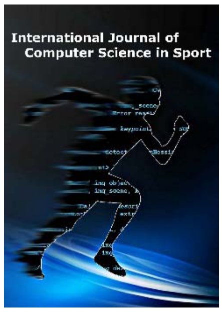
International Journal of Computer Science in Sport

Volume 17, Issue 2, 2018

Journal homepage: http://iacss.org/index.php?id=30

sciendo

DOI: 10.2478/ijcss-2018-0010



Geometric simplification of a new penalty area for football

Morales, C. A.

Facultad de Ingeniería, Universidad Peruana de Ciencias Aplicadas, Perú

Abstract

A geometric simplification of a recent proposal for a new geometry of the penalty area for football (soccer) is presented. A simplified line is necessary because the fully mathematical curve previously proposed can be difficult to implement in the real world of football, which has thousands of tournaments at all –economic and generational– levels of the sport. The idea behind the proposal is that the game and its fairness can be improved if the penalty area is drawn according to mathematics or a measure of actual scoring chance.

KEYWORDS: PENALTY AREA, FOOTBALL, SOCCER, MATHEMATICS, DIVING

Introduction

Football international tournaments watched by hundreds of millions such as the FIFA World Cup, Copa América and UEFA Euro, commonly have plays in and around the penalty area (PA) that spark worldwide debate, and one of the most debated type of controversy is related to a granted −or not− penalty kick. These controversial plays, most often than not, are initiated by a vague or subtle dive (simulation), an unclear tackle or a blatant and clear dive (Morales, 2016; Morris & Lewis, 2010); in general, any mistake by the referee inside the penalty area provokes such a big debate and a related sense of unfairness. Now, clear dives that result in a most unfair penalty kick must be greatly reduced in the sport if its popularity, quality and marketability are to be maintained or improved (David et al., 2011; Morales, 2016), and the same should be said about clear referee mistakes in and around the PA.

Recently, a new penalty area was proposed based on mathematics and behavioral science (psychology) with the goal of reducing diving in football; that is, considering the fact that a player has an increased tendency to deceive or cheat when at a low-scoring-chance position (David et al., 2011; Morales, 2016), a measure of scoring chance was presented: inversely proportional to distance to goal and directly proportional to angle towards goal (Morales, 2016). This theoretical proposal for measuring mathematically scoring probability according to position has been lately and independently backed by *experiments* or analysis of real data from professional level football games where *scoring-chance density maps* are presented (Caley, 2015; Mackay, 2016; Scisports, 2016). Moreover, the proposed PA not only would reduce the problem of cheating or diving by attackers but also the more general one of unfair outcomes of referee mistakes close to goal (Morales, 2016).

However, the proposal needs to be improved or simplified geometrically because it will not be easy and feasible, in practice or on the field to paint the fully mathematical curve. This is in turn because in Cartesian coordinates the proposed curve involves the functions *tangent* and *square root* and even in (inverse) polar coordinates we have *arccosine* and a *tangent* functions (Morales, 2016); these functions are basic and simple in Analytical Geometry but very difficult to paint on the field or pitch even in a much resourceful tournament as the World Cup.

In this work, that simplification or improvement of the previously proposed PA is presented. The perimeter of the new PA has been engineered or approximated by means of a couple of ready and easy *circular arcs*. In addition, the problem or impracticality of the mathematical boundary starting at the post (Morales, 2016) is solved. It is emphasized that the idea behind the proposal is that football and its fairness can be enhanced if the penalty area is painted according to mathematics or a measure of scoring capacity.

Method

A new boundary or periphery for the penalty box in association football has been lately proposed (Morales, 2016), it is shown herein in Fig. 1, where the position of the goal posts is at both ends of the curve or perimeter (the current PA is also depicted for comparison, dashed line). The proposal has also been presented outside scientific realms, and while presenting it in public television and radio, an explanation that was missed in the academic article arose; it is explained or written now and herein: one of the main advantages or difference of the new penalty area (PA) is that as an attacking football or point moves along the limits of the current or *old* PA, the probability or chance of scoring varies quite wildly, whereas along the new perimeter the possibility of scoring is mathematically the very same. This notable and mathematical advantage makes it possible –it is argued– much fairer outcomes of different situations on the pitch close to the goal, where it is more critical. The fact that a shot ball has very different chances of being scored along a rectangular PA line is very intuitive or clear to see: at the intersection of the current PA border and the goal line the probability or potential is *zero* (at least according to our mathematical model) and right in front of the middle of the goal it is maximum. Nonetheless, it can also be shown through observation of the 3D plot of the scoring potential function or its contour plot, which are figures 3 and 4 in the original article (Morales, 2016); note that the proposed or new PA is, of course, one of the contours. Moreover, the fact that probability of scoring varies on the 2 dimensions (football field) according to those plots or our mathematical results, has been lately and independently proven by analysis of actual professional football data, or real scoring-chance *density maps* (Caley, 2015; Charles, 2014; Mackay, 2016; Scisports, 2016; Sumpter, 2017a); also, other analysis have recently resulted in similar scoring potential functions (Knutson, 2016; Sumpter, 2017b).

