Performance Analysis in Table Tennis - Stochastic Simulation by Numerical Derivation

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Abstract

The aim of this study was to identify the impact of different tactical behaviors on the winning probability in table tennis. The performance analysis was done by mathematical simulation using a Markov chain model. 259 high-level table tennis games were evaluated by means of a new simulation approach using numerical derivation to remove the necessity to perform a second modeling step in order to determine the difficulty of tactical behaviors. Based on the derivation, several mathematical constructs like directional derivations and the gradient are examined for application in table tennis. Results reveal errors and long rallies as the most influencing game situations, together with the positive effect of risky play on the winning probability of losing players.

KEYWORDS: MARKOV CHAIN, PERFORMANCE ANALYSIS, TABLE TENNIS, SIMULATION, NUMERICAL ANALYSIS

Introduction

There has been growing interest in tactical and technical analysis in sports science (Lees, 2003). This process, also called performance analysis, aims to provide coaches with the information to optimize their decision-making during competitions as well as in training. Hughes and Bartlett (2002) analyzed the structure of different sports resulting in a classification of performance indicators into the three categories technical, tactical and biomechanical. Both O'Donoghue (2004, 2008) and Hughes (1998) examined match analysis in racket sports and suggested methods for the selection of indicators.

There exist several studies that aim to apply performance analysis to table tennis. Malagoli-Lanzoni, Di Michele and Merni (2011) summarize the different performance indicators commonly used in table tennis literature. The process of data collection was also analyzed and enhanced with the help of software tools by different authors (Baca, Baron, Leser & Kain, 2004; Juan, Zhang & Hu, 2008). Leser and Baca (2009) additionally developed a software for the qualitative analysis of performance in table tennis by collecting numerous types of data for

the stroke, ball positions, the players and the match. It is immediately obvious that performance analysis in table tennis is a large topic, with a wide range of research covering many aspects of performance to improve training and competition.

Most of the aforementioned studies try to assess performance by using notational analysis represented by statistics, indices and coefficients, which can be summarized as the absolute individual balance of successful and unsuccessful game actions (Lames et al., 1997). These indicators examine the individual performance outside of the context of the game actions, losing an important feature for evaluation. Hughes and Bartlett (2002) specifically emphasize that the use of match classification indicators, when observed in isolation, can be misleading.

To treat this, studies employed analysis which view the game as a system in which the behavior of the system is influenced by the interaction of the two playing parties, employing probabilistic models to calculate the outcome of matches (Strauss & Arnold, 1987; Newton & Aslam, 2009). Other research also included technical and tactical behavior into the stochastic models, first to predict playing patterns (McGarry & Franks, 1994) and second to determine the performance relevance of tactical behavior (Lames, 1991; Pfeiffer, 2005; Zhang, 2006; Pfeiffer & Hohmann 2008).

Lames (1991) proposes a performance analysis by mathematical simulation in tennis, which is based on transition matrices and tries to support the evaluation of game situations within their respective context. By assuming specific properties of the game actions, described by the Markov chain model, transition matrices can be treated as stochastic processes which then allow the application of different computations on the matrices. The impact of a tactical behavior hereby is described by the change in the winning probability induced by the changes to the transition matrices done through the simulation. Lames and McGarry (2007) use this simulative approach to determine the types of tactical behaviors relevant to athlete's performance in volleyball, while Pfeiffer, Zhang and Hohmann (2010) apply it to table tennis.

In addition to the first step of modeling, where the game actions are modeled by a Markov chain, these simulative approaches need a second step to determine the relative difficulty of tactical behaviors so the changes to the transition matrices fit into the semantics of the sport. The aim of this study was to improve the original simulation method by replacing the second modeling step through application of mathematical derivation. This substitutes the needed difficulty function with well-known mathematical principles.

Methods

Participants

259 matches of world class table tennis players, both male and female, were examined in this study. The evaluated games took place over the span of four years (2008-2012) in official tournaments of the ITTF, ETTU and IOC. The dataset consists of the length, server and winner per rally, together with the final result of each match.

