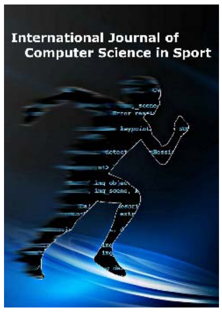
International Journal of Computer Science in Sport

Volume 17, Issue 2, 2018

Journal homepage: http://iacss.org/index.php?id=30

sciendo

DOI: 10.2478/ijcss-2018-0008



Probabilistic models comparing Fast4 and traditional tennis

Simmonds, E., O'Donoghue, P.

Cardiff Metropolitan University, Cyncoed Campus, Cardiff, Wales, CF23 6XD, UK.

Abstract

Tennis matches are hierarchies made up of sets containing games which, in turn, contain points. Traditional tennis games and tiebreakers could theoretically be infinite in length because a player needs to be at least 2 points ahead of the opponent to win. Fast4 tennis is a newer format of tennis that has been used at a number of levels of the sport including professional tennis where it has been used in Next Generation Finals events since 2017. The purpose of the current investigation is to compare the traditional tennis format to Fast4 tennis in terms of the probability of different players winning matches and the duration of matches. Probabilistic models of Fast4 tennis games and tiebreakers were developed. These models allowed the probability of winning games and tiebreakers to be compared between the two formats of tennis for a range of probabilities of players winning points. The models were then used within a series of simulations to determine the probability of winning sets and matches as well as the durations of games, tiebreakers, sets and matches in the two formats. Each component of the two formats of tennis was simulated 100,000 times revealing a reduced impact of serve, greater chance of upsets and shorter matches in Fast 4 tennis than in traditional tennis. The probability of players of differing abilities winning matches as well as the duration of tennis matches should be considered by those making decisions on the format of matches to be applied in tennis tournaments as well as by those preparing to compete in such tournaments.

KEYWORDS; RULE CHANGES IN SPORT, PROBABILITY, SIMULATION, MATCH DURATION.

Introduction

This paper uses an example of a rule change in tennis to illustrate how simulation can be used to provide useful information to decision makers in sport. Simulation is an area of computer science that has been applied in sports science research (Link and Lames, 2015). Simulation of sport requires a model of the area of interest within the particular sport. This model is typically constructed using a reductive approach (Perl, 2015). Modelling and simulation can be used to determine probabilities of winning and other characteristics of tennis matches (Carter and Crews, 1974; Morris, 1977; Croucher, 1986; Newton and Aslam, 2006). These characteristics can be used to compare tennis matches played under different rules and scoring systems. The algorithmic structure of a simulator typically follows the structure of the sport event that is being simulated. For example, simulating a tennis match may involve creating subroutines for sets, games, tiebreakers and points. Data structures represent features of the sport event that is being simulated as well as accumulating statistics that are of interest to decision makers. The models, and their representation within simulation packages, can be adjusted to allow comparison between situations of interest.

To accommodate the competitive environment, sports must constantly evolve and adapt as rules and regulations become pushed to the limit. Rules in sport are modified to prevent injuries, attract athletes, improve players' performances, attract new spectators and adapt sports to children (Arias et al., 2011). Rule changes made in tennis include the introduction of the tiebreaker (Croucher, 1982), racket head size and racket length limitations (Brody, 1996; Miller, 2006), the foot fault rule (Higgins and Lees, 1995) and the use of Hawk-Eye for challenging umpire and line judge calls (Baodong, 2014).

The traditional structure of tennis matches into sets, games and points has remained the same for many decades with only small rule changes being applied to alter the game. Players take turns to serve, with the serving player alternating with games. The first player to win 4 or more points and be at least two points ahead wins the game. Both players start a game at 0-0 or love-love, the first point won is 15, followed by 30, 40 then game. However if both players are level at 40-40 (deuce) then the game continues (advantage and possibly further deuce score-lines) until one player is 2 points ahead. To win a set within tennis, players will need to win 6 games with the opponent winning fewer than 5 games, or win by a score of 7-5 or win by a tiebreaker if the set reaches a score of 6-6. A tiebreaker can occur at the end of any set within a match except the final set within a match. The US Open is an exception to this because a tiebreaker is played in the deciding set at the US Open. The tiebreaker is won by winning at least 7 points and winning at least 2 more points than the opponent. In a final set, at tournaments other than the US Open, if the score is 6-6, the set continues until it is won by 2 clear games. To win a match, players compete in the best of 3 (female matches at Grand Slam tournaments) or 5 (male matches at Grand Slam tournaments) set matches requiring the winning player will win 2 or 3 sets respectively.

Whilst the traditional format is played on both the professional male and female tour events and at Grand Slams, alternate formats are increasing in popularity as the sport attempts to gain a wider audience, enhance the game and make it more exciting to the spectator. One popular variation is Fast4 tennis which is similar to the traditional format with a match being a hierarchy of sets, games and points. In accordance to the LTA (Lawn Tennis Association), Fast4 tennis matches are the best of 5 sets with sets being won by the first player to win 4 games. If a set reaches a score of 3-3 then a tiebreaker game is played (Lawn Tennis Association, 2017). The score in games progresses 0, 1, 2, 3 and 4 points rather than the 0, 15, 30, 40 and "game" which are used in traditional tennis. Players alternate service games within sets of Fast4 tennis matches, but a player does not need to win by at least 2 points; if the score reaches 3-3 within a game, the next point determines the winner of the game. Tiebreakers within Fast4 tennis are won by the first player to reach 5 points. A further difference between tiebreakers in Fast4 and traditional tennis matches is that the player serving first in the tiebreaker serves in the first 2 points, the 5th and 6th point while the other player serves during the 3rd, 4th, 7th and 8th points of the tiebreaker. If the Fast4 tiebreaker reaches a score of 4-4 then a coin is tossed to determine which player serves in the decisive final point of the tiebreaker. Further rule changes implemented within Fast4 tennis include let points being played; if the ball clips the net when serving and lands within the service box the point continues and the ball is in play. In some variations of Fast4 tennis, power play points can be used. Power play can be used once per set when serving. The power play must be announced by the chair umpire and the winner of the power play point receives two points. Other rule changes implemented within Fast4 tennis are coaches being permitted to communicate with players during matches, a shorter warm-up time of 5 minutes with other time limits applying to inter-point breaks, inter-set breaks and medical time-outs. Fast4 tennis has mainly been played at amateur level but has started to be used in professional tennis since the Next Generations ATP (Association of Tennis Professionals) Finals in Milan in November 2017.

The purpose of the current investigation is to compare traditional tennis and Fast4 tennis in terms of the probability of players winning games, tiebreakers, sets and matches and the duration of games, tiebreakers, sets and matches for a range of probabilities of the two players within matches winning points of serve. The study assumes that the probability of a player winning a point on serve is independent of score-line and the outcome of previous points. These assumptions have been supported by some research into tennis (O'Donoghue, 2001; Pollard, 2002) and challenged by other research (Klaasen and Magnus, 2001). The scope of the current investigation restricts the traditional format of tennis to that used in the US Open; where tiebreakers are used in all sets that reach a score of 6-6. The scope of the current study is also restricted to the scoring system of Fast4 tennis and does not include power plays, time limits, communication with coaches or let serves counting.

Methods

This study is a theoretical comparison of traditional and Fast4 tennis formats. The formats include the traditional match format, which is currently played at the US Open and the alternative Fast4 format being played at the Next Generation finals. Equations for the probability of winning a game and a tiebreaker in traditional tennis have been published previously (Croucher, 1986; O'Donoghue, 2013). These equations represent all possible paths of winning games and tiebreakers as enumerated in Figures 1 to 2. This paper determines equivalent equations within Fast4 tennis. Equations for the probabilities of winning sets and matches in terms of the probability of winning points are possible. However, these equations become very complicated very quickly. Therefore, the equations for winning games and tiebreakers were used as the underlying models of simulations that played 100,000 sets and matches to determine the probabilities and durations of sets and matches in each format of tennis.

Traditional Tennis Games

Figure 1 shows the different ways in which the serving player (Player A) and receiving player (Player B) can win a game in traditional tennis. The equation used as the basis of this research is the equation specified by Croucher (1986) which covers all of the paths illustrated in Figure 1.