We simplify the new penalty area markings presented lately in a sport science journal; the method or simplification is analytic; in fact, that proposal was theoretical as well, whose result is a curve or perimeter written mathematically in *closed form*. In this regard, the functions involved in the 2-variable (x,y) expression are indeed simple and common, at least in geometry or trigonometry: the tangent function and distance to origin (root of $x^2 + y^2$) (Morales, 2016). Nevertheless, there is a problem in that the absolute mathematical marking would be difficult to implement on a real football field; in fact, the world cup, continental championships and top leagues are a very small fraction of all tournaments, which are very diverse –for example– economically and generationally. Therefore, the pure mathematical proposal –which is in part also based on human behavior science– must be now *engineered*.

The proposition is to approximate the mathematical curve of the new PA, Fig. 1, simply by means of a full quarter circle plus another circular arc, whose centers and radii are different. As a matter of fact, the first analysis is to find the “major and minor axes” of the PA boundary, where these terms appear if one considers that the PA line resembles an ellipse. Because the PA, or any PA in similar sports, is symmetric, the *minor axis* is easy to define, it is collinear with the x axis and its value was found or established previously (Morales, 2016): 21.96 m ≈ 22.0 m.

The *major axis* requires more work: to find the point (x,y) where the curve has zero slope. Thus, we derive implicitly equation 9 by Morales (2016) to obtain

$$\frac{0.015\,(x + y\,y')}{\sqrt{x^2 + y^2}} = \frac{b\left(y^2 - x^2 - (b/2)^2 - 2\,x\,y\,y'\right)}{\left(x^2 + y^2 - (b/2)^2\right)^2 + (bx)^2} \tag{1}$$

where y' is the derivative of y with respect to x, and b is the distance between posts, 7.32 m. Now, the condition is zero slope or $y' = 0$, so

$$\frac{7.32\,(y^2 - x^2 - 13.4)\sqrt{x^2 + y^2}}{x\,((x^2 + y^2 - 13.4)^2 + 53.58\,x^2)} = 0.015$$

This is the nonlinear implicit equation that must be solved; we employed a multivariate 2x2 Newton-Raphson method using an initial guess of (9,14) that is actually not difficult to be guessed as we have the plot or Fig. 1 (the other equation is number 9 of the previous work). The result of the computational procedure for that *maximum* point is (9.49,13.89). As this is more an engineering work than a mathematics one, we define or round off the semi-major axis as 13.9 m, or major axis = 27.8 m, and the *critical point* will be 9.5 m.

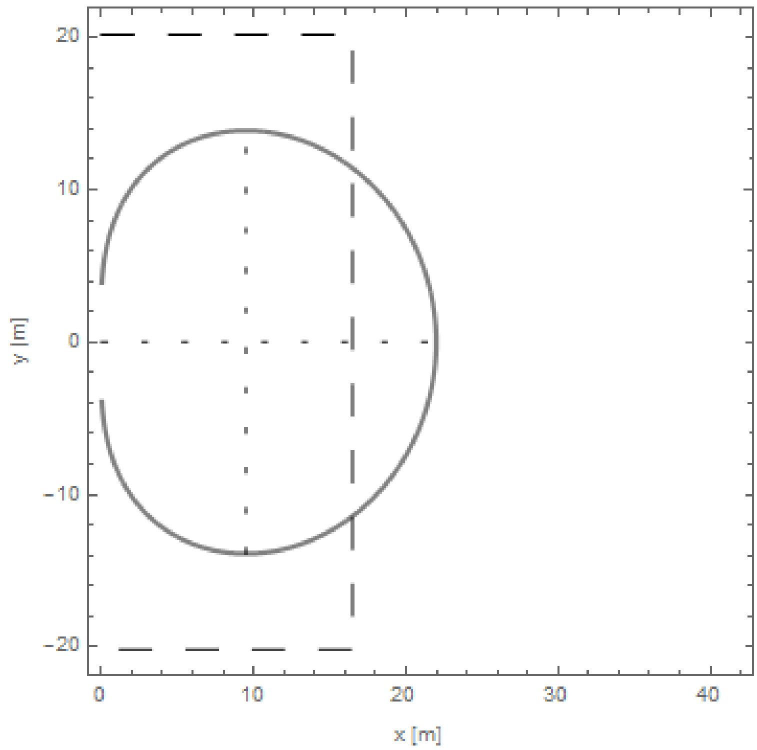


Fig. 1. Mathematical (solid) and rectangular (dashed) penalty areas

Results

The previous two axes are the dotted lines shown in Fig. 1 and these intersect at (9.5,0). Now, based on this *center*, we paint the main or right quarter circle of the simplified PA, with a *mean* radius equal simply to