Design

A table tennis rally was classified by the following discrete states: First to fifth stroke, more than five strokes, point and error, each for both players. Players are named "A" and "B", with player A to be known as the winner of the match. In context of the observation system, a single rally can be seen as a system that alternates between the stroke states of the players. This system starts with the serve (first stroke) and ends with a point/error for either player A or B.

Markov Simulation

Markov chains

A Markov chain is a special type of stochastic process, with a stochastic process defined as a system that moves stepwise through a defined set of states. The transitions between these states then occur with certain probabilities. A finite stochastic process is a stochastic process that can only reach a finite amount of states. The Markov property of a stochastic process defines the conditional probability for a state to be reached in the next step, given all previous steps of the process, as equal to the conditional probability for this event with only the current state given. Simply said, the Markov property assumes that each state of a rally only depends on the last former state and is independent of all earlier states. The system is assumed to have no memory. A Markov process is further called Markov chain, if the transition probability between two states is independent from the position of these states in the chain of events. Markov chains also know two types of states: Transient states, which cannot be returned to by the system, and absorbing states, which cannot be left (the transition probability to the own state equals 1). A Markov chain can then be expressed by transition matrices as seen in figure 1.

%	B2	B3	B4	B5	B>5	PA	PB
A1	82,0					5,0	13,0
A2		90,0				2,0	8,0
A3			88,4			2,3	9,3
A4				84,2		0,0	15,8
A5					76,7	10,0	13,3
A>5					83,3	0,9	15,7
	A2	A3	A4	A5	A>5	PB	PA
B1	100,0					0,0	0,0
B2		89,6				4,2	6,3
B3			84,4			4,4	11,1
B4				78,9		2,6	18,4
B5					75,0	0,0	25,0
B>5					74,3	1,8	23,9

Figure 1: Markov transition matrix for a table tennis game, with submatrices Q (yellow) and R (green)

These matrices consist of several submatrices, with Q and R being the most relevant. Q contains all transition probabilities of the transient (non-absorbing) states (the transitions between the strokes) among each other, while R contains the transitions from the transient states to the absorbing states (point/error), (Lames, 1991). The theory of Markov chains also allows the computation of several properties of the evaluated process. The normal matrix $N = (I - Q)^{-1}$, with I as the identity matrix, contains the mean frequency of occurrence for each transient state. From that, the matrix $B = N \cdot R$, an $s \times r$ matrix which contains the probabilities to be absorbed in the absorbing state r for each of the s transient states. The winning probability for a rally for each player can then be taken from the first two rows of the matrix B (Lames, 1991).

Simulation

After the observation system has been modeled as a Markov chain, the winning probabilities of the unmodified transition matrices can be calculated. In the second step the transition matrices have to be manipulated in such a way, that the new transition probabilities express the desired deviation in tactics. Additionally, in order to retain the stochastic nature of the transition

matrices, the tactical deviations have to be compensated by other transition probabilities, so that the probabilities per row still add up to one. Figure 2 shows an example manipulation on the transition probability of the serve of player A to the return of player B from the matrix in figure 1. In this case, the transition probabilities to the direct point and error states were reduced by 5%, while the transition from the serve of player A to the return of player B was increased by the resulting 10% in turn. This type of manipulation will later be defined as “safer play”, since it conveys the tendency of a player to trade a higher direct point rate with a lower direct error rate. In the last step, the winning probabilities have to be calculated again for the manipulated transition matrix. The resulting difference in the winning probability gives an estimate of the importance of the modeled behavior. However, the deflection of tactical behavior requires an additional modeling step, because the amount of the manipulation has to account for the relative difficulty of the behavioral change. As an example, if a player already achieves an extraordinary point rate of 90% at a certain stroke, increasing that point rate by 5% is very difficult in reality. Increasing the point rate from 50% to 60% is considerably easier though. Hence the simulation requires a “difficulty function”, which conducts small deviations for difficult behavioral changes, and bigger variations for easier tactical behaviors.

