INTERACTION DUE TO MECHANICAL SOURCE IN GENERALIZED THERMO MICROSTRETCH ELASTIC MEDIUM

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The eigen value approach, following the Laplace and Hankel transformation has been employed to find a general solution of the field equations in a generalized thermo microstretch elastic medium for an axisymmetric problem. An infinite space with the mechanical source has been applied to illustrate the utility of the approach. The integral transformations have been inverted by using a numerical inversion technique to obtain normal displacement, normal force stress, couple stress and microstress in the physical domain. Numerical results are shown graphically.

Key words: generalized thermo-microstretch, eigen value, Laplace transform, Hankel transform, mechanical source.

1. Introduction

The classical theory of elasticity successfully explains the behavior of construction materials (various sorts of steel, aluminum, concrete) provided the stresses do not exceed the elastic limit and no stress concentration occurs. But it is inadequate to model the modern engineering components which possess internal structure such as polycrystalline materials and materials with fibrous or coarse grain structure. Eringen (1966) coined a term micropolar elasticity and used this theory to explain the deformation of elastic media with such oriented particles. A micropolar continuum is a collection of interconnected particles that are made up of dipole atoms or dumb-bell molecules and are subjected to surface and body couples which is capable of translational as well as rotational motion.

The governing equations of thermoelasticity in the usual framework of linear coupled thermoelasticity consists of the wave type (hyperbolic) equations of motion and the diffusion type (parabolic) equation of heat conduction. But it was observed that if an isotropic, homogeneous, elastic material is subjected to thermal or mechanical disturbances, the effects in the temperature and displacement field are felt immediately at an infinite distance from the source of disturbance. This implies that a part of the disturbance has an infinite velocity of propagation which is physically impossible. To overcome these discrepancies two generalizations to the coupled theory were introduced. The first is due to Lord and

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Shulman (1967), who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier’s law. The second generalization is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Green and Lindsay obtained an explicit version of the constitutive equations in 1972.

Nowacki (1966) and Eringen (1970) extended the linear theory of micropolar continua to include the thermal effect and formulated the micropolar thermoelasticity theory. The linear theory of elastic materials with stretch is one of the generalizations of the classical theory of elasticity. Eringen (1971) developed the theory of micropolar elastic solid with stretch which included the effect of axial stretch during the rotation of molecules. Microstretch solids are capable of stretching and contraction independent of their translation and rotation. Thus, in these solids, the motion is characterized by seven degrees of freedom, namely three for translation, three for rotation and one for stretch. Porous media whose pores are filled with gas inviscid liquid, asphalt and composite fibrous materials are some examples of microstretch elastic solids. Eringen (1990) also developed a continuum theory of thermo-microstretch elastic solids. Green and Naghdi (1993) proposed the theory of thermoelasticity without energy dissipation and presented the derivation of a complete set of governing equations of the linearized version of the theory for homogeneous and isotropic materials in terms of displacement and temperature fields and proved the uniqueness of the solution of the corresponding initial mixed boundary value problem. Iesan and Neppa (1995) contributed to this field by studying a problem on extension and bending of a microstretch elastic circular cylinder. A problem of bending of microstretch elastic plates was investigated by Ciarletta (1999). Chandrasekhar and Srinath (2000) studied the problem of thermoelastic waves without energy dissipation in an unbounded body having a spherical cavity. Aouadi (2008) studied the linear theory of microstretch thermoelastic bodies with microtemperature and proved the existence of coupling of microrotation vector field with the microtemperatures for isotropic bodies. Kumar and Partap (2009) investigated the propagation of axisymmetric free vibrations in microstretch thermoelastic homogeneous isotropic solids which were subjected to stress free thermally insulated and isothermal conditions. Othman and Lotfy (2010) applied the normal mode analysis on the general model of the equations of generalized thermo-microstretch for a homogeneous isotropic elastic half-space of different theories. Othman et al. (2013) analyzed the effect of gravity on the same model for generalized thermo-microstretch for a homogenous isotropic elastic half-space solid subjected to a Mode-I crack problem in the context of Green Naghdi theory.

