novel method has been proposed that enables correlated patterns onto associative memories with low computational cost [4]. The method, which is called pseudo-orthogonalization, first prepares a random pattern (mask pattern) of which length is the same as that of a memory pattern, and element-

wise exclusive not-or (XNOR) operation is applied between the random pattern and the memory pattern. The pattern to be embedded in the network is a concatenation of XNORed pattern (masked pattern) and the corresponding random pattern. These pseudo-orthogonalized patterns have low correlation, thus, they can be embedded by using Hebbian learning rule without a degradation of storage capacity. The length of these patterns become double, however, the embedding process is simple and requires rather low computational cost.

Recently, the applications of complex or hypercomplex number systems to neural networks have been studied extensively [5, 6]. The complex-valued or quaternionic extensions would be suitable for the pseudo-orthogonalization scheme; the pair of a pattern can be naturally embedded by utilizing imaginary part(s). The pseudo-orthogonalization method has been extended based on complex numbers and quaternions [7]. In previous study, the numerical simulation results show that the proposed method can store the patterns better than the conventional (real-valued) method from the viewpoint of the loading rate, which is defined as the ratio of the number of embedded memory patterns to the number of neurons in the network. However, the performance for embedding correlated patterns has not been investigated in details. In this paper, the performance for the extended pseudo-orthogonalization method is examined from the view point of correlations in memory patterns. We show that the extended method has stable recall performance on highly correlated memory patterns compared to the conventional real-valued method does.

2 Preliminaries

In this Section, we explain Hebbian learning rule and the network dynamics for real-valued, complex-valued, and quaternionic Hopfield neural networks.

2.1 Real-Valued Hopfield Neural Network

Let $(\xi_1^\mu, \dots, \xi_N^\mu)$, $\xi_m^\mu \in \{+1, -1\}$ be the μ -th learning pattern. Hebbian learning rule for real-valued Hopfield neural network (RHNN) is represented as

$$w_{mn} = \frac{1}{N} \sum_{\mu=1}^P \xi_m^\mu \xi_n^\mu, \quad (1)$$

where w_{mn} is a synaptic weight between m -th and n -th neurons, which satisfies the conditions $w_{mm} = 0$ and $w_{mn} = w_{nm}$ for all m and n , and P is the number of the learning patterns. The dynamics of the network is given as

$$x_m(t+1) = \text{sgn} \left(\sum_{n=1}^N w_{mn} x_n(t) \right), \quad (2)$$

where $x_m(t) \in \{+1, -1\}$ denotes the output of the m -th neuron at the time step t and N is the total number of neurons in the network. The function $\text{sgn}(\cdot)$ is an activation function which is defined by $\text{sgn}(u) = 1$ when $u \geq 0$, and $\text{sgn}(u) = -1$ when $u < 0$.

2.2 Complex-Valued Hopfield Neural Network

In complex-valued Hopfield neural network (CHNN), the inputs, output, and synaptic weights are encoded by complex values [8, 6, 9]. The m -th element of a μ -th complex-valued learning pattern is given as $\xi_m^\mu = \xi_m^{\mu(e)} + \xi_m^{\mu(i)}i$, where $\xi_m^{\mu(e)}, \xi_m^{\mu(i)} \in \{+1, -1\}$. Hebbian learning rule for CHNN is defined as

$$w_{mn} = \frac{1}{2N} \sum_{\mu=1}^P \xi_m^\mu \xi_n^{\mu*}, \quad (3)$$

where synaptic weights satisfy the conditions $w_{mn} \geq 0$ and $w_{mn} = w_{nm}^*$. Here, the asterisk denotes the complex conjugation. The dynamics of the network is given as follows

$$x_m(t+1) = \text{csgn} \left(\sum_{n=1}^N w_{mn} x_n(t) \right). \quad (4)$$

The function $\text{csgn}(\cdot)$ is an activation function for complex-valued neurons which is defined by $\text{csgn}(s) = \text{sgn}(s^{(e)}) + \text{sgn}(s^{(i)})i$.