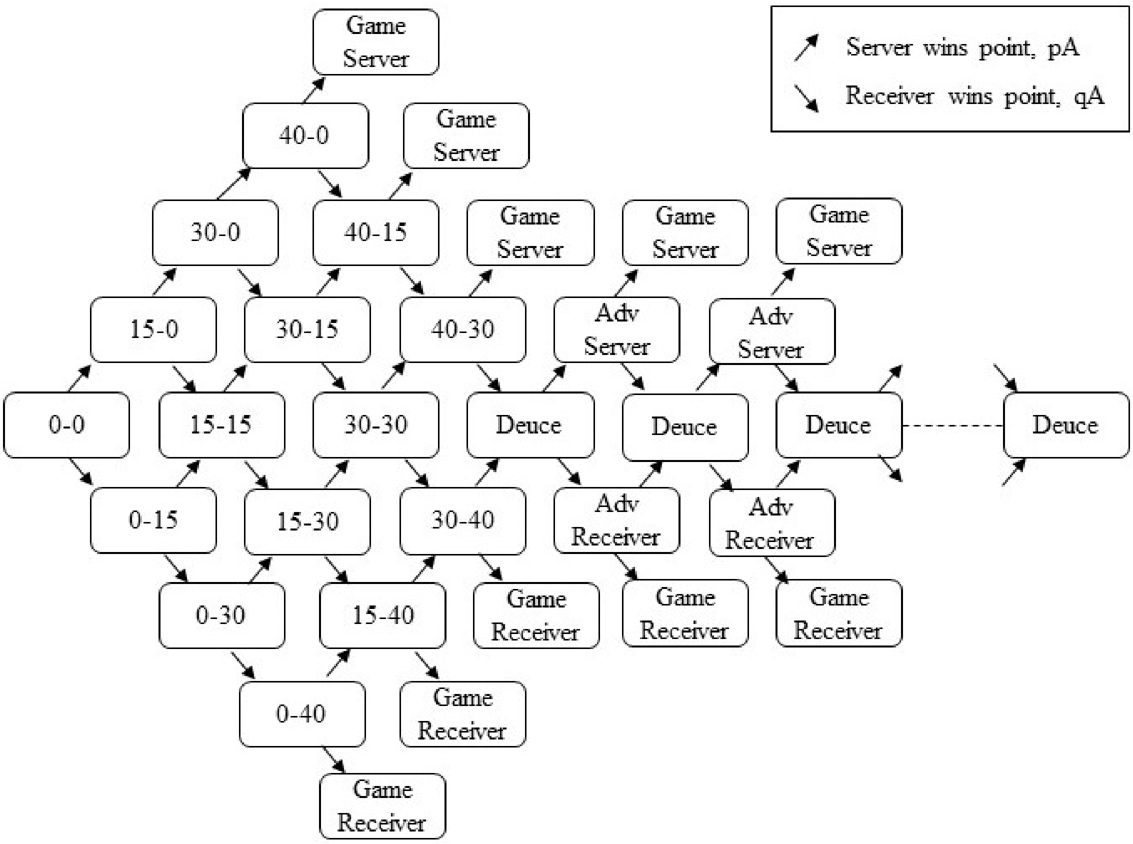


Figure 1. Paths for winning or losing a game in traditional tennis.

The probability of Player A winning and losing a point on serve are represented by pA and qA respectively (qA = 1 – pA). Similarly, the probability of Player B winning and losing a point on serve are represented by pB and qB respectively (qB = 1 – pB). Equation (1) represents the probability of Player A winning their own service game, PA_{Game}; if we change pA and qA to pB and qB this would give us the probability of Player B winning Player B's service game.

$$PA_{Game} = pA^4(1 + 4qA + 10qA^2) + 20pA^5qA^3/(1 - 2pAqA) \quad (1)$$

Traditional Tennis Tiebreakers

Tiebreakers are played at the end of every set if the score is 6-6. Figure 2 shows the different paths to winning or losing the tiebreaker where Player A serves first. Both players serve during the tiebreaker which means that pA, qA, pB and qB are all involved in the model.

The equation used to determine Player A's probability of winning a tiebreaker where they serve first, $PA_{Tiebreaker}$, was specified by O'Donoghue (2013) and is shown in equation (2).

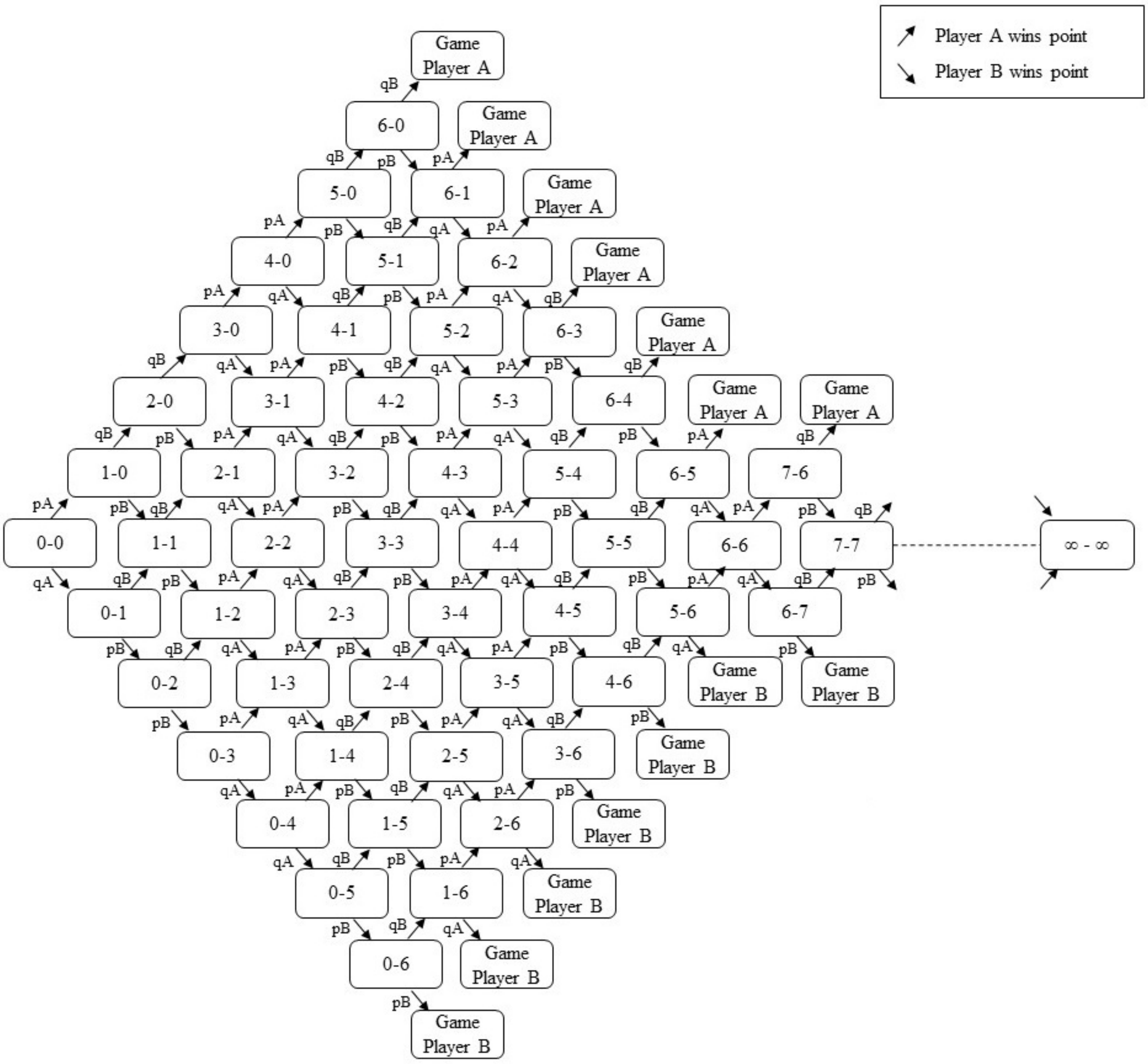


Figure 2. Paths for winning or losing a tiebreaker in traditional tennis.

$$
\begin{aligned}
\text{PATiebreaker} = {} & pA^3qB^4 \\
& + 4pA^4qB^3pB + 3pA^3qAqB^4 \\
& + 6pA^5qB^2pB^2 + 16pA^4qAqB^3pB + 6pA^3qA^2qB^4 \\
& + 4pA^5qB^2pB^3 + 30pA^4qAqB^3pB^2 + 40pA^3qA^2qB^4pB + 10pA^2qA^3qB^5 \\
& + 5pA^5qB^2pB^4 + 50pA^4qAqB^3pB^2 + 100pA^3qA^2qB^4pB^2 + 50pA^2qA^3qB^5pB + 5pAqA^4qB^6 \\
& + 6pA^6qBpB^5 + 75pA^5qAqB^2pB^4 + 200pA^4qA^2qB^3pB^3 + 150pA^3qA^3qB^4pB^2 + 30pA^2qA^4qB^5pB + pAqA^5qB^6 \\
& + (pA^6pB^6 + 36pa^5qApB^5qB + 225pA^4qA^2pB^4qB^2 + 400pA^3qA^3pB^3qB^3 \\
& + 225pA^2qA^4pB^2qB^4 + 36pAqA^5pBqB^5 + qA^6qB^6)\, pAqB / (1 - pApB - qAqB)
\end{aligned}
\tag{2}
$$