2. Formulation and solution of the problem

We consider a homogeneous, isotropic generalized thermo-microstretch elastic medium of infinite extent pointing vertically into the medium. Field equations and the constitutive relations without body forces, body couples, heat sources and stretch force are given by Eringen (1990), Lord and Shulman (1967) and Green and Lindsay (1972) as

\[
\lambda \phi^* + (\lambda + 2\mu + K) \nabla (\nabla \cdot u) - (\mu + K) \nabla \times \nabla \times u + K \nabla \phi - \nu \left( I + \tau_1 \frac{\partial}{\partial t} \right) \nabla T = \rho \frac{\partial^2 u}{\partial t^2},
\]

(2.1)

\[
(\alpha + \beta + \gamma) \nabla (\nabla \cdot \phi) - \gamma \nabla \times \nabla \times \phi + K \nabla \times u - 2K \phi = \rho j \frac{\partial^2 u}{\partial t^2}.
\]

(2.2)

\[
\rho C^* \left( \frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right) + \nu T_0 \left( \frac{\partial}{\partial t} + \Xi \tau_0 \frac{\partial^2}{\partial t^2} \right) \nabla \cdot u + \nu T_0 \frac{\partial \phi^*}{\partial t} = K^* \nabla^2 T,
\]

(2.3)
Interaction due to mechanical source in generalized …

\( \alpha \nabla^2 \varphi^* + \frac{1}{3} \nu_i \varphi^* - \frac{\lambda}{3} \nabla \cdot \mathbf{u} = \frac{3}{2} \rho \frac{\partial^2 \varphi^*}{\partial t^2}, \)  

(2.4)

\( t_{ij} = \lambda \ n_r \ n_r \ \delta_{ij} + \mu \left( u_{i,j} + u_{j,i} \right) + K \left( u_{j,i} - \varepsilon_{ijr} \varphi_r \right) - \left( T + \tau_0 \frac{\partial T}{\partial t} \right) \delta_{ij}, \)

(2.5)

\( m_{ij} = \alpha \varphi_{r,r} \delta_{ij} + \beta \varphi_{r,i} + \gamma \varphi_{j,i}, \)

(2.6)

\( \lambda_i = \alpha_0 \varphi_{j,i}^* \)

(2.7)

where

\( \nu = (3\lambda + 2\mu + K) \alpha_{ij}, \quad \nu_j = (3\lambda + 2\mu + K) \alpha_{ij}. \)

Since we are considering a two-dimensional axisymmetric problem, so we assume the components of the displacement vector \( \mathbf{u} \) and microrotation vector \( \varphi \) are of the form

\( \mathbf{u} = (u_r, 0, u_z), \quad \varphi = (0, \varphi_\theta, 0). \)  

(2.8)

Here due to symmetry about the \( z \)-axis all the quantities are independent of \( \theta \), so that \( \frac{\partial}{\partial \theta} \equiv 0 \). With these considerations and using Eqs (2.8), the system of Eqs (2.1) to (2.7) reduces to

\( (\lambda + \mu) \left[ \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial \theta^2} + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right] + (\mu + K) \left[ \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial \theta^2} \right] + \)

\( -K \frac{\partial^2 \varphi_\theta}{\partial \theta^2} - \nu \left( 1 + \tau_0 \frac{\partial^2 T}{\partial \theta^2} \right) \frac{\partial^2 \varphi_\theta}{\partial r^2} = \frac{\partial^2 u_r}{\partial r^2}, \)

(2.9)

\( (\lambda + \mu) \left[ \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial \theta^2} \right] + (\mu + K) \left[ \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial \theta^2} \right] + \)

\( -K \frac{\partial^2 (\tau_0 \varphi_\theta)}{\partial \theta^2} - \nu \left( 1 + \tau_0 \frac{\partial^2 T}{\partial \theta^2} \right) \frac{\partial^2 \varphi_\theta}{\partial r^2} = \rho \frac{\partial^2 u_r}{\partial r^2}, \)

(2.10)

\( \frac{\gamma}{c_r^2} \varphi_{\theta,\theta} + \frac{1}{c_r^2} \frac{\varphi_{\theta,\theta}}{c_r^2} - \frac{\varphi_{\theta,\theta}}{c_r^2} - 2K \varphi_\theta + K \left[ \frac{\partial \varphi_\theta}{\partial z} - \frac{\partial \varphi_\theta}{\partial z} \right] - K \frac{\partial \varphi_\theta}{\partial z} = \rho \frac{\partial^2 \varphi_\theta}{\partial c_r^2}, \)