2.3 Quaternionic Hopfield Neural Network

Quaternions [10] are a class of hypercomplex numbers that consist of a real number and three

kinds of imaginary number, i , j , and k . Formally, a quaternion is defined as a vector in a four-dimensional vector space

$$x = x^{(e)} + x^{(i)}i + x^{(j)}j + x^{(k)}k, \quad (5)$$

where $x^{(e)}, x^{(i)}, x^{(j)}$, and $x^{(k)}$ are real numbers. Quaternion bases satisfy the following identities: $i^2 = j^2 = k^2 = ijk = -1$. Quaternions are also written using 4-tuple or 2-tuple notations as follows

$$x = (x^{(e)}, x^{(i)}, x^{(j)}, x^{(k)}) = (x^{(e)}, x), \quad (6)$$

where $x = (x^{(i)}, x^{(j)}, x^{(k)})$. In this representation $x^{(e)}$ is the scalar part of x , and x forms the vector part. The quaternion conjugate is defined as

$$x^* = (x^{(e)}, -x) = x^{(e)} - x^{(i)}i - x^{(j)}j - x^{(k)}k. \quad (7)$$

We define the operation between quaternions, $p = (p^{(e)}, p) = (p^{(e)}, p^{(i)}, p^{(j)}, p^{(k)})$ and $q = (q^{(e)}, q) = (q^{(e)}, q^{(i)}, q^{(j)}, q^{(k)})$. The addition and subtraction of quaternions are defined in the same manner as that of complex numbers or vectors by

$$\begin{aligned} p \pm q &= (p^{(e)} \pm q^{(e)}, p \pm q) \\ &= (p^{(e)} \pm q^{(e)}, p^{(i)} \pm q^{(i)}, p^{(j)} \pm q^{(j)}, p^{(k)} \pm q^{(k)}). \end{aligned} \quad (8)$$

(9)

With regard to the multiplication, the product of p and q is represented as follows

$$pq = (p^{(e)}q^{(e)} - p \cdot q, p^{(e)}q + q^{(e)}p + p \times q). \quad (10)$$

In Quaternionic Hopfield Neural Network (QHNN), all neuronal parameters in the network are encoded by quaternions [11, 12, 13]. Let the m -th element of a μ -th quaternionic learning pattern be $\xi_m^\mu = \xi_m^{\mu(e)} + \xi_m^{\mu(i)}i + \xi_m^{\mu(j)}j + \xi_m^{\mu(k)}k$ where $\xi_m^{\mu(e)}, \xi_m^{\mu(i)}, \xi_m^{\mu(j)}, \xi_m^{\mu(k)} \in \{+1, -1\}$. Hebbian learning rule for QHNN is represented as

$$w_{mn} = \frac{1}{4N} \sum_{\mu=1}^P \xi_m^\mu \xi_n^{\mu*}, \quad (11)$$

where synaptic weights satisfy the conditions $w_{mm} \geq 0$ and $w_{mn} = w_{nm}^*$. The dynamics of the network is given as follows

$$x_m(t+1) = \text{qsgn} \left(\sum_{n=1}^N w_{mn} x_n(t) \right). \quad (12)$$

The function $\text{qsgn}(\cdot)$ is an activation function for quaternionic neurons which is defined by $\text{qsgn}(s) = \text{sgn}(s^{(e)}) + \text{sgn}(s^{(i)})i + \text{sgn}(s^{(j)})j + \text{sgn}(s^{(k)})k$.

3 Pseudo-Orthogonalization based on Complex Numbers and Quaternions

The purpose of the pseudo-orthogonalization method is to randomize memory patterns, so that they can be stored by Hebbian learning [4]. We present a pseudo-orthogonalization method based on complex numbers and quaternions in this Section [7].

First, let us recapitulate the basic pseudo-orthogonalization method. In the pseudo-orthogonalization, the memory patterns are masked by using random patterns as shown in Fig. 1. The pseudo-orthogonalized patterns which are the concatenation of the random patterns and the masked patterns are embedded to the network. Fig. 2 shows the generation method for the masked patterns. The masked patterns are obtained by element-wise multiplication of the memory patterns and random patterns. Thus, the original patterns can be reconstructed from the masked patterns and the random patterns as shown in Fig. 3.

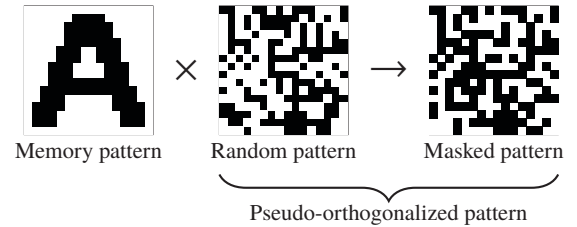


Figure 1. Schematic of pseudo-orthogonalization.

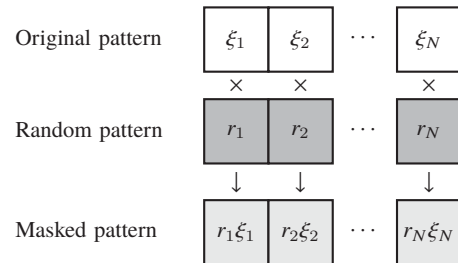


Figure 2. Generation of masked patterns.