Fast4 Tennis Games

Figure 3 shows the various paths to winning or losing games in Fast4 tennis. As with traditional tennis, there is only one path by which a player can win a game 4-0. Hence expression (3). Expressions (4) and (5) show that there are 4 and 10 paths to winning the game 4-1 and 4-2 respectively. There are 20 different paths that can result in the game reaching a score of 3-3; this is represented by expression (6). Where the score does reach 3-3, Player A wins the game by winning the next point. The conditional probability of this is pA as shown in expression (7).

$$pA^4 \text{ (A wins 4-0)} \quad (3)$$

$$4pA^4qA \text{ (A wins 4-1)} \quad (4)$$

$$10pA^4qA^2 \text{ (A wins 4-2)} \quad (5)$$

$$20pA^3qA^3 \text{ (The game reaches 3-3)} \quad (6)$$

$$pA \text{ (A wins 4-3 given that the game reached 3-3)} \quad (7)$$

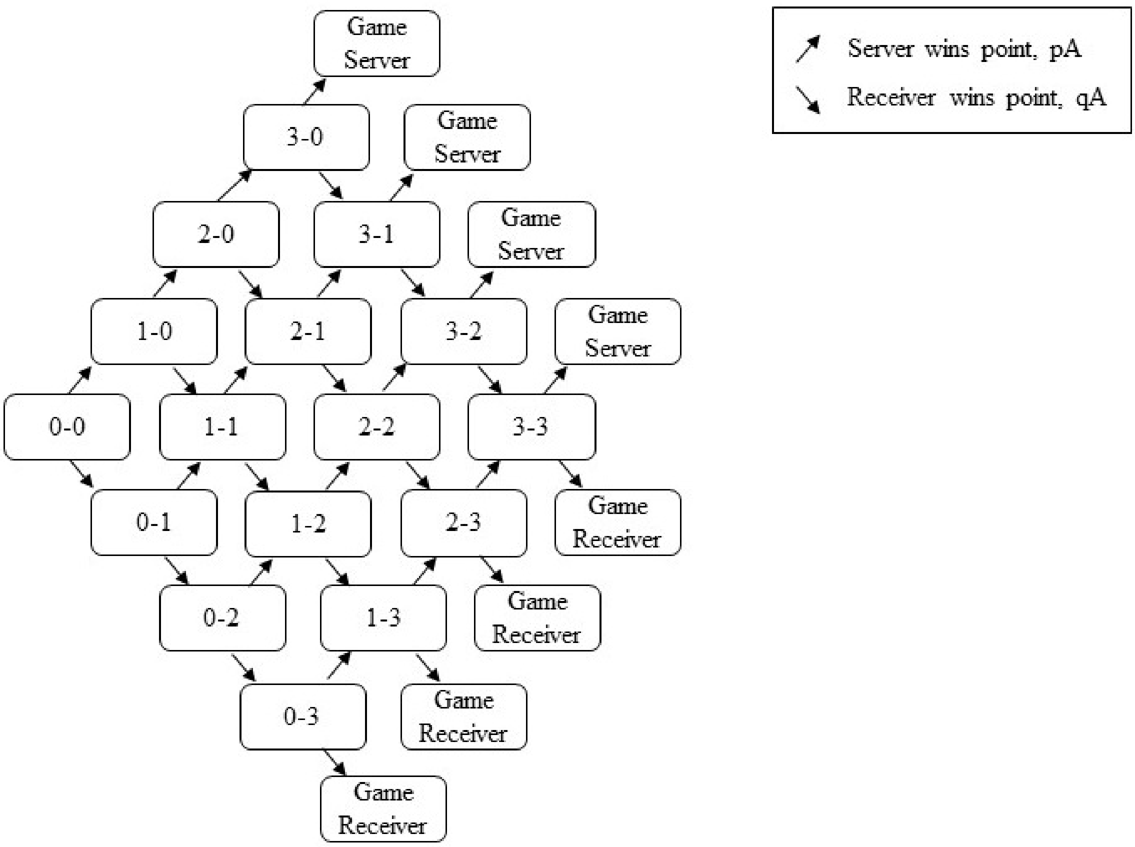


Figure 3. Paths for winning or losing a game in Fast4 tennis.

Equation (8) represents the probability that Player A wins their own service game in Fast4 tennis combining expressions (3) to (7). Equation (8) only differs from equation (1) for traditional tennis by replacing pA^2 / (1 - 2pAqA) with pA because only one point will be played after the score reaches 3-3.

$$PA_{Game} = pA^4(1 + 4qA + 10qA^2) + 20pA^4qA^3 \qquad (8)$$

Fast4 Tennis Tiebreakers

Figure 4 shows all of the paths to winning or losing a tiebreaker in Fast4 tennis where Player A serves first. As in traditional tennis, both players serve during a Fast4 tiebreaker. Therefore, pA, qA, pB and qB are all included within the model. The two probabilities of 0.5 at the score of 4-4 represent the coin toss where we assume that the probabilities of Player A and Player B serving the 9th point are equal.

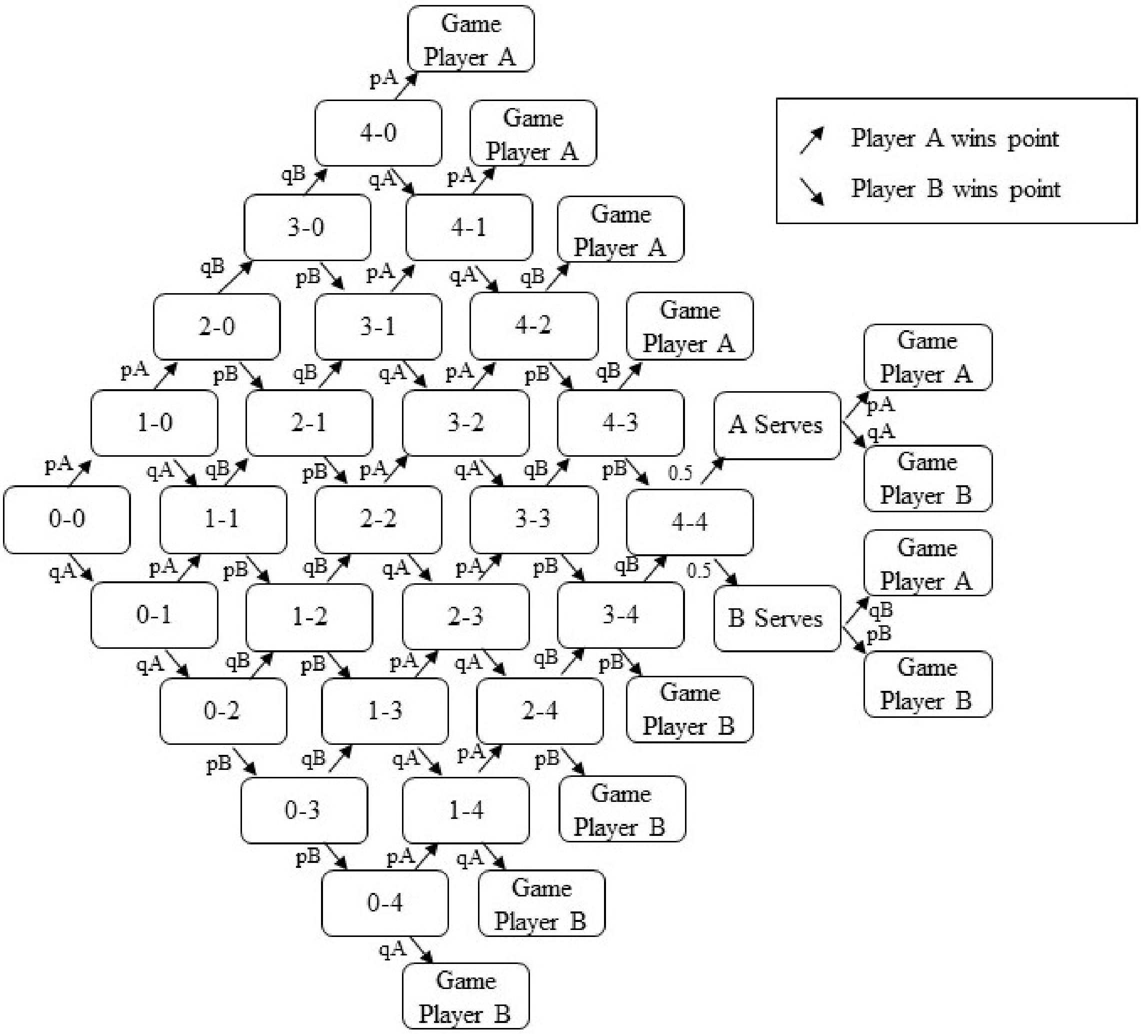


Figure 4. Paths for winning or losing a tiebreaker in Fast4 tennis.