(2.11)

\( K^* \left[ \frac{\partial^2 T}{\partial c_r^2} + \frac{1}{c_r^2} \frac{\partial T}{\partial c_r} + \frac{\partial^2 T}{\partial \theta^2} \right] = \rho C^* \left[ \frac{\partial T}{\partial c_r} + \tau_0 \frac{\partial^2 T}{\partial c_r^2} \right] + \)

\( + \nu T_0 \left[ \frac{\partial}{\partial c_r} + \tau_0 \frac{\partial^2}{\partial \theta^2} \right] \left( \frac{\partial u_r}{\partial c_r} + \frac{u_r}{c_r} + \frac{\partial u_r}{\partial c_r} \right) + \nu T_0 \frac{\partial \varphi}{\partial c_r}, \)  

(2.12)
\[ \alpha_0 \left[ \frac{\partial^2 \varphi^*}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi^*}{\partial r} + \frac{\partial^2 \varphi^*}{\partial z^2} \right] + \frac{1}{3} \nu_j T - \frac{1}{3} \lambda_j \varphi^* + \frac{1}{3} \nu_j T - \frac{1}{3} \lambda_j \varphi^* = 2 \rho_j \frac{\partial^2 \varphi^*}{\partial t^2} \]

where

\[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \]

Introducing dimensionless quantities as

\[ z' = \frac{\omega^* z}{C_i}, \quad t' = \omega^* t, \quad \tau'_0 = \omega^* \tau_0, \quad \tau'_l = \omega^* \tau_l, \]

\[ T' = \frac{T}{T_0}, \quad u'_r = \frac{\rho_0^* C_i u_r}{\nu T_0}, \quad u'_z = \frac{\rho_0^* C_i u_z}{\nu T_0}, \]

\[ \varphi'_0 = \frac{\rho}{T_0^2} \varphi_0, \quad t'_rr = \frac{t_{rr}}{T_0}, \quad t'_zz = \frac{t_{zz}}{T_0}, \quad m'_0 = \frac{\omega^*}{C_i \nu T_0} m_0, \]

\[ \omega^* = \frac{C^* (\lambda^* + 2 \mu^*)}{K^*}, \quad C_j^* = \frac{\lambda^* + 2 \mu^*}{\rho}, \quad \varphi^* = \frac{\rho c_j^2}{\nu T_0} \varphi^*, \quad \lambda'_z = \frac{\omega^*}{c_j \nu T_0} \lambda_z. \]

After suppressing the primes for convenience Eqs (2.9)-(2.13) reduce to

\[ m_1 \left[ \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right] + m_2 \frac{\partial^2 u_z}{\partial r \partial z} + m_3 \frac{\partial^2 u_r}{\partial z^2} + \]

\[ -m_4 \frac{\partial \varphi_0}{\partial z} - \left( 1 + \tau_j \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial r} + m_5 \frac{\partial \varphi^*}{\partial z} = \frac{\partial^2 u_r}{\partial t^2}, \]

\[ m_1 \frac{\partial^2 u_r}{\partial r^2} + m_2 \left( \frac{\partial^2 u_z}{\partial r \partial z} + \frac{1}{r} \frac{\partial u_z}{\partial r} \right) + m_3 \left( \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} \right) + \]

\[ -m_4 \frac{1}{r} \frac{\partial (r \varphi_0)}{\partial r} - \left( 1 + \tau_j \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial z} + m_5 \frac{\partial \varphi^*}{\partial z} = \frac{\partial^2 u_z}{\partial t^2}, \]

\[ \left[ \frac{\partial^2 \varphi_0}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_0}{\partial r} - \frac{\varphi_0}{r^2} + \frac{\partial^2 \varphi_0}{\partial z^2} \right] - 2m_6 \varphi_0 + m_6 \left[ \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right] = m_7 \frac{\partial^2 \varphi_0}{\partial t^2}. \]
\[
\left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial \theta^2} \right] - \left[ \frac{\partial^2 T}{\partial \tau_\theta \partial \tau^2} \right] = \varepsilon \left[ \frac{\partial}{\partial \tau} \left( \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_\theta}{\partial \theta} \right) \right] + \Xi \tau_\theta \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{u_r}{r} + \frac{\partial u_\theta}{\partial \theta} \right) + \varepsilon m_\theta \frac{\partial \phi^*}{\partial \tau},
\]
(2.18)