%	B2	B3	B4	B5	B>5	PA	PB
A1	↑ 92,0 (+10)					↓ 0,0 (-5)	↓ 8,0 (-5)

Figure 2: Transition matrix row after the manipulation of transitions, with the amount of manipulation in brackets

Numerical Derivation

The idea of this study was to replace the additional modeling step to determine the difficulty of tactical behavior with the evaluation of the derivative of the function for the winning probability at this point of the Markov chain. However, since there is no analytical definition for the derivative of the matrix operations involved in the computation of the winning probability, numerical derivation has to be used. This means that an appropriate numerical approximation for the derivative of markov winning probabilities has to be found.

Winning Probability as a Function

To compute the derivative of the winning probability of a player for a particular stroke number, we first have to define the function to derivate. The function $P_{Win} = f(\hat{x}) = f(x_1, \dots, x_{36})$ in the multi-dimensional state space of the Markov model takes the Markov transition matrices as input parameters and returns the winning probabilities for both players in a two-dimensional vector. In general, this function can be regarded as a blackbox around the matrix operations of the traditional simulation, so $f(x)$ essentially performs the computation of the matrix N and consequently matrix B . The wanted winning probability is then calculated from the matrix B under the assumption that the total number of services is distributed evenly among both players using the equation $f(\hat{x}) = 0.5 * (P_{ServeA} + P_{ServeB})$. Here, the point probabilities P_{ServeA} and P_{ServeB} denote the probability to end up in the corresponding absorbing state from either the serve of player A or the serve of player B. The overall winning probability for a rally is therefore composed of the probability to win the rally after ones own serve and the probability to win the rally after the opponent serves, both weighted equally. Although this does not entirely match the distribution of services between players, it gives a reasonable accurate estimation of service behavior over all games. The function $f = P_{Win}$ now has to be numerically derived to receive the impact of a tactical change on the winning probability.

Derivation

The original simulation used the absolute difference between the simulated winning probability $f(\hat{x} + \hat{h})$ and the observed winning probability $f(\hat{x})$ as an indicator for the impact of the simulated behavior. The vector $\hat{x}$ describes the elements in the unmodified matrix, while the step size $\hat{h}$ determines the amount of the manipulation to that vector. For the example shown in figure 2, $\hat{h}$ would contain the value 10 for the transition to B2 and the value -5 for the transitions to PA and PB respectively, with all other values set to zero. The non-zero elements in $\hat{h}$ therefore determine the behavior that is simulated by the derivation. The numerical derivation replaces this absolute difference with the slope of the winning probability. This slope can be interpreted as the ratio of vertical change (winning probability) to the horizontal change (manipulated transition probability).

There are several possibilities to compute the derivative of a function. The most common definition for the derivative is given by the forward difference

$$f' = \lim_{h \rightarrow 0} \frac{f(\hat{x} + \hat{h}) - f(\hat{x})}{|\hat{h}|}$$

which is already in numerically computable form. Another method is the backward difference

$$f' = \lim_{h \rightarrow 0} \frac{f(\hat{x}) - f(\hat{x} - \hat{h})}{|\hat{h}|}$$

that inverts the direction in which the slope is computed. However, in comparison to the forward difference and the backward difference, the central difference formula

$$f' = \lim_{h \rightarrow 0} \frac{f(\hat{x} + \hat{h}) - f(\hat{x} - h)}{2 * |\hat{h}|}$$

produces results with up to two orders of magnitude better precision (Press, Teukolsky, Vetterling & Flannery, 2007) and is hence used in most cases of the simulation. There exist two edge cases that require the application of the forward and backward difference, however. The forward difference is used to compute the derivative for matrix rows, where the transition probabilities have a value of zero, since it is impossible to apply the central difference at this point, because further subtraction of the step size would result in negative probabilities. Correspondingly, the backward difference is used for transitions that have the maximum probability value 1.