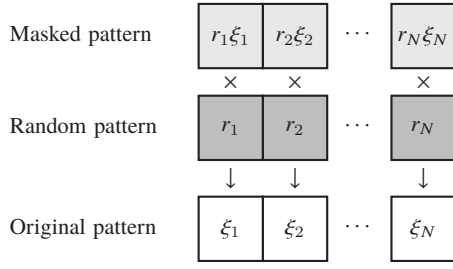


Figure 3. Reconstruction of original patterns.

Let (ξ_1, \dots, ξ_N) where $\xi_m \in \{+1, -1\}$ be the original memory pattern and (r_1, \dots, r_N) where $r_m \in \{+1, -1\}$ be a random pattern corresponding to the original pattern. The m -th element of the real-valued pseudo-orthogonalized pattern is generated by

$$\eta_m^r = \begin{cases} r_n, & \text{if } m = 2n - 1 \\ r_n \xi_n, & \text{if } m = 2n \end{cases}. \quad (13)$$

This is the concatenation of the random pattern and the XNOR masked pattern between the original memory pattern and the random pattern (see Fig. 4(a)). Thus, the element η_m takes either $+1$ or -1 and the length of the pseudo-orthogonalized pattern, denoted by N' , becomes twice as the original one, i.e. $N' = 2N$.

Next, we show a method of pseudo-orthogonalization by using complex numbers. The random pattern is assigned to the real part of the complex values and the masked pattern is assigned to the imaginary part in the method utilizing complex numbers. Thus, the m -th element of the complex-valued pseudo-orthogonalized pattern is defined by

$$\eta_m^c = r_m + r_m \xi_m i. \quad (14)$$

Therefore, the original patterns can be reconstructed by

$$\xi_m = \eta_m^{c(r)} \eta_m^{c(i)}. \quad (15)$$

In this case, the length of the original and randomized patterns are same, i.e. $N' = N$ (see Fig. 4(b)).

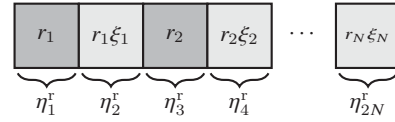
Finally, a method of pseudo-orthogonalization by using quaternions is described. The m -th element of the quaternionic pseudo-orthogonalized pattern is defined by

$$\eta_m^q = r_{2m-1} + r_{2m-1} \xi_{2m-1} i + r_{2m} j + r_{2m} \xi_{2m} k. \quad (16)$$

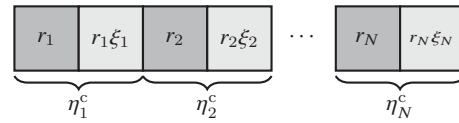
That is, odd numbered elements in random patterns are assigned to the real part and the rest is assigned to the imaginary part j . Also, odd numbered elements in masked patterns are assigned to the imaginary part i and the rest is assigned to the imaginary part k . Therefore, the original patterns can be reconstructed by

$$\xi_m = \begin{cases} \eta_n^{q(e)} \eta_n^{q(i)}, & \text{if } m = 2n - 1 \\ \eta_n^{q(j)} \eta_n^{q(k)}, & \text{if } m = 2n \end{cases}. \quad (17)$$

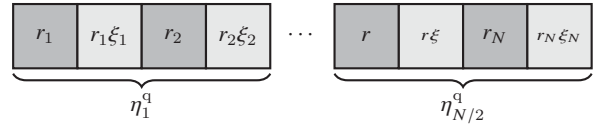
$N' = N/2$ is obtained in the quaternionic pseudo-orthogonalization method (see Fig. 4(c)).



(a) Real-valued pseudo-orthogonalization



(b) Complex-valued pseudo-orthogonalization



(c) Quaternionic pseudo-orthogonalization

Figure 4. Pseudo-orthogonalized patterns generated from random patterns and masked patterns utilizing (a) real numbers, (b) complex numbers, and (c) quaternions.

4 Retrieval Dynamics for Pseudo-Orthogonalized Patterns

In this Section, we describe the retrieval dynamics for pseudo-orthogonalization.

The recall of the stored patterns in pseudo-orthogonalization method needs the random mask patterns that is used in the pseudo-orthogonalization process. However, there is no information about the random mask patterns in the retrieval process, so that the initial state of the network cannot be determined for a cue signal pattern.

Therefore, the network dynamics are extended for retrieving memory patterns without random patterns by using a simulated annealing method [4].