\[
\frac{\partial^2 \phi^*}{\partial r^2} + \frac{1}{r} \frac{\partial \phi^*}{\partial r} + \frac{\partial^2 \phi^*}{\partial \theta^2} + Tm_\theta - m_{10} \phi^* - m_{11} \left( \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_\theta}{\partial \theta} \right) = m_{12} \frac{\partial^2 \phi^*}{\partial \tau^2}
\]
(2.19)

where

\[
m_i = \frac{\lambda + 2 \mu + K}{\rho C_i^2}, \quad m_2 = \frac{\lambda + \mu}{\rho C_i^2}, \quad m_3 = \frac{\mu + K}{\rho C_i^2}, \quad m_4 = \frac{K}{\rho C_i^2},
\]

\[
m_5 = \frac{\lambda_0}{\rho C_i^2}, \quad m_6 = \frac{K C_i^2}{\gamma \omega^2}, \quad m_7 = \frac{\rho C_i^2}{\gamma}, \quad m_8 = \frac{\nu I}{\nu},
\]

\[
m_9 = \frac{\nu \varepsilon_i^2}{3 \nu \omega \omega^2}, \quad m_{10} = \frac{\lambda \varepsilon_i^2}{3 \alpha_0 \omega^2}, \quad m_{11} = \frac{\lambda_0 \varepsilon_i^2}{3 \alpha_0 \omega^2}, \quad m_{12} = \frac{3 \beta \varepsilon_i^2}{2 \alpha_0}.
\]

Applying the Laplace and Hankel transforms on Eqs (2.15)-(2.19) defined by

\[
\bar{f}(r, z, p) = L\{f(r, z, t)\} = \int_0^\infty f(r, z, t) \exp(-pt) dt ,
\]
(2.20)

\[
\tilde{f}(\xi, z, p) = H\{\bar{f}(x, z, p)\} = \int_0^\infty r \bar{f}(x, z, p) J_n(\xi r) dr ,
\]
(2.21)

we obtain

\[
D^2 \ddot{u}_r = \frac{1}{m_3} \left[ (m_1 \xi^2 + p^2) \ddot{u}_r - \xi m_2 D\ddot{u}_\theta + m_4 D\ddot{\phi}_0 + \nu \xi (1 + \tau_\theta) \dddot{\phi} + m_5 \dddot{\phi}^* \right],
\]
(2.22)

\[
D^2 \ddot{u}_\theta = \frac{1}{m_1} \left[ (m_1 \xi^2 + p^2) \ddot{u}_\theta - \xi m_2 D\ddot{u}_r - \xi m_4 D\ddot{\phi}_0 + (1 + \tau_\theta) D\dddot{\phi} + m_5 \dddot{\phi}^* \right],
\]
(2.23)

\[
D^2 \ddot{\phi}_0 = - m_6 D\ddot{u}_r + \xi m_6 \ddot{u}_\theta + (2 m_6 + \xi^2 + m_7 p^2) \dddot{\phi}_0.
\]
(2.24)

\[
D^2 \ddot{\phi} = \varepsilon p (1 + \tau_\theta) \dddot{u}_r + \varepsilon \dddot{u}_r (1 + \tau_\theta) \dddot{\phi} + \left( \dddot{\phi}^2 + p (1 + \tau_\theta) \dddot{\phi} \right) + \varepsilon p m_6 \dddot{\phi},
\]
(2.25)

\[
D^2 \ddot{\phi}^* = \xi m_{12} \dddot{u}_r + m_{12} D\ddot{u}_\theta - m_9 \dddot{\phi} + \left( m_{12} \dddot{\phi}^2 + \xi^2 + m_{10} \right) \dddot{\phi}^*.
\]
(2.26)
The system of Eqs (2.22)-(2.26) can be written as

\[
\frac{dW(\zeta, z, p)}{dz} = A(\zeta, p)W(\zeta, z, p)
\]