The probability of Player A winning the tiebreaker is given by equation (15) which is made up of the different ways of winning a tiebreaker to 5 points shown in expressions (9) to (14).

There is only one possible way for Player A to win 5-0 and that is by winning all three points on their serve and two points on Player B's service as shown in expression (9).

$$pA^3qB^2 \text{ (A wins 5-0)} \quad (9)$$

To win a tiebreaker 5-1, six points are played, four of which are on Player A's serve, the other two on player B's serve; Player A must win the last point which is on their serve. In total, there are two ways in which Player A can win all four of their own service points and lose one of the opponents serve. However, if Player A loses one of their own service points (one of the first three) they would need to win both of Player B's service points as shown in expression (10).

$$2pA^4qBpB + 3pA^3qAqB^2 \text{ (A wins 5-1)} \quad (10)$$

The final point of a tiebreaker won 5-2 is on Player B's serve, therefore Player A must win this point. Thus Player A can win the tiebreaker 5-2 by winning 2, 3, or all 4 of their own service points. If Player A wins all of their service points they would have lost Player B's first two serves; there is only 1 way in which this could happen.

Where Player A loses one of their four service points they would be required to win two of the three service points Player B plays including last one; thus there are 4 x 2 = 8 ways that Player A can win 5-2 having lost one service point.

If Player A loses two of their service points they would be required to win all three of Player B's service points to win 5-2. There are six combinations of two out of four service points that Player A could lose. To summarise, the probability of winning 5-2 when Player A loses 0, 1, or 2 of their service points is shown in expression (11).

$$pA^4qBpB^2 + 8pA^3qAqB^2pB + 6pA^2qA^2qB^3 \text{ (A wins 5-2)} \quad (11)$$

When Player A wins a tiebreaker 5-3, each player has four service points with Player A winning the last point on Player B's serve; Player A can therefore win the tiebreaker by winning 4, 3, 2, or 1 of their own service points. There is only one way for Player A to win the tiebreak 5-3 not having lost a service point, winning the last point on Player B's serve but losing the other 3 points where Player B served.

If Player A is to lose one of their four service points and still win the tiebreaker 5-3, they must win one of the first three service points from Player B's serve, there are 4 x 3 = 12 ways in which Player A can do this.

If Player A is to lose two of their four service points, they must win two of the first three service points from Player B's serve, there are six pairs of Player A's service points that Player A could win and there are three pairs of Player B's first three service points, with Player A always winning the last of Player B's service points.

For Player A to win the tiebreaker 5-3 only winning one of their own service points would require them to win all four of Player B's service points to win the tiebreaker. There are four combinations of one out of four service points being won on Player A's own serve. To summarise, if Player A were to win the tiebreak 5-3 by winning 1, 2, 3 or 4 service points the probability of this would be as in expression (12).

$$pA^4qBpB^3 + 12pA^3qAqB^2pB^2 + 18pA^2qA^2qB^3pB + 4pAqA^3qB^4 \text{ (A wins 5-3)} \quad (12)$$

If Player A were to win the tiebreaker 5-4, the score must first reach 4-4. In order to get to 4-4, there is only one way in which both players win all 4 of their own service points and only one way for both players to lose all of their own service points.

There are 4 x 4 = 16 ways for each player to lose one of their own service points and win the other three. There are 6 x 6 = 36 ways for both players to lose two of their own service points and win the other two. There are 4 x 4 = 16 ways for each player to lose three of their own service points and win the other one. Thus the probability of the score reaching 4-4 is given by expression (13).

$$pA^4pB^4 + 16pA^3qApB^3qB + 36pA^2qA^2pB^2qB^2 + 16pAqA^3pBqB^3 + qA^4qB^4 \text{ (the tiebreaker reaches 4-4)} \tag{13}$$

After reaching 4-4 a coin is tossed, the winner will decide whether they wish to serve or receive for the final point of the tiebreaker. We assume Player A has a chance of 0.5 of serving, they can win the point if serving or receiving. The conditional probability player A winning the tiebreaker given that the score has reached 4-4 is given in expression (14).

$$0.5pA + 0.5qB \text{ (A wins the tiebreaker given the score reached 4-4)} \tag{14}$$

Combining expressions (9) to (14), the overall probability of Player A winning the tiebreaker when serving first is given by equation (15).

$$\begin{aligned} PA_{Tiebreaker} = {} & pA^3qB^2 \\ & + 2pA^4qBpB + 3pA^3qAqB^2 \\ & + pA^4qBpB^2 + 8pA^3qAqB^2pB + 6pA^2qA^2qB^3 \\ & + pA^4qBpB^3 + 12pA^3qAqB^2pB^2 + 18pA^2qA^2qB^3pB + 4pAqA^3qB^4 \\ & + (pA^4pB^4 + 16pA^3qApB^3qB + 36pA^2qA^2pB^2qB^2 + 16pAqA^3pBqB^3 + \\ & qA^4qB^4)(0.5pA + 0.5qB) \end{aligned} \tag{15}$$

Simulation

The equations for winning games and tiebreakers were used as the underlying models of simulations for sets and matches. The longest matches of those analysed in the current study were best of 5 set matches in traditional tennis where both players had a probability of 0.7 of winning a point on serve. Player A won 50.18% of the 100,000 simulated matches (indicating a probability of 0.5018) and the mean match duration was 272.8 points. This case also showed the greatest variability in match duration. Therefore, convergence values and 95% confidence intervals were determined for this particular scenario. Table 1 shows how many simulations were required to reduce the absolute error of the estimated match parameters to different levels with respect to the values achieved after 100,000 simulations.

An absolute error of 5 for the number of points played (mean = 272.8) is acceptable for decision making purposes suggesting that around 100 simulations might be sufficient. The absolute error for the probability of winning the match exceeds 0.05 after 100 simulations. This absolute error is not acceptable for the expected probability of winning of 0.5. The absolute error approaches a more acceptable value of 0.01 after 1000 simulations. Therefore, 95% confidence limits for the estimated match parameters were determined for 1000 sets of 1000 simulations. The 95% confidence limits for the probability of Player A winning and the mean number of points played in a match were 0.4996 ± 0.003 and 272.83 ± 0.37 respectively.

Table 1. The number of simulations required for the absolute error of estimated parameters to stabilise within given levels (best of 5 traditional set matches where pA =0.7 and pB = 0.7).

Probability of Player A winning		Number of points played	
Absolute Error	**Simulations**	**Absolute Error**	**Simulations**
0.1	18	10	48
0.05	136	5	75
0.01	2,644	1	1,511
0.005	4,254	0.5	1,648
0.001	63,726	0.1	95,343

The absolute errors in Table 1 show that there is an accuracy versus simulation time trade off which would be important to consider if such simulations were being carried out regularly in practical contexts. Given that the current simulation exercise was for a single scientific study, accuracy was prioritised over simulation time. Therefore, matches were simulated 100,000 times within the current investigation. Games and tiebreakers were also simulated 100,000 times in order to determine their duration. There were a total of 9 simulators as games, tiebreakers, sets and matches were simulated for both formats of tennis with traditional tennis matches being simulated when best of 3 sets and best of 5 sets are played. The simulation packages were developed in Matlab 2017a (The MathWorks Inc., Natick, MA). The duration of given game units is reported as the mean number of points within the 100,000 simulations of those units.

Results

Games

Figure 5 shows that where players win the majority of points on serve, the chance of the serving player winning the game is reduced in Fast4 tennis compared to traditional tennis. Figure 6 shows the mean number of points per game determined by the simulator for a range of probabilities of the serving player winning a point. Games are longest in both current and Fast4 formats where the probability of the serving player winning a point is 0.5. In theory, games under the current system could be of infinite length with repeated deuces while Fast4 tennis limits the maximum length of a game to 7 points. As a consequence, the simulator has shown that the mean number of points per game is lower in Fast4 tennis than in traditional tennis.