The retrieval dynamics for CHNN has been formulated as following equations [7]

$$h_m(t) = \sum_{n=1}^N w_{mn} x_n(t) + s z_m \left(x_m^{(i)}(t) + x_m^{(e)}(t) i \right), \quad (18)$$

$$\text{Prob}(x_m^{(*)} = 1) = \frac{1}{1 + \exp(-\beta(t) h_m^{(*)}(t))}, \quad (19)$$

$$(*) \in \{(e), (i)\},$$

where $z_m \in \{+1, -1\}$ denotes the m -th element in an input cue pattern, s is the strength of the input cue signal part, and $\beta(t+1) = \gamma\beta(t)$ is the inverse temperature parameter that increases with time step t . γ with ($\gamma > 1$) is the increase rate for β . The states of the neurons $x_m(t)$ are initialized randomly at $t = 0$ and they evolve stochastically by using Eq. (19). Here, the real part and the imaginary part of the internal state $h_m(t)$ are separately updated.

Similarly, the retrieval dynamics for QHNN is defined as follows

$$h_m(t) = \sum_{n=1}^N w_{mn} x_n(t) + s z_{2m-1} \left(x_m^{(i)}(t) + x_m^{(e)}(t) i \right) + s z_{2m} \left(x_m^{(k)}(t) j + x_m^{(j)}(t) k \right), \quad (20)$$

$$\text{Prob}(x_m^{(*)} = 1) = \frac{1}{1 + \exp(-\beta(t) h_m^{(*)}(t))}, \quad (21)$$

$$(*) \in \{(e), (i), (j), (k)\}.$$

5 Experiments

In this Section, we investigate the retrieval performance in CHNN and QHNN by using the proposed pseudo-orthogonalization method. For this experiment, random patterns are used as the original memory pattern which are obtained by using the following probability

$$\text{Prob}(\xi_m = \pm 1) = (1 \pm \sqrt{b} \zeta_m) / 2, \quad (22)$$

where ζ_m is the m -th element in a random pattern generated according to the uniform probability

$$\text{Prob}(\zeta_m = \pm 1) = 1/2. \quad (23)$$

b is a correlation parameter for the random patterns which satisfies $E[\text{Corr}(\xi_m, \zeta_m)] = b$.

In order to evaluate the stability and retrieval performance, we define the overlap between the current network state $x_i(t)$ and the memory pattern ξ_m . For RHNN, CHNN, and QHNN, we define the m_r, m_c and m_q , respectively, by

$$m_r(t) = \frac{1}{N} \sum_{m=1}^N \xi_m x_m(t), \quad (24)$$

$$m_c(t) = \frac{1}{2N} \sum_{m=1}^N \left(\xi_m^{(e)} x_m^{(e)}(t) + \xi_m^{(i)} x_m^{(i)}(t) \right), \quad (25)$$

$$m_q(t) = \frac{1}{4N} \sum_{m=1}^N \left(\xi_m^{(e)} x_m^{(e)}(t) + \xi_m^{(i)} x_m^{(i)}(t) + \xi_m^{(j)} x_m^{(j)}(t) + \xi_m^{(k)} x_m^{(k)}(t) \right). \quad (26)$$

First, we investigated how the correlation in the original memory patterns affects the retrieval performance in RHNNs, CHNNs, and QHNNs. Figure 5 shows the retrieval success rates with changing the correlation parameter for original memory patterns. In this experiments, the length of the original memory patterns was set to 1000, so that the number of neurons in RHNNs, CHNNs, and QHNNs were 2000, 1000, and 500, respectively. The loading rate, which is the ratio of the number of the stored patterns P and the number of neurons N , was fixed to 0.13, and the correlation parameter was set to 0.1 from 0.0 to 0.5 with a step of 0.05. The parameters for the recall process were set as $s = 0.9$, $\gamma = 1.002$, $\beta(0) = 1.0$. We obtained the retrieval pattern after 1000 iterations of updates and 100 trials were conducted. For each experiment, a random pattern is used as an initial configuration of the network, and cue of the embedded original patterns is used as an external cue signal. The recall was considered to be successful when the overlap between the retrieved pattern and its true pattern achieved 0.95. From the Figure 4, we find that the retrieval success rates are decreased with the increase of the correlation for the original memory patterns in all types of networks. However, the success rates in CHNNs and QHNNs are decreased slower than those in RHNNs.

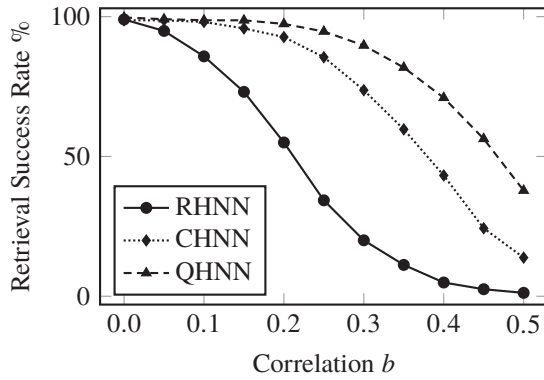


Figure 5. Retrieval success rates with changing the correlation in the original memory patterns.