(2.27)

where

\[
W = \begin{bmatrix} U \\ U' \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ A_2 & A_1 \end{bmatrix}, \quad U = \begin{bmatrix} \ddot{u}_r & \ddot{u}_z & \ddot{\phi}_0 & \ddot{\Phi} & \ddot{\Phi}^* \end{bmatrix},
\]

\[
A_1 = \begin{bmatrix} 0 & \frac{\zeta m_2}{m_3} & \frac{m_4}{m_3} & 0 & 0 \\ -\frac{\zeta m_2}{m_1} & 0 & 0 & \frac{1 + \tau_1 p}{m_1} & -\frac{m_5}{m_1} \\ -m_6 & 0 & 0 & 0 & 0 \\ 0 & e p(1 + \tau_0 p \Xi) & 0 & 0 & 0 \\ 0 & m_{11} & 0 & 0 & 0 \end{bmatrix},
\]

\[
A_2 = \begin{bmatrix} \frac{(m_2^2 + p^2)}{m_3} & 0 & 0 & -\frac{\zeta (1 + \tau_1 p)}{m_3} & 0 & 0 \\ 0 & \frac{(m_3^2 + p^2)}{m_1} & -\frac{m_5}{m_1} & 0 & 0 \\ 0 & -m_6 \zeta & \left(2m_6 + \zeta^2 + m_7 p^2\right) & 0 & 0 \\ \zeta m_{11} & 0 & 0 & -m_9 & \left(m_{12} p^2 + \zeta^2 + m_{10}\right) \end{bmatrix}
\]

where \(O\) is the null matrix, \(I\) is the unit matrix of order 4, \([\cdot]'\) is the transpose of matrix \([\cdot]\) and \(D = \frac{d}{dz}\).

To solve Eq.(2.27), we take

\[
W(\zeta, z, p) = W(\zeta, p)e^{qz},
\]

(2.28)

for some parameter \(q\) so that

\[
A(\zeta, p)W(\zeta, z, p) = qW(\zeta, z, p),
\]

(2.29)

which leads to the eigen value problem. The characteristic equation corresponding to the matrix \(A\) is given by

\[
det(A - qI) = 0.
\]

(2.30)
On expanding Eq.(2.30), we get

\[ q^{10} - \sigma_1 q^8 + \sigma_2 q^6 - \sigma_3 q^4 + \sigma_4 q^2 + \sigma_5 = 0 \]  
(2.31)

where \( \sigma_i \)s \( (i=1,2,\ldots,5) \) are functions of \( \xi \) and \( p \).

The eigen values of the matrix \( A \) are characteristic roots of Eq.(2.31). The eigen vectors \( X(\xi, p) \) corresponding to the eigen value \( q_s \) can be determined by solving the system of homogenous equations

\[ [A - qI]X(\xi, p) = 0. \]  
(2.32)

The set of eigen vectors \( X_s(\xi, p); s=1,2,\ldots,10 \) may be defined as

\[ X_s(\xi, p) = \begin{bmatrix} X_{i1}(\xi, p) \\ X_{i2}(\xi, p) \end{bmatrix} \]  
(2.33)

where

\[ \begin{align*}
X_{i1} &= [a_i q_i \ b_i \ -\xi \ c_i \ d_i]^T, \\
X_{i2} &= [a_i q_i^2 \ b_i q_i \ -\xi q_i \ c_i q_i \ d_i q_i]^T, \\
q &= q_i; \quad i = 1, 2, 3, 4, 5, \\
X_{j1} &= [-a_i q_i \ b_i \ -\xi \ c_i \ d_i]^T, \\
X_{j2} &= [a_i q_i^2 \ -b_i q_i \ \xi q_i \ -c_i q_i \ -d_i q_i]^T, \\
j &= i + 5; \quad q = -q_i; 
\end{align*} \]

for \( i = 1, 2, 3, 4, 5, \)

\[ \Delta_s = m_s \left( F_1 - F_2 q_s^2 \right), \quad a_s = \frac{q_s}{\Delta_s} \left( m_6 g_{11} F_3 + F_2 H_3 \right), \quad b_s = \frac{-1}{\Delta_s} \left( m_6 g_{11} F_3 q_s^2 + F_1 H_3 \right), \]