Tiebreakers

Both players serve within a tiebreaker. Therefore, Figure 7 contains 3 charts showing the probability that the tiebreaker is won by the player who served first in the tiebreak (Player A) for different probabilities of Player B winning points on serve. This reveals that the probability of Player A winning the tiebreaker is greater than 0.5 when their probability of winning a service point is 0.5 or more and they have a greater probability of winning points on serve than Player B. The probability of winning a tiebreaker is not amplified by increases in probability of a player winning a point on serve above 0.5 as much as the probability of winning other games. For example, consider the situation where Player B has a 0.5 probability of winning a point on serve. Where Player A has a probability of winning a service point of 0.7, the probability of Player A winning a tiebreaker is 0.792 in traditional tennis and 0.737 in Fast4 tennis. These probabilities are noticeably below the corresponding probabilities of 0.902 and 0.872 for winning a non-tiebreaker games in traditional and Fast4 tennis respectively.

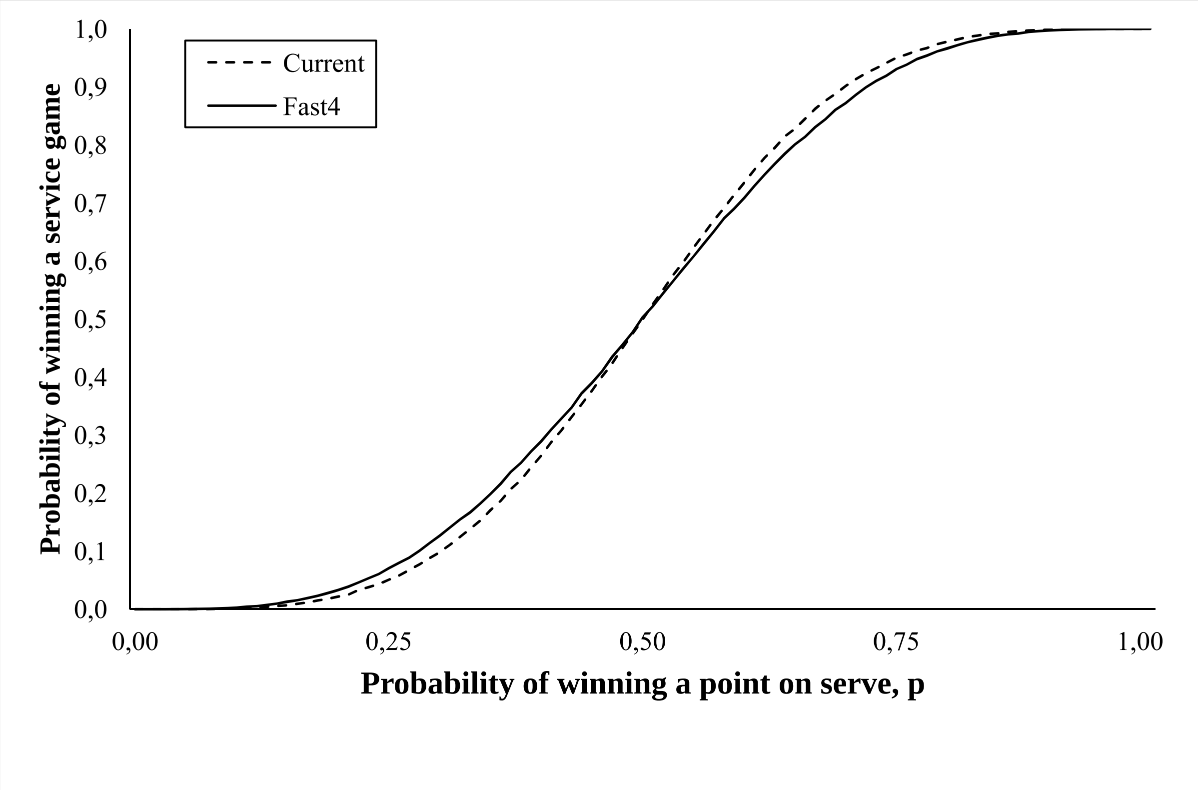


Figure 5. The probability of winning games in traditional and Fast4 tennis.

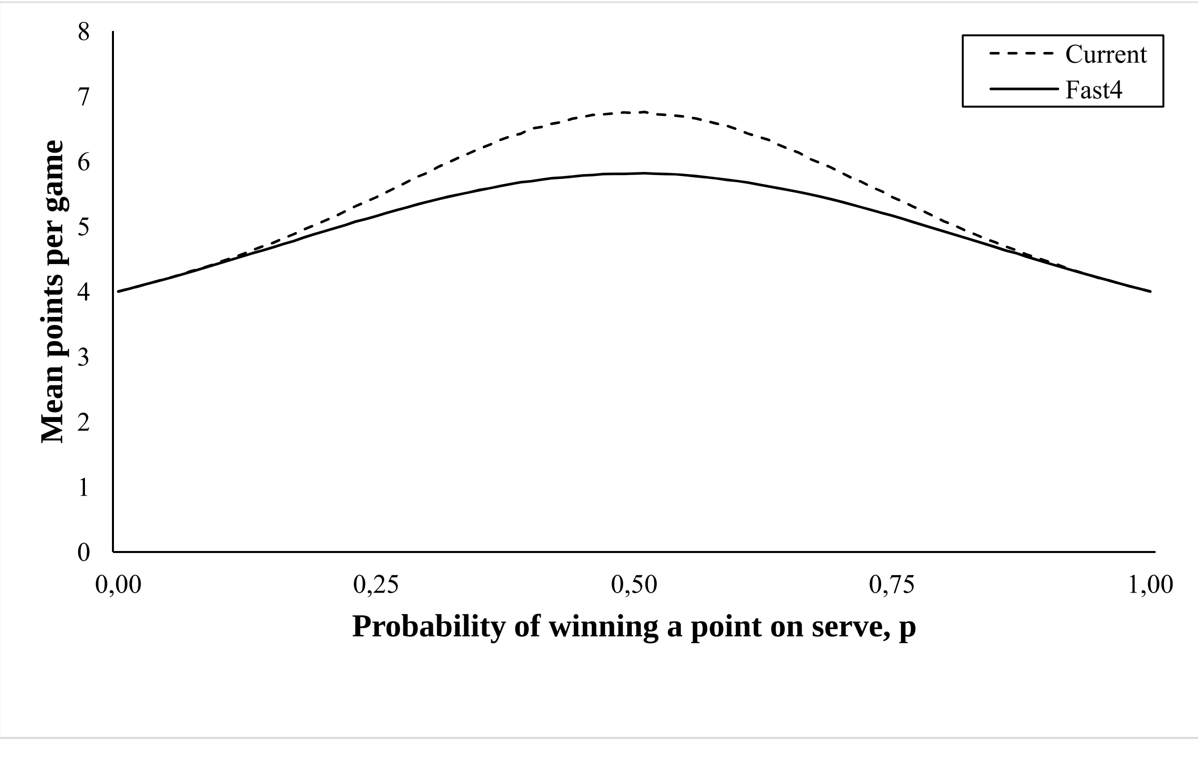


Figure 6. The duration of games in traditional and Fast4 tennis.

Figure 8 shows the mean number of points per tiebreaker according to the simulations. The longest tiebreakers occur when the two players have higher probabilities of winning points on serve and when these probabilities are similar. For example, when both players' probabilities of winning a point on serve increase from 0.5 to 0.7, the simulator revealed the mean number of points in traditional tiebreakers to increase from 11.75 to 12.27 while the mean number of points in Fast4 tiebreakers increased from 7.54 to 7.64. The lower volume of points in Fast4 tiebreakers is due to the reduced number of points required to win the tiebreaker and because a player does not have to be 2 points ahead of the opponent to win a Fast4 tiebreaker.

Sets

Figure 9 shows the probability of winning a set for the player who served first in the set for three different probabilities of the opponent winning a point on serve. When the probability of a player winning a point on serve increases from 0.5 to 0.7 while the opponent's probability of winning a point on serve remains at 0.5, the probability of the player winning a set increases from 0.5 to 0.957 in traditional sets and from 0.5 to 0.887 in Fast4 tennis sets.

Figure 10 shows the mean number of points per set in the two formats of tennis for the same range of probabilities of the players winning points on serve. Sets are longer when the two players have similar probabilities of winning points on serve. Where the two players' probabilities of winning a point on serve are the same, the value of this probability does not make too much difference to the duration of sets in traditional tennis (a mean of about 66 points when the probability of winning a point on serve is 0.5, 0.6 and 0.7). The mean number of points played per Fast4 set is 35.2 when both players have a 0.7 probability of winning a point on serve. This is only marginally greater than the 34.3 points when both players have a probability of 0.5 or 0.6 of winning a point on serve.