In order to achieve a high precision for the numerical computation, it is important to choose the step size $\hat{h}$ so that it minimizes the round-off as well as the fractional error of the derivation computation. To this end, $\hat{h}(x) = \sqrt[4]{eps * x}$ has proven to be a suitable estimate for the central difference formula (Press et al., 2007). Here, x denotes the single transition probability for the currently simulated stroke, while eps describes the smallest machine-representable floating-point number. This is the point in the simulation, where the sports-specific difficulty function gets replaced by $\hat{h}(x)$, which only depends on numeric values present in the transition matrices and doesn't require further knowledge of the relative difficulty of tactical behaviors.

A special strategy to compute the step size $\hat{h}(x)$ is required in case the transition probability to simulate is zero. In that case the value of x is replaced by the mean value of all transition probabilities for the current dimension to provide a plausible step size for the central difference.

The value of the derivative now directly provides an indicator for the relevance of the simulated behavior.

Partial Derivation

In mathematics, the partial derivative of a multi-variable function is its derivative with respect to a single of those variables, while the rest is held constant. This means that only a single transition probability will be manipulated, or in other words, $\hat{h}$ only contains one non-zero component for the respective transition. The goal of this analysis was to study the impact of point and error rates per stroke on the winning probability in table tennis. This is done by manipulating the transitions to the absorbing states “point” and “error” (PA or PB, depending on the player of the current stroke) respectively for each stroke. In order to reach a uniform distribution of the tactical change over the remaining variables, we chose to compensate it using the equation given by Lames (1991):

$$\delta P_{yi} = -\frac{P_{yi}}{1 - P_x} * \delta P_x, i = 1 \dots n$$

Here P_{yi} describes the n transition probabilities used to compensate for the change δP_x in the investigated probability P_x . The compensation also results in a directional derivative overall ($\hat{h}$ contains > 1 non-zero elements) even though in theory the single change in the error / point probability represents a partial derivative.

Directional Derivation

The second tactical behavior examined in this study is the effect of safer/riskier play. The behavior “safer play” is defined as a decrease of both absorbing states at the current stroke. This roughly expresses the fact that a player is less likely to commit errors if he plays with more caution, but at the same time also scores less direct points. The term “riskier play” is defined accordingly by an increase of the error and point transition probabilities, which in turn expresses that the error rate is expected to grow at the same rate as the direct point rate if a player plays more aggressively. The necessary compensation for these manipulations was accounted for by the de- or increase of the total amount of change for the transition to the next state (e.g. from the third stroke of player A to the fourth stroke of player B) of that row, which is $2 * |\hat{h}|$ for the central difference formula and $|\hat{h}|$ for the forward and backward differences. Figure 2 shows an example of the manipulation of a matrix row for the simulation of safer play. To get the values for the simulation of risky play, the signs of the deflected values need to be switched.

Gradient

The mathematical gradient of a scalar function f is a vector operator, commonly denoted with the symbol ∇ . In a cartesian coordinate system it is a vector field whose components are the partial derivatives of f . Applied to the function for the winning probability the gradient results in the equation

$$\nabla f(x_1, \dots, x_{36}) = \frac{\partial f}{\partial x_1} e_1 + \dots + \frac{\partial f}{\partial x_{36}} e_{36}$$

where e_i are the orthogonal unit vectors of the coordinate system (Kaplan, 1992).

The orientation of ∇f is the direction in which the directional derivation of f has the largest value and $|\nabla f|$ (the length of the gradient vector) is the value of that directional derivative. When applied to the winning probability function $f(x_1, \dots, x_{36})$ it results in a vector of length