\[ \begin{align*}
c_s &= \frac{q_s}{\Delta_s} \left[ e \ pm_8 \left( a_s H_1 + \xi b_s g_{10} - \xi g_{11} \right) - \xi g_7 g_9 \left( b_s + \xi a_s \right) \right], \\
d_s &= \frac{c_s m_9 - m_{11} q_s \left( b_s + \xi a_s \right)}{H_3}, \\
F_1 &= a^* kH_1 - b^* k\xi^2 + b^* f\xi^2 H_5 + gH_1 H_2 H_3 - a^* fh\xi^2 - gh\xi^2 H_4, \\
F_2 &= a^* kc^* - b^* k + b^* fh \xi + gc^* H_4 H_5 - a^* fh - ghH_4, \\
F_3 &= a^* k + ghH_4 H_5, \\
H_1 &= g_1 - q^2, \quad H_2 = g_2 - q^2, \quad H_3 = g_3 - q^2, \quad H_4 = g_4 - q^2, \quad H_5 = g_5 - q^2, \\
a^* &= \epsilon pm_8, \quad b^* = g_7 g_9, \quad c^* = g_{10}, \quad f = g_6, \quad g = g_7, 
\end{align*} \]
Thus the solution of Eq.(2.27) as given by Sharma and Chand (1992) is

$$W(\zeta, z, q) = \sum_{i=1}^{5} \left[ B_i X_i \exp(q_i z) + B_{i+5} X_{i+5} \exp(-q_i z) \right]$$

(2.34)

where $B_i$s ($i=1,2,\ldots,10$) are arbitrary constants. Equation (2.34) represents the solution of the generalized thermo microstretch elastic medium for the axisymmetric case and gives displacement, microrotation, temperature distribution and scalar microstretch in the transformed domain.

3. Application

We consider an infinite generalized thermo microstretch elastic space in which a concentrated force is $F = -F_0 \frac{\delta(r)\delta(t)}{2\pi r}$ where $F_0$ is the magnitude of the force, acting in the direction of the $z$-axis at the origin of the cylindrical polar co-ordinate system as shown in the Fig.1.
The boundary conditions for the plane \( z=0 \) are given by,

\[
\begin{align*}
&u_r(r,0^+,t) - u_r(r,0^-,t) = 0, \quad u_z(r,0^+,t) - u_z(r,0^-,t) = 0, \quad (3.1) \\
&\varphi_0(r,0^+,t) - \varphi_0(r,0^-,t) = 0, \quad \varphi^*(r,0^+,t) - \varphi^*(r,0^-,t) = 0, \quad (3.2) \\
&T(r,0^+,t)T(r,0^-,t) = 0, \quad \frac{\partial T}{\partial z}(r,0^+,t) - \frac{\partial T}{\partial z}(r,0^-,t) = 0, \quad (3.3) \\
&t_{2r}(r,0^+,t) - t_{2r}(r,0^-,t) = 0, \quad t_{zz}(r,0^+,t) - t_{zz}(r,0^-,t) = -F_0 \frac{\delta(r)\delta(t)}{2\pi r}, \quad (3.4) \\
&m_{z0}(r,0^+,t) - m_{z0}(r,0^-,t) = 0, \quad \lambda_{z0}(r,0^+,t) - \lambda_{z0}(r,0^-,t) = 0. \quad (3.5)
\end{align*}
\]

**For \( z>0 \):** Making use of Eqs (2.8) and (2.14) on Eqs (2.5)-(2.7) and \( F'_0 = \frac{F_0}{K} \), we get the stresses in the non-dimensional form with primes. After suppressing the primes and applying the Laplace and Hankel transforms defined by Eqs (2.20) and (2.21) on the resulting equations and using boundary conditions Eqs (3.1)-(3.5), we get the transformed components of displacement, microrotation, scalar microstretch, temperature distribution, tangential force stress, normal force stress, tangential couple stress and microstress for \( z>0 \), given by

\[
\begin{align*}
\tilde{u}_r(\xi,z,p) &= -\sum_{s=1}^{5} a_s q_s B_s + z e^{-q_s z}, \\
\tilde{u}_z(\xi,z,p) &= \sum_{s=1}^{5} b_s B_s z e^{-q_s z},
\end{align*}
\]
\[ \tilde{\phi}_0(z, z, p) = -\xi \sum_{s=1}^{5} B_s s e^{-q_s z}, \] (3.8)