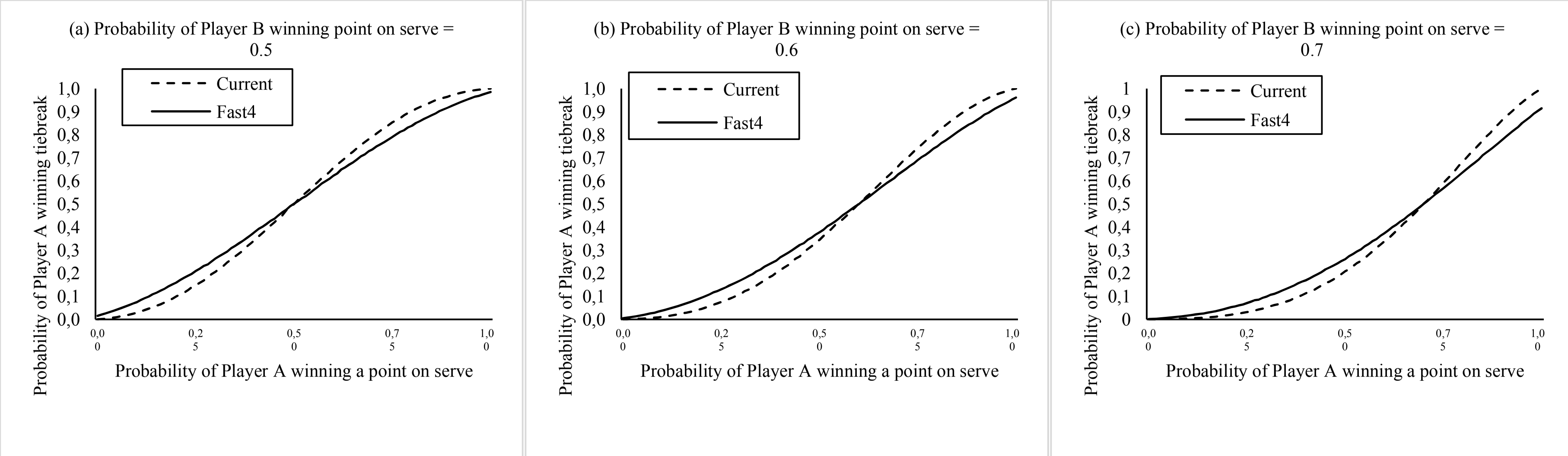


Figure 7. The probability of a tiebreaker being won by the player who served first (Player A).

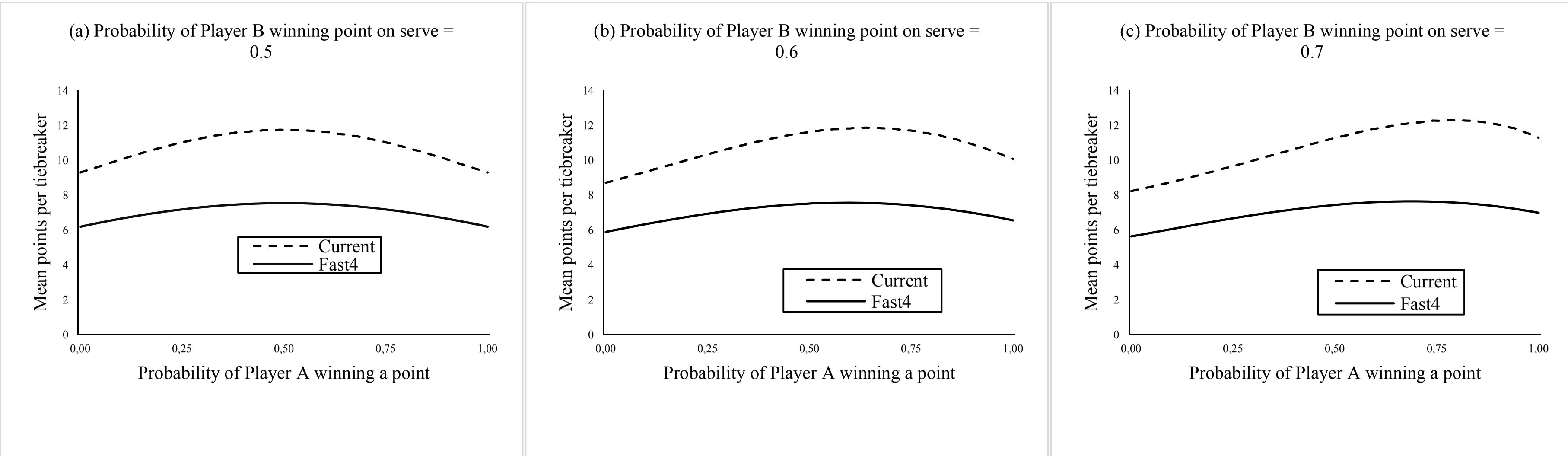


Figure 8. The duration of tiebreakers in traditional and Fast4 tennis

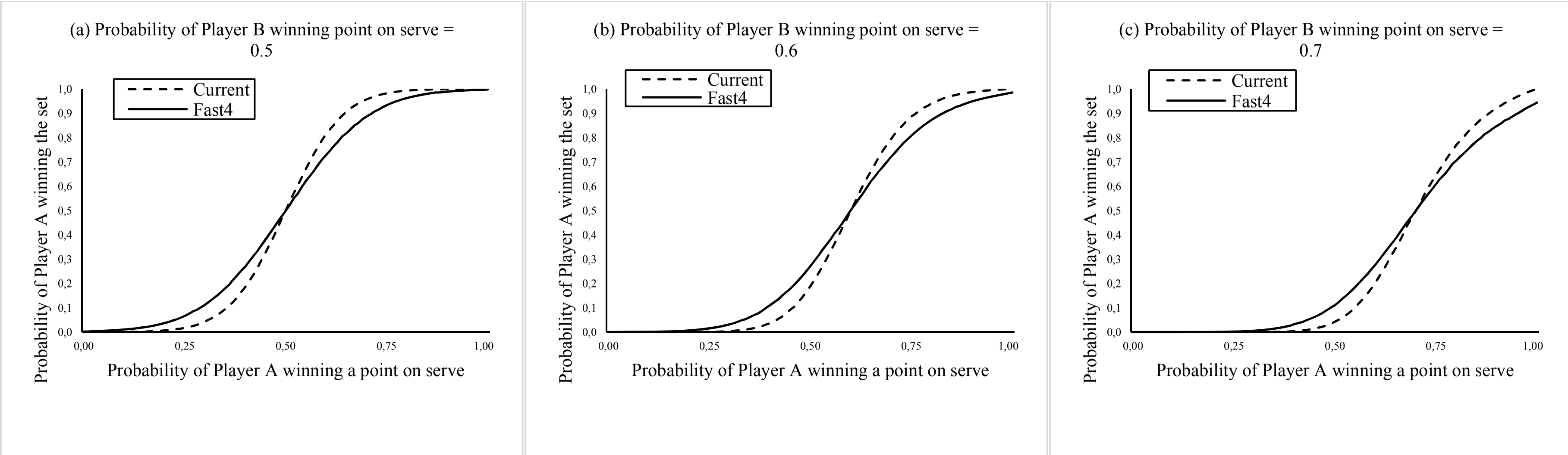


Figure 9. The probability of winning a set for the player who served first (Player A).

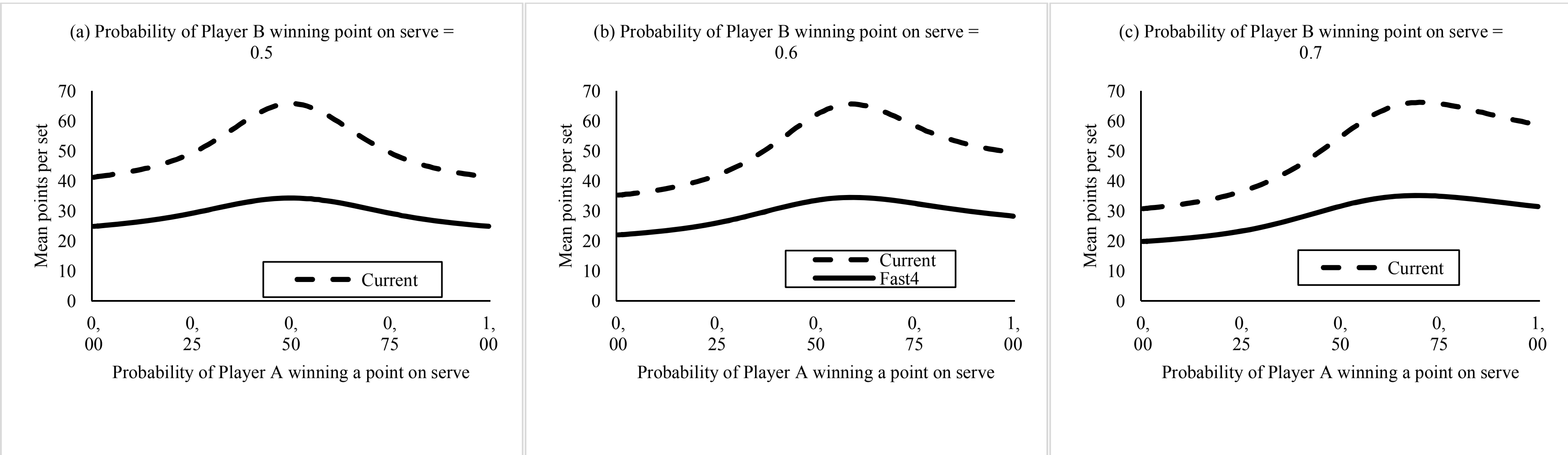


Figure 10. The duration of sets in traditional and Fast4 tennis.