36 that contains the partial derivation of each possible transition in the original matrix. The main goal of the gradient analysis is to find the strokes with an outstanding influence on the outcome of a game. Since the gradient points in the direction of “steepest” change in the winning probability, it provides the possibility to automatically identify these transitions for each game and player. To that end, the simulation procedure was changed by a small step. In contrast to the aforementioned simulations, the service probability was not considered equally distributed over both players, but a game was simulated two times, one time for each player exclusively being at the serve. This changes the function for the winning probability to either $P_{Win} = P_{ServeA}$ or $P_{Win} = P_{ServeB}$, depending on which player is currently simulated. It also means that some transition probabilities can never be reached in a simulation. For example, if a player is serving, it can never happen that the same player hits the second or fourth ball in a rally. ∇f therefore contains the partial derivations of all transitions that can take place in an alternating state sequence, starting with the simulated player. As an example, for the simulation of player A as the serving player, this sequence would consist of the following states: (A1, B2, A3, B4, A5, B >5, A>5, ...). To obtain gradient vectors that contain only strokes of a single player (A1, A2, A3, ...), these alternations have to be taken into account. This is done by switching the corresponding derivation values between the gradient vectors. For example, to get the gradient values for the return of player A, the values from the simulation with player B as the serving player have to be considered. It also has to be taken into account that the states “point for player A” (PA) and “point for player B” (PB) have different semantics depending on which stroke is currently simulated. For example if player A is serving, PA stands for the service aces. However if player B is the serving player, PA represents the service errors.

This separated simulation of the players also causes the gradient vector to lose dimensions. Since not all 36 states are reachable from each configuration, the vector is reduced to the size of 21. Each stroke class adds three elements to the vector: The partial derivations of its transition probability to the next stroke and the two transitions to the absorbing point and error states. This adds up to 15 elements for the first five strokes of a player. The sixth stroke has to be split up in two classes, since it can be reached from both starting points of the simulation. This results in two subclasses for the sixth stroke: The sixth stroke after one’s own serve, and the sixth stroke after the serve of the opponent.

The separation of simulation for each player sounds like a contradiction to the original goal of the Markov model, to include the interaction process between players into the performance analysis. However, the interaction between the players is not omitted, but shifted so that only a subset of the original states is considered in the simulation.

The gradients for each game were analyzed using a factor analysis. The factor extraction was done by principal component analysis and Varimax rotation with Kaiser Normalization.

Results

Comparison between the methods

To validate the new simulation method, the partial derivation values for each game were compared with the values produced by the original method introduced by Lames (1991). Therefore the simulation was performed with the classical model (difficulty function) for the change in tactical behavior for $\hat{h}$.

$$\hat{h}(x) = K + B * 4 * x * (1 - x)$$

The constant K describes the amplitude of change in the edge case of the simulation, for example when the transition probability is zero, and was chosen with a value of 0.01. The other constant, B, determines the maximum amplitude of change and was chosen with the value of 0.05 in order to achieve the maximum change of 6% in transition probability, which was found to be stable in terms of its behavior by Lames (1991) and Pfeiffer (2005).

Since the original method works with the absolute change in the winning probability, in contrast to the slope generated by the Markov simulation, a direct comparison of both methods is difficult, however. The correlation between the derivation values and the absolute results showed a mean value of 0.457. A better way to compare both methods is to convert the absolute differences in the winning probability from the original method to slope values, which makes both methods easily comparable. This is done by dividing the results by the absolute amount of change $|\hat{h}|$ for each simulated value. The mean correlation between the original and the new method for the slope values then becomes 0.995, with a mean absolute difference of 0.022 between the values for each stroke.

Error and Winner rates

The impact of error and winner rates was simulated by the application of partial derivations to the winning probability function. A three-factor variance analysis with repeated measurements with the derivation direction (point/error) and the stroke number as within-subject effects and the gender as between-subject effect was used to analyze the resulting derivatives. Since the data did not comply with the sphericity criteria, the Greenhouse-Geisser corrected values had to be considered. The analysis shows significant results for both the stroke number and the interaction of stroke number with gender and point/error probability (table 1). The biggest share on the variance was found for the stroke number, with a partial eta-square value of .441, followed by the combinations of stroke number with gender and point. Between-subject tests indicate that the variance can also be explained by the gender with a significance of $p < .001$ and a partial eta square of .119 (table 2).