$$\frac{(22-9.5)+13.9}{2}=13.2\ m$$

The other or left quarter circle will have its center point at the intersection of the major axis and a perpendicular to the goal line starting at a post; this makes good sense because the mathematical or original curve starts at the post (Fig. 1) and because it is also very convenient as posts already exist and there is no need to define a more abstract point on the pitch. The radius of this quarter circle is 13.2 – b/2 = 9.5 m, which advantageously coincides with a previous measure so there is conveniently no need for another one. The mathematical PA and its engineered or practical counterpart are shown in Fig. 2 along with the current *goal area*. It is noted that the maximum difference or separation between PAs is 0.7 m; it is interesting that it is the same separation (0.7 m) along both axes.

Now, another problem with the mathematical PA is that a boundary starting at the posts is impractical because a) free kicks could be called too close to the posts, which would imply very awkward *wall* formations or in general, awkward defense-attack formations, and b) it would be very difficult for referees to decide if a foul (close to the post) is outside of the field, inside the PA or in between. To solve this, we suggest to paint the shorter quarter circle only up to the intersection with the goal area line; in this manner, we solve the problem discussed just previously, not only that, an important use is given to that almost forgotten goal area, which does not have many uses in modern football.

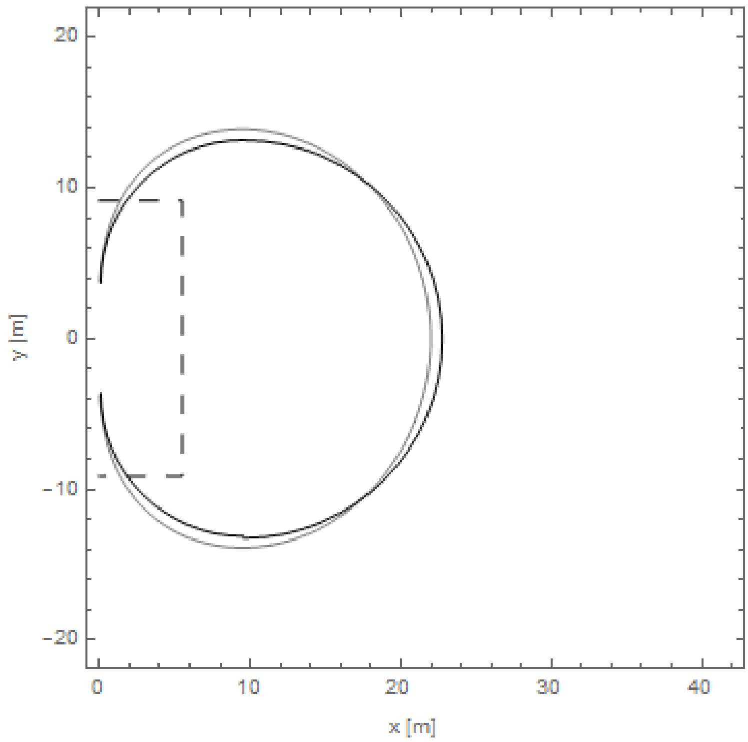


Fig. 2. The simplified or engineered PA (black), the mathematical PA (grey) and the goal area (dashed)

As final result, the new or proposed markings of the PA are shown in Fig. 3; the important or new result is that those can be painted with the same current and international field tools and techniques because circles and circular arcs are currently marks on the football field. Furthermore, note that similar sports, as futsal and field hockey, have for a long time been played with penalty areas made up of circular quarters and straight lines.