Matches

Figure 11 shows the probability of winning three different types of tennis match for given probabilities of players winning points when serving. When a player's probability of winning a point on serve increases from 0.5 to 0.7 while the opponent's probability of winning a point on serve remains at 0.5, the chance of the player winning the match increases to 0.995 in the best of 3 traditional sets, 0.999 in the best of 5 traditional sets and 0.988 in the best of 5 Fast4 sets. Figure 7 shows that where players win a majority of points on serve, Fast4 tennis reduces the impact of the serve for the full range of probabilities of winning a point on serve from greater than 0.5 to less than 1.0.

Figure 12 shows the mean duration of tennis matches determined by the simulator. The longest matches occur when the two players have equal probabilities of winning points on serve. The best of 3 set matches have a mean of 165.5 and 165.6 where both players have probabilities of 0.5 and 0.6 of winning a point on serve respectively. The length of these matches increases to 177.1 points when the players have a 0.7 probability of winning a point on serve. Similarly, the best of 5 set matches have a mean number of points of 272.1 and 271.9 for probabilities of winning points on serve of 0.5 and 0.6 but involve a mean of 280.3 points when the players' probability of winning points on serve increases to 0.7. The mean number of points in the best of 5 Fast4 tennis sets is 141.7, 142.4 and 145.0 when the players' probability of winning points on serve is 0.5, 0.6 and 0.7 respectively. The reduced increase in the length of Fast4 tennis matches as serve dominance increases is explained by the maximum number of points in a 5 set match being 255 under the Fast4 format. Theoretically, matches could be infinite under the traditional format.

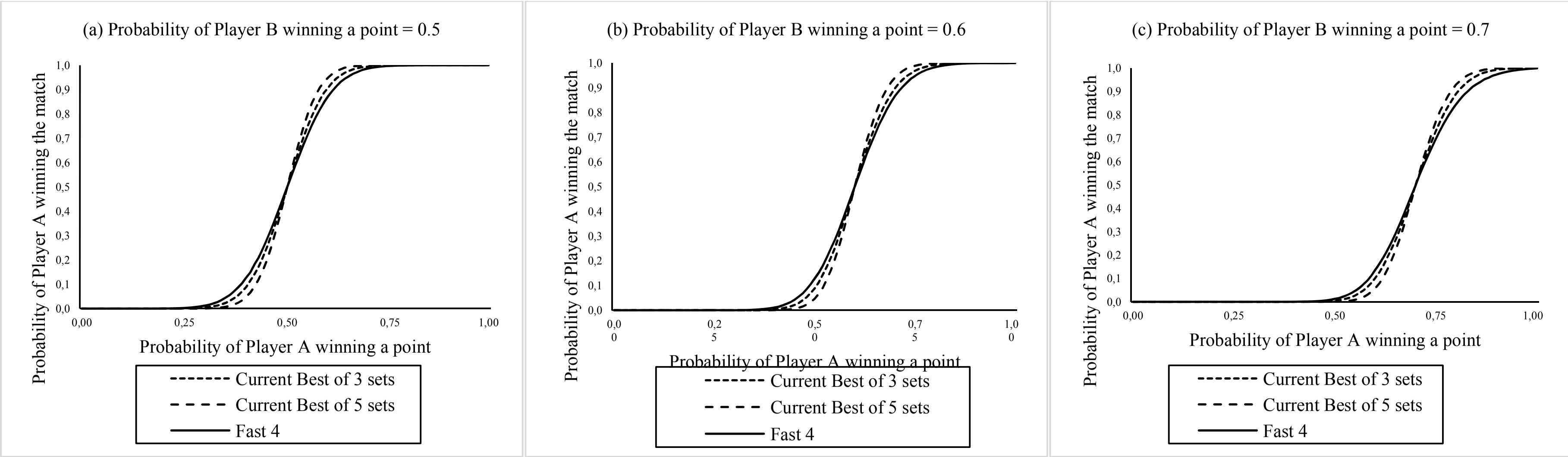


Figure 11. Probability of winning a match for the player who served first (Player A).

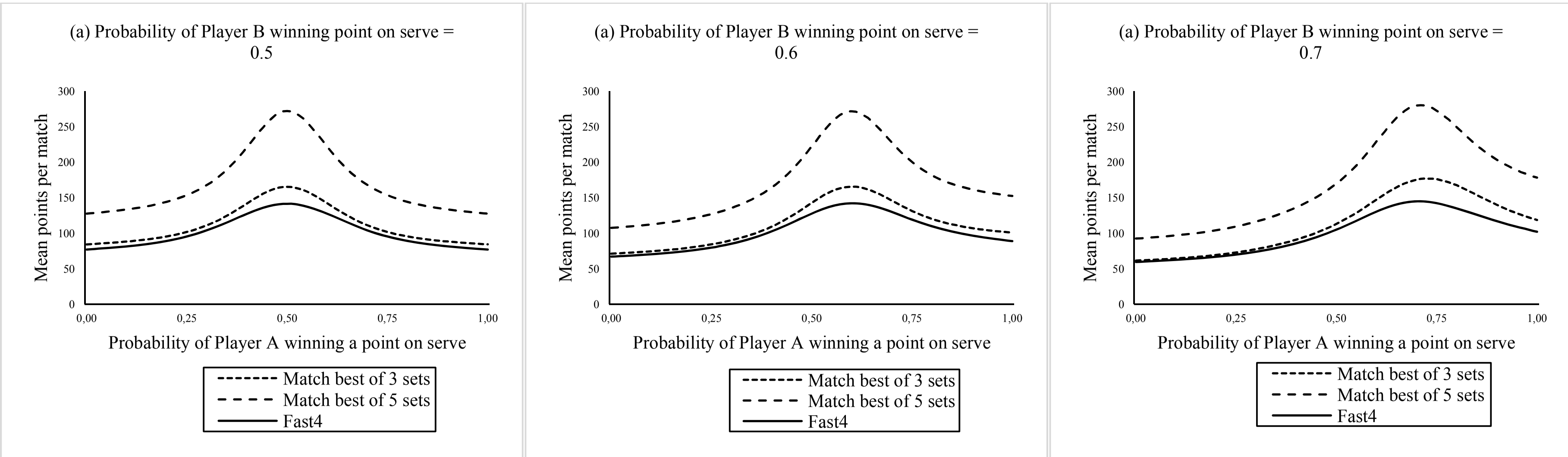


Figure 12. The duration of matches in traditional and Fast4 tennis.

Discussion

Computer simulation has provided estimates of parameters of interest allowing traditional tennis and Fast4 tennis to be compared in terms of players' chances of winning matches and match durations. Modern computing power allows sufficient simulation runs to be executed for estimates of parameters to converge within tolerable limits for decision making purposes. There is an accuracy versus simulation time trade off. Therefore, those using simulators to generate decision support information need to consider the number of simulation runs required to provide accurate enough information in good time for use by decision makers. Decision makers need to be aware of confidence limits of parameter estimates that they are using.

The simulation study conducted in the current investigation has compared traditional and Fast4 tennis formats. The serving player in tennis has an advantage over the opponent (Fischer, 1980). The current investigation has shown that the effect of the serve is reduced in Fast4 tennis when compared with traditional tennis. The same probability of winning a point on serve (assumed to be greater than 0.5) amplifies to lower probabilities of winning games, sets and matches in Fast4 tennis than it does in traditional tennis. This may be explained by the receiving player not needing to be 2 points ahead to break serve making service breaks more likely in Fast4 tennis than in traditional tennis. This may have implications for the types of player who are best suited to different tennis formats. Taller players have been found to win more points on serve and play more aces than shorter players (Söğüt et al., 2018). Given the reduced dominance of the serve in Fast4 tennis, shorter players may have a greater chance of success in this format of tennis than in traditional tennis.