Table 1. 3-factor variance analysis, within-subject tests (Greenhouse-Geisser)

Source	Square sum	df	Mean squares	F	Sig	Partial eta ²
Point	.040	1.000	.040	1.275	.259	.002
Point * Gender	.000	1.000	.000	.006	.936	.000
Stroke Nr.	54.608	1.051	51.969	406.478	.000	.441
Stroke Nr. * Gender	9.355	1.051	8.903	69.638	.000	.119
Point * Stroke Nr.	.300	1.699	.176	15.608	.000	.029
Point * Stroke Nr. * Gender	.028	1.699	.017	1.467	.230	.003

Lastly, figure 3 compares the absolute mean values of the derivation of the point and error probability between stroke number and gender. The long rallies with more than five strokes show simulation values with an exceedingly higher absolute derivative than the rest of the strokes. With the absolute amount of change as important indicator for tactical behavior, the

long rallies can be considered as the most impacting rallies in a game.

Table 2. 3-factor variance analysis, between-subject tests

Source	Square sum	df	Mean squares	F	Sig	Partial eta ²
Constant term	399.568	1	399.568	11782.146	.000	.958
Gender	2.358	1	2.358	69.544	.000	.119
Error	17.499	516	.034

Figure 3. Impact of Point and Error probability per stroke and gender on the winning probability of a player. Absolute slope values are shown for better comparison of stroke types.

Gradient

This study used the gradient to identify the most important strokes for a player in terms of absolute impact on the winning probability. The factor analysis of the gradient clusters the elements of the transitions matrices into groups to find independent factors that explain the variance in stroke impact. The resulting groups of strokes with similar importance then represent the main clusters of performance in a table tennis game.

The scree plot for the computed factor analysis shows reason to differentiate four main components. However, these components are only able to explain 58.5% of the total variance, with the first component explaining 24% of the variance after rotation, the second component taking 16.8% and the last two components 9.6% and 8% respectively.

Table 3. Total Variance Explained

Component	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	6,148	29,277	29,277	5,050	24,048	24,048
2	2,641	12,575	41,852	3,528	16,800	40,849
3	1,973	9,397	51,248	2,018	9,608	50,457
4	1,533	7,299	58,547	1,699	8,090	58,547

Table 4 displays the rotated factor loadings for all factors with a correlation higher than .30. The first component features a high correlation with the long stroke classes, where the rallies last 6 strokes or longer. Component three is also clearly correlated to the service, while the high correlations of component four can be attributed to the return strokes. Finally, with the exception of the factor “Point 3.”, the second component exclusively contains noticeable correlations for error and transition probabilities.

Table 4. Rotated Component Matrix

	Component			
	1	2	3	4
Serve			,956	
Point Serve				
Error Serve			-,943	
Return				,895
Point Return				-,873
Error Return	,426	-,326		
3. Stroke		-,631		
Point 3.		,459		
Error 3.		,635		
4. Stroke		,680		
Point 4.				
Error 4.		-,747		
5. Stroke	-,321	-,510	-,302	
Point 5.				
Error 5.	,429	,605		
6 Opp. Serve	,818	,366		
Point 6 Opp. Serve	,873			
Error 6 Opp. Serve	-,837	-,411		
6 Own Serve	-,853	-,354		
Point 6 Own Serve	-,890			
Error 6 Own Serve	,860	,375		

Risk-taking

Directional derivatives were conducted to examine the importance of the tactical behaviors “risky play” and “safe play”. The results show a substantially higher amount of positive derivation values for the losing players, while winning players exhibit mostly negative values. Table 5 shows the results of the paired t-test for dependent samples for each stroke.

Table 5. T-Test between Winner and Loser for all strokes

Stroke	Mean	Std.dev	T	df	Sig(2-sided)	r	Cohen's d
Total	-.154	.201	-30.25	1553	.000	-.704	-1.417
1	-.159	.113	-22.531	258	.000	-.164	-2.136
2	-.154	.115	-21.566	258	.000	-.216	-2.090
3	-.137	.117	-18.747	258	.000	-.339	-1.906
4	-.111	.121	-14.927	258	.000	-.571	-1.645
5	-.085	.102	-13.482	258	.000	-.864	-1.617
5+	-.276	.395	-11.265	258	.000	-.940	-1.379

The data shows a significant difference between the means of the derivatives of the winning and losing player for each stroke. To compute the effect sizes for the dependent tests, the formula given by Dunlap, Cortina, Vaslow and Burke (1996) was used. All effect sizes indicate a large effect of the difference between winner and loser, although the effect strength diminishes with growing stroke number. Furthermore the sign of the correlation r validates the assumption of an inverse relation between the winning and the losing player.