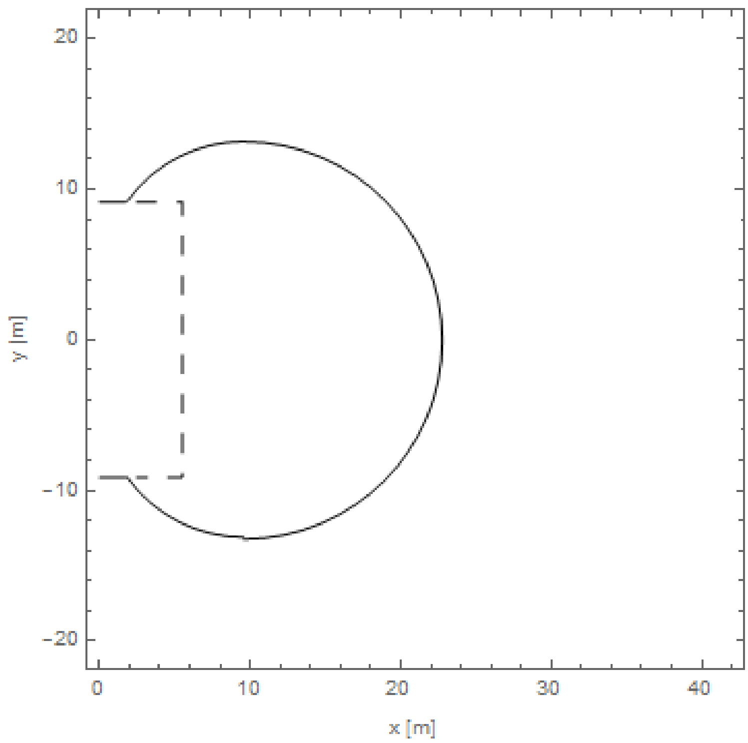


Fig. 3. New PA markings

Discussion

To implement the mathematical PA proposed in previous work would be difficult or unfeasible in the real world of football which has thousands of tournaments literally all over the world and at all levels; thus, a simplification is considered based also on mathematics and the result is a PA line that is not only made up of two simple circular arcs, but also a very good approximation of the fully mathematical PA as shown by Fig. 2 and by the fact that the maximum separation between both curves is only 0.7 m. The implication is that this PA can be easily painted on the field by means of the same tools and techniques already available at all levels of the sport; it is finally shown in Fig 3 after another inconvenience of the mathematical PA is solved, that of the PA marking starting and ending at the posts. Nevertheless, the implementation of this proposal will still require an adjustment period, mainly for goalkeepers and referees.

Conclusion

An improvement and simplification of a previously proposed penalty area has been engineered and achieved. The original PA was based on a measure of scoring chance which implied tangent, square and arccosine functions (in Cartesian or polar coordinates) that are basic and simple in Analytical Geometry but very difficult to paint even in a much resourceful tournament as the UEFA Champions League. The result is a PA that can be drawn by just two radii of 13.2 and 9.5 m. The idea is that this new PA can a) reduce the possibility of games being decided unfairly and b) diminish diving by players near the goal (Morales, 2016).

FIFA is looking for new, even radical, ideas for positive change in the beautiful game (Mirror, 2017), one coming from the scientific and engineering community should also be considered.

Acknowledgement

Thanks to Prof. Andrés Granados (Universidad Simón Bolívar, Caracas), a fluid dynamicist – very interested in mathematics– for having indicated, after looking at the mathematical penalty area (Morales, 2016), that he saw something similar to “two quarters” of a *golden spiral*.

References

Caley, M. (2015) Premier league projections and new expected goals. https://cartilagefreecaptain.sbnation.com/2015/10/19/9295905/premier-league-projections-and-new-expected-goals (retrieved August 2018).

Charles, N. (2014) How can an attacking team get close enough to expect a goal? http://www.wallpaperingfog.co.uk/2014/02/how-can-attacking-team-get-close-enough.html (retrieved August 2018).

David, G.K., Condon, C.H., Bywater, C.L., Ortiz-Barrientos, D., Wilson, R.S., & Korb J (2011) Receivers limit the prevalence of deception in humans: Evidence from diving behaviour in soccer players. *PLoS ONE*; 6: e26017.

Knutson, T. (2016) Explaining and training shot quality. https://statsbomb.com/2016/04/explaining-and-training-shot-quality/ (retrieved August 2018).

Mackay, N. (2016) What is a possession-based model? http://mackayanalytics.nl/2016/11/11/what-is-a-possession-based-model-and-why-does-it-matter/ (retrieved August 2018).

Mirror (2017) http://www.mirror.co.uk/sport/football/news/fifa-technical-director-marco-van-9647981 (retrieved August 2018).
Morales, C.A. (2016) A mathematics-based new penalty area in football: tackling diving. *Journal of Sports Sciences*, *34*(24), 2233-2237.
Morris, P.H., & Lewis, D. (2010) Tackling diving: The perception of deceptive intentions in association football (soccer). *Journal of Nonverbal Behavior, 34*, 1 –13.
Scisports (2016) Expected goals model: an alternative approach. http://www.scisports.com/news/2016/expected-goals-model-an-alternative-approach (retrieved August 2018).
Sumpter, D. (2017a) The geometry of shooting. https://medium.com/@Soccermatics/the-geometry-of-shooting-ae7a67fdf760 (retrieved August 2018).
Sumpter, D. (2017b) *Soccermatics*. UK: Bloomsbury Sigma.