There is more chance of a match being an upset in Fast4 tennis; an upset is where a superior player loses to an inferior opponent. The quality of a tennis player can be represented by the proportion of points they win when serving and when receiving. Cui et al. (2017) found that higher quality tennis players tended to perform better than their lower quality counterparts in terms of percentage of points won on serve and percentage of points won when receiving. Consider Player A's probability of winning a point on serve to be somewhere between 0.6 and 1.0 while the opponent (Player B) has a probability of winning a service point of 0.6. Figure 11(b) shows that for the full range of probabilities of Player A winning a point on serve from 0.6 to 1.0, the probability of Player A winning the match is lower in Fast4 tennis than in traditional tennis. One of the explanations for the greater chance of an upset in Fast4 tennis is the greater chance of Simpson's Paradox matches (Wright et al., 2013) where one player wins the match despite winning a minority of the points in the match. Consider a traditional best of 5 sets match where a player wins 0-6, 0-6, 7-6, 7-6, 7-6 where each game they lose is lost to love (0-4), each game that is won requires one deuce (5-3) and the three tiebreakers are won 7-5. This match is won with 111 (37%) of the 300 points played. It is possible that superior players can win Simpson's Paradox matches against inferior opponents. However, Simpson's Paradox matches are more likely to be upsets than matches won by the superior player (Wright et al., 2013). Fast4 tennis can involve Simpson's Paradox matches where matches can be won with an even lower percentage of points than in traditional tennis. Consider a Fast4 tennis match where a player wins 0-4, 0-4, 4-3, 4-3, 4-3 where each game they lose is lost 0-4, each game they win is won 4-3 and the three tiebreakers are won 5-4. This match is won with 51 (32.8%) points out of the 158 points that are played which is a lower than the 37% in the most extreme case in traditional tennis. Players should be aware of the greater opportunity for upsets in Fast4 tennis and adjust their strategy to enhance their chances of defeating higher ranked opponents or to avoid losing matches to lower ranked opponents. This may involve implementing a strategy with a higher than average risk (Skinner, 2011; Rodenberg, 2014).

Figure 12 shows that the duration of Fast4 matches is lower than that of best of 3 or best of 5 traditional set matches. It should also be noted that the maximum number of points in Fast4 tennis matches is 255 whereas traditional tennis matches could contain an infinite number of points. Figure 12 also reveals that the longest matches in each format of the game are between evenly matched opponents, especially where both players have high probabilities (>= 0.7) of winning points on serve. In preparing for tournaments, players should ensure they are capable of playing longer matches in traditional tennis than in Fast4 tennis. Lower ranked players cover a greater distance during tennis matches than their higher ranked opponents (Cui et al., 2017). This can make traditional tennis particularly taxing for lower ranked players. Youth tennis involves acceleration and deceleration during points (Galé-Ansodi et al., 2016). Therefore, shorter Fast4 tennis matches may reduce injury risk for young players. In designing youth tennis tournaments, organisers should also consider the number of matches players compete in per day as multiple matches increase fatigue and pain ratings of players while also decreasing the accuracy of the serves (Marage et al., 2018). The length of matches should be considered together with the physical demand of playing on different court surfaces because energy expenditure is greater on clay courts than on hard courts (Chapelle et al., 2017). Fast4 tennis may also reduce the physiological demands on players at the game level due to the maximum number of points in a game being 7. Higher physiological responses have been found in service games than in receiving games (Mendez-Villanueva et al., 2007; Kilit et al., 2016). Avoiding lengthy service games with multiple deuces could reduce the demands on players.

The findings of the current investigation have implications for player preparation and tournament design. In traditional tennis, 30-40 is considered to be the most important point in tennis where the serving player wins the majority of points. Morris (1977) defined the importance of a point as being the difference in the conditional probability of winning a game if the point is won and if the point is lost. This method reveals that 30-40 is the most important point with importance values of 0.692 and 0.845 when the probability of the server winning a point is 0.6 and 0.7 respectively. Deuce is the second most important point (0.462) when the probability of the server winning a point is 0.6. However, when the probability of the server winning a point is 0.7, Deuce is only the 6th most important point (0.415) behind 30-40, 15-40, 15-30, 0-30 and 0-40. In Fast4 tennis, 3-3 is the most important point no matter what the probability of the server winning a point is. The importance value of 3-3 in Fast4 tennis is 1.0 because winning this point means winning the game and losing this point means losing the game. The point at 4-4 in a tiebreaker in Fast4 tennis is even more important because the set is either won or lost on this point. The importance of a point should dictate the strategy adopted during the point. There is an accuracy versus power trade-off when serving where a powerful serve is less likely to be in but more likely to lead to the point being won if it is in. In professional men's tennis, 60% of first serves are in and 95% of second serves are in while 75% of points where the first serve is in are won compared to 50% of the points where the second serve is in (Gerchak and Kilgour, 2017). Transforming these values into probabilities and applying Gale's (1971) equation suggests that these players should win 64% of their service points. However, if the second serve were played as powerfully as the first, even though this results in more double faults, these players could expect to win 63% of service points. There may be some individual players who would win more service points by going for power rather than accuracy on the second serve (King and Baker, 1979). While such a strategy may be beneficial over the course of a match, when facing an individual 3-3 point that will win or lose a service game the player needs to manage risks in choosing an optimal serve with the best chance of winning the point. Players have a preference for serving to the opponent's backhand side but need to use different serves to prevent their serve being anticipated by opponents (Unierzyski and Wieczorek, 2004; O'Donoghue, 2009). Serves to the opponent's backhand could be saved for the most critical points; Brabenec (1996) recommended serving to the backhand side at 30-40 in traditional tennis games and also recommended that receiving players may be able to run around the backhand to play forehand returns when the score is 30-40. These strategies may be more appropriate at 3-3 in Fast4 tennis. Male players tend to use the review system to challenge umpire and line judge decisions more on important points than female players do (Kovalchik et al., 2017). Female players may benefit from a more strategic use of the review system and both female and male players should consider the importance of points in the different formats of tennis matches when deciding whether to make a challenge or not.

The chances of upsets should also be considered by those designing tennis tournaments. One of the reasons for applying rule changes in sport is to increase entertainment and spectator interest (Williams, 2008). The tournament structure should give the best players a good chance of reaching the latter stages and winning the tournament while also introducing reasonable uncertainty as to the outcome of matches. The chances of the best players winning tournaments will be reduced in Fast4 tennis due to the reduced chance they have of winning individual matches. Consider a series of four matches where a top seeded player has probabilities of 0.70, 0.68, 0.66 and 0.64 of winning points of serve while their opponents have probabilities of 0.5, 0.5, 0.6 and 0.6 of winning points on serve. When the probabilities of winning these 4 matches shown in Figures 11(a) and 11(b) are combined, the overall probability of winning all 4 matches is 0.488 in Fast4 tennis which is lower than the 0.515 in the best of 3 traditional sets and 0.610 in the best of 5 traditional sets. Thus lower ranked players have more of an opportunity to advance to the latter stages of Fast4 tennis tournaments than they do in the traditional tennis format.

The Fast4 tennis format could help with the organisation of Grand Slam tournaments by shortening matches and creating greater opportunity to rearrange matches should play be delayed due to rain. This could reduce the physical strain placed on a player who is forced to play back to back match sessions without a rest day. A “wash out” day at the French Open in 2016 caused matches to be moved from the second Monday to an already busy Tuesday, with players having to play on consecutive days with no rest. The Director of the French Open, Guy Forget, stated “if they do have, eventually at one point, to play two matches, then I guess the fittest guy will be rewarded for it” (Tennis, 2016); this could have given an unfair advantage to players whose matches had not been delayed. There was a similar story a few weeks later at Wimbledon, with matches being delayed and match congestion forming on outside courts (Standard, 2016). The shorter matches of the Fast4 tennis format would help to alleviate the pressure on the tournament schedule should it be delayed by rain. Fast4 tennis may also increase the percentage of matches that are completed in tournaments played in hot conditions.

Currently, women play the best of three traditional sets at Grand Slam tournaments while men play the best of five traditional sets. This may make it more difficult for a male player to win singles and doubles titles at the same tournaments than it is for their female counterparts. On the other hand, the greater amount of court time enjoyed by male players during singles matches could lead to greater sponsorship earnings than enjoyed by female players. The introduction of the best of five Fast4 sets in both men’s and women’s singles would eliminate these differences between men’s and women’s tennis.

In conclusion, simulation has provided useful information about the probability of winning and the duration of Fast4 tennis matches allowing this format of the game to be compared with traditional tennis. The Fast4 format of tennis matches reduces the impact of the serve, leads to more upsets and shorter matches than the traditional format of tennis matches. These findings should be considered by players and coaches preparing for different formats of tennis used at tournaments they are entering as well as organisers who make decisions about the format of tennis matches to use within tournaments.

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