Since only the impact of single behaviors was examined in previous sections, a multiple regression analysis was performed to expose any relation between the change in the winning probability (D5+) and risky/safe play (E5+, P5+) for rallies lasting longer than five strokes (tables 4 and 5). For the regression the point and error probabilities were chosen as predictors, and the derivative from the simulation as the dependent variable.

Table 6. Regression Summary

R	R²	Adjusted R²	Std.Error of the estimate
.444	.197	.194	.219
Predictors: (Constant), E5+, P5+			
Dependent Variable: D5+			

The regression summary reveals a rather small value with only 19.4% of the variance being defined by the predictors. The generated model can be trusted with values of $F = 63.230$ and $p < 0.01$ in an analysis of variance, however.

Table 7 reveals the impact of each predictor on the change in the winning probability.

Table 7. Regression coefficients

Model	B	Std.Error	Beta	T	Sig
(Constant)	-.278	.027		-10.268	.000
P5+	.197	-.758	-.121	-3.069	.002
E5+	1.021	.095	.425	10.756	.000

Both point and error probability have a significant effect on the winning probability. In comparison, the error probability $Beta = .425$ contributes almost four times as much to the explanation of the variance as the point probability $Beta = .121$.

Discussion

In general, the simulation using numerical derivation can be established as an effective alternative to the traditional simulation method. The criticism regarding the original method, especially the Markov-chain property, can still be cause for concern, since it demands stability of the transition probabilities regardless of the time the transition takes place within a rally or game. With the transitions set to the different stroke numbers, this point in time is fixed at least over the span of a rally for each transition in our Markov model.

As the comparison between both methods points out, the resulting values are highly correlating ($r = 0.995$) when brought onto the same scale, thus reinforcing the usability of the numerical method. This scale leads to one of the difficulties of the developed method. Unlike the traditional simulation, which used absolute differences in the winning probability as indicator for the performance, the derivation produces slopes that may be hard to interpret in the context of a table tennis game at first. In addition, the numerical simulation is susceptible to errors like rounding and machine imprecision. However, this is a well studied problem in numerics and can be minimized by the careful selection of the step size for the derivation according to Press et al. (2007). Ultimately, the stable properties of the simulation allow the application of several mathematical constructs, such as the partial derivation, directional derivation or the gradient.

The *partial derivation* analysis reveals significant differences between gender, stroke numbers, in regards to points and errors, with the higher influence of errors compared to the direct point rate especially apparent. The difference between the stroke numbers confirm that even though the model itself is rather simple, with only the length and result of rallies included and finer details as for example the technique of each stroke ignored, significant findings about the individual stroke lengths are possible. The high impact of long rallies with more than five strokes, as well as the higher impact of errors in comparison to direct points are notable findings of the analysis. While the partial derivation focuses solely on the influence of the point and error rates, the *gradient analysis* includes information about the relevance of all reachable transitions of the Markov simulation and hence provides an insight into the general structure of performance in a table tennis game. Four main components that contribute to the winning probability of a player were revealed by the factor analysis of the gradient. These components represent independent factors of performance and can give an indication, on which strokes to focus in more detailed analyses. In compliance with the findings of the partial derivation, the long rallies lasting more than five strokes were identified as the most important independent class of strokes during a match. The second component almost exclusively contains transient transitions (transitions to the next stroke), as well as transitions to the error states (direct errors). Because these behaviors are contradicting in the tactical context of a table

tennis game, the extraction of a general term that includes both of these stroke classes is quite difficult. One way of classifying these strokes could be to formulate a reverse definition and label them as the strokes, where no direct point is made. On the contrary, the third and fourth components are clearly related to the serve and the return strokes again.