\[ \tilde{\phi}'(z, z, p) = \sum_{s=1}^{5} c_s B_s s e^{-q_s z}, \] (3.9)

\[ \tilde{T}(z, z, p) = \sum_{s=1}^{5} d_s B_s s e^{-q_s z}, \] (3.10)

\[ \tilde{t}_{z\ell}(z, z, p) = \sum_{s=1}^{5} \left( m_s a_s q_s^2 - \xi b_s m_{14} + \xi m_4 \right) B_s s e^{-q_s z}, \] (3.11)

\[ \tilde{t}_{z\ell}(z, z, p) = -\sum_{s=1}^{5} \left[ \xi m_{15} a_s q_s + m_1 b_s q_s - m_{15} c_s + (l + \tau_0 p) d_s \right] B_s s e^{-q_s z}, \] (3.12)

\[ \tilde{m}_{z\ell}(z, z, p) = \xi m_{13} \sum_{s=1}^{5} q_s B_s s e^{-q_s z}, \] (3.13)

\[ \tilde{h}_{z}(z, z, p) = -m_{16} \sum_{s=1}^{5} c_s q_s B_s s e^{-q_s z}. \] (3.14)

For \( z<0 \): the above expressions get suitably modified, e.g.,

\[ \tilde{u}_p(z, z, p) = \sum_{s=1}^{5} a_s q_s B_s s e^{q_s z}. \] (3.15)

where

\[ m_{13} = \frac{\gamma_0^2}{\rho C_l^2}, \quad m_{14} = \frac{\mu}{\rho C_l^2}, \quad m_{15} = \frac{\lambda}{\rho C_l^2}, \quad m_{16} = \frac{\alpha_0 v^2}{\rho C_l^2}. \]

Making use of the transformed displacement, microrotation, temperature distribution, scalar microstretch and stress components given by Eqs (3.6)-(3.14) in region \( z>0 \) and equations for the region \( z<0 \) in the boundary conditions, we obtain ten linear relations between \( B_i \) \((i=1,2,\ldots,10)\) which on solving give

\[
B_i = B_0 = \frac{F_0}{4\pi a_0 \Delta_j} \left[ (a_3 - a_1)((c_4 - c_2)(d_5 - d_2) - (d_4 - d_2)(c_5 - c_2)) + \\
+ (a_4 - a_2)((c_4 - c_2)(d_3 - d_2) - (d_3 - d_2)(c_4 - c_2)) + \\
+ (a_5 - a_2)((c_3 - c_2)(d_4 - d_2) - (d_3 - d_2)(c_4 - c_2)) \right],
\]
\[
B_{3} = B_{7} = \frac{F_{0}}{4 \pi q_{2} A_{1}} \left[ (a_{3} - a_{1})(c_{5} - c_{1})(d_{4} - d_{1}) - (d_{5} - d_{1})(c_{4} - c_{1}) \right] + \\
+ (a_{4} - a_{1})(c_{5} - c_{1})(d_{3} - d_{1}) - (d_{4} - d_{1})(c_{2} - c_{1}) \right] + \\
+ (a_{3} - a_{1})(c_{4} - c_{1})(d_{3} - d_{1}) - (d_{4} - d_{1})(c_{3} - c_{1}) \right].
\]

\[
B_{3} = B_{8} = \frac{F_{0}}{4 \pi q_{3} A_{1}^{*}} \left[ (a_{2} - a_{1})(c_{5} - c_{1})(d_{3} - d_{1}) - (d_{4} - d_{1})(c_{5} - c_{1}) \right] + \\
+ (a_{4} - a_{1})(c_{5} - c_{1})(d_{2} - d_{1}) - (d_{5} - d_{1})(c_{2} - c_{1}) \right] + \\
+ (a_{3} - a_{1})(c_{2} - c_{1})(d_{2} - d_{1}) - (d_{4} - d_{1})(c_{4} - c_{1}) \right].
\]