Next, we show the dependence of the critical loading rate on the correlation in the memory patterns. The critical loading rate is defined as the loading rate when the overlaps of the retrieved pattern and its original pattern is lower than a threshold. Figure 6 shows the critical loading rates against the correlations. The threshold was set to 0.95 in this result. From this Figure, we find that the critical loading rates in CHNNs and QHNNs are also decreased slower than those in RHNNs.

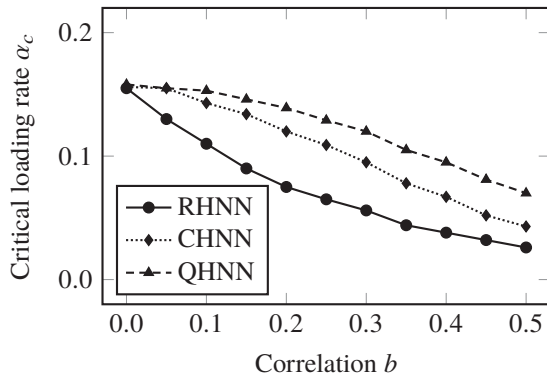


Figure 6. Critical loading rates with changing the correlation in the original memory patterns.

6 Discussion

We discuss reasons why the retrieval performance in pseudo-orthogonalization is maintained by utilizing complex numbers and quaternions. We first examined the storage capacity of pseudo-orthogonalized patterns against correlations in the original memory patterns. Figure 7 shows the changes of critical loading rates with increasing the correlations for the memory patterns. We performed this experiment under the same conditions

in the previous Section. The overlaps for calculating the critical loading rate were obtained by averaging 100 trials in 1000 updates. In each of these trials, an initial configuration are set to one of the memory patterns, by using Eqs. (2),(4), and (12). From the results, the critical loading rates for RHNNs were decreased with the increase of the correlations, however the critical loading rates for CHNNs and QHNNs were not changed under the same conditions. The memory patterns become more unstable with the increase of the correlation for the original memory patterns in RHNNs. In contrast, the memory patterns in CHNNs and QHNNs are stable when the correlation is increased. The complex-valued and quaternionic pseudo-orthogonalization methods can stabilize highly correlated memory patterns better than conventional real-valued method does. Therefore, the retrieval performance of CHNNs and QHNNs are maintained even if the correlation in the memory patterns is increased.

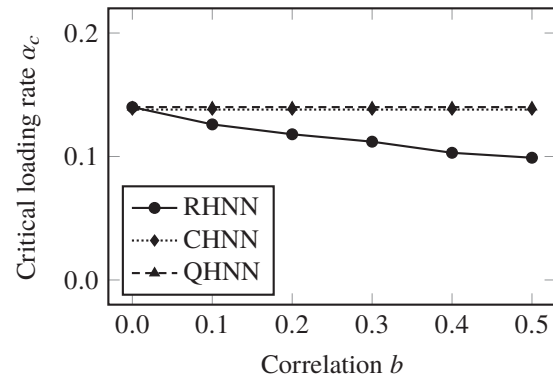


Figure 7. Effect of the correlation of the original memory patterns on the stability of stored patterns.

7 Conclusion

In this paper, we have investigated the stability and retrieval performances for the pseudo-orthogonalization from the viewpoint of correlations in memory patterns. The extended pseudo-orthogonalization method based on complex numbers and quaternions has been evaluated by changing the correlation of memory patterns.

The experimental results show that the memory patterns tend to be more unstable with the increase of the correlation in the original memory patterns in conventional real-valued pseudo-

orthogonalization method. On the other hand, the memory patterns by using the extended pseudo-orthogonalization are stable even if the correlation is increased. Thus, the extended method can stabilize highly correlated memory patterns better than conventional real-valued one. On retrieving the stored patterns from a cue input pattern, the performance of the extended pseudo-orthogonalization method is maintained compared to the real-valued method under the condition of high loading rate and strong correlation in the memory patterns. This is because the memory patterns stored by the proposed method are stable even if the correlation in memory patterns is increased.

Parameter dependencies for the storing and retrieval performances, such as the strength of input stimuli on the retrieval stage, should be explored in details. Also, it is important to investigate the structure on basins of attractors for the complex-valued and quaternionic networks, as compared to the real-valued networks. These remain for our future work.

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