The results of the *directional derivation* analysis show that the impact of the behavior “risky play” depends on the evaluated player. While risky play mostly results in a disadvantage for the winning player, losing players can benefit from a more risky playstyle. A possible explanation for this result could be that winning players already perform on a near optimal risk/reward level, while losing players still have room to improve in that regard. Again, although both error and direct point rates are found to have significant impact, the error rate contributes with an extraordinary share to the result.

Since all of the conducted analyses determine the *error rate* as highly impacting factor on the winning probability of a player and hence the result of a table tennis game, the prevention of errors, especially at highly impacting stroke types like the long rallies, appears to be a desirable goal for high-level players. Pfeiffer et al. (2010) find a similar effect in their study, where the influence of direct points is found smaller than the error rate. They explain this behavior by the outstanding defensive performance of world-class players, which induces a greater willingness to take risks. This outstanding defensive performance could also be the reason for the high impact of long rallies that we found in the partial derivation as well as in the gradient vectors. Since the possibility to score points early in a rally against world-class defensive players is relatively low, rallies tend to last longer and hence the performance in the long rallies becomes more important. This is further supported by the components of the gradient analysis, where the serve and return strokes showed the smallest contribution to the variance. Another explanation is given by Malagoli-Lanzoni, Di Michele and Merni (2014), who found that the stroke types that are mostly used at the beginning of a rally (push and flick) show a rather high proportion of returns. Again, this means that rallies tend to last longer and the performance in these long rallies has greater impact on the outcome of a game. They also discovered that aggressive stroke types like the top forehand and top-counter-top forehand, which are considered risky stroke techniques, are more associated to rally winners than the other strokes. This supports our assumption that the winners operate on a better risk/reward level than the losers of a game.

In general, the simulation using numerical derivation brings the advantage that a second modeling step to determine the difficulty of tactical behaviours can be skipped. This removes the need of game-specific knowledge to apply the simulation method on different sports. In fact, the simulation itself can be seen as a closed black-box, which gets the transition matrices as input, computes the desired numerical derivation on the matrices and returns the slopes of the winning probability as output, regardless of sport or behavior to examine. Naturally, this means that the resulting analyses are limited by the information contained in the input matrices, as their variables determine which behaviors can be examined and to what extent. Considering that the input data in this study is relatively simple, with only the rally lengths, server and winner known, it is possible to draw conclusions about the general structure of a table tennis game. Some interesting questions that are relevant in table tennis practice, like the impact of transitions between stroke techniques or stroke positions, as examined in other studies (Zhang, 2006; Pfeiffer et al., 2010; Malagoli-Lanzoni et al, 2014), can not be answered with the available data. However, since the simulation method itself is independent from the information contained in the input data, it would be possible to examine these features if it were included in the dataset.

Especially if you see the mathematical constructs we presented as a framework to automatically evaluate different sports on a mathematical basis without the need to adapt the simulation, this independence could prove to be advantageous. Given an arbitrary set of transition matrices, it would be possible to apply the gradient analysis on that set to automatically identify the most important transitions, then run consecutive simulation passes with more sophisticated partial or directional derivations on these transitions, resulting in a fully automatic analysis of the most important game behaviors. Future work on the method could also implement an improved way to compute the winning probability by including the actual distribution of services between both players. Additionally, the gradient vectors could be used for a cluster analyses on a per game basis to find similar game types, players or even match-ups. Finally, the differences between winning and losing players encourage further studies.

Conclusion

In the present study, stochastic simulation using a Markov chain model of table tennis and numerical derivation was used for the first time. In comparison with the traditional methods, it retains the advantages of the original Markov simulation, but skips the necessity to perform a second modeling step at the same time. This enables the application of the simulation in different fields without the need to adapt it to the difficulty of the sports-specific tactical behaviors, which makes it more approachable. The use of the numerical derivative also allows the employment of mathematical constructs that are based on derivation. The extension of the Markov simulation through numerical derivation hence is a profitable procedure in table tennis, allowing the computation and interpretation of the relevance of tactical behavior patterns.

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