\[
B_{4} = B_{9} = \frac{F_{0}}{4 \pi q_{4} A_{1}^{*}} \left[ (a_{3} - a_{1})(c_{3} - c_{1})(d_{4} - d_{1}) - (d_{5} - d_{1})(c_{4} - c_{1}) \right] + \\
+ (a_{3} - a_{1})(c_{4} - c_{1})(d_{2} - d_{1}) - (d_{4} - d_{1})(c_{2} - c_{1}) \right] + \\
+ (a_{4} - a_{1})(c_{2} - c_{1})(d_{3} - d_{1}) - (d_{2} - d_{1})(c_{3} - c_{1}) \right].
\]

\[
B_{4} = B_{10} = \frac{F_{0}}{4 \pi q_{5} A_{1}^{*}} \left[ (a_{2} - a_{1})(c_{3} - c_{1})(d_{4} - d_{1}) - (d_{5} - d_{1})(c_{4} - c_{1}) \right] + \\
+ (a_{3} - a_{1})(c_{4} - c_{1})(d_{3} - d_{1}) - (d_{4} - d_{1})(c_{2} - c_{1}) \right] + \\
+ (a_{4} - a_{1})(c_{2} - c_{1})(d_{3} - d_{1}) - (d_{2} - d_{1})(c_{3} - c_{1}) \right].
\]

where

\[
\Delta_{1}^{*} = m_{1} \left[ (c_{2} - c_{3}) \left( a_{1}(d_{3}b_{4} - d_{4}b_{3}) + a_{3}(d_{4}b_{1} - d_{1}b_{4}) + a_{4}(d_{1}b_{3} - d_{3}b_{1}) \right) \right] + \\
+ (c_{2} - c_{3}) \left( a_{1}(d_{4}b_{5} - d_{5}b_{4}) + a_{4}(d_{5}b_{1} - d_{1}b_{5}) + a_{5}(d_{1}b_{2} - d_{2}b_{1}) \right) \right] + \\
+ (c_{4} - c_{5}) \left( a_{1}(d_{2}b_{3} - d_{3}b_{2}) + a_{2}(d_{3}b_{1} - d_{1}b_{3}) + a_{3}(d_{1}b_{2} - d_{2}b_{1}) \right) \right] + \\
+ (c_{2} - c_{4}) \left( a_{1}(d_{3}b_{5} - d_{5}b_{3}) + a_{3}(d_{5}b_{2} - d_{2}b_{5}) + a_{5}(d_{2}b_{1} - d_{1}b_{2}) \right) \right] + \\
+ (c_{4} - c_{5}) \left( a_{1}(d_{3}b_{2} - d_{2}b_{3}) + a_{2}(d_{2}b_{5} - d_{5}b_{2}) + a_{3}(d_{5}b_{1} - d_{1}b_{5}) \right) \right] + \\
+ (c_{3} - c_{5}) \left( a_{1}(d_{2}b_{4} - d_{4}b_{2}) + a_{4}(d_{4}b_{1} - d_{1}b_{4}) + a_{5}(d_{1}b_{3} - d_{3}b_{1}) \right) \right] + \\
+ (c_{3} - c_{5}) \left( a_{1}(d_{2}b_{5} - d_{5}b_{2}) + a_{4}(d_{5}b_{1} - d_{1}b_{5}) + a_{5}(d_{1}b_{4} - d_{4}b_{1}) \right) \right] + \\
+ (c_{4} - c_{5}) \left( a_{1}(d_{3}b_{4} - d_{4}b_{3}) + a_{4}(d_{4}b_{2} - d_{2}b_{4}) + a_{5}(d_{2}b_{3} - d_{3}b_{2}) \right) \right] + \\
+ (c_{4} - c_{5}) \left( a_{1}(d_{3}b_{5} - d_{5}b_{3}) + a_{3}(d_{5}b_{2} - d_{2}b_{5}) + a_{5}(d_{2}b_{1} - d_{1}b_{2}) \right) \right] + \\
+ (c_{2} - c_{1}) \left( a_{2}(d_{4}b_{3} - d_{3}b_{4}) + a_{3}(d_{3}b_{2} - d_{2}b_{3}) + a_{4}(d_{2}b_{4} - d_{4}b_{2}) \right) \right].
\]

Thus functions \( \tilde{u}_{x}, \tilde{u}_{z}, \tilde{\phi}_{0}, \tilde{T}, \tilde{T}_{x}, \tilde{T}_{z}, \tilde{m}_{a}, \tilde{\phi}^{*} \) and \( \tilde{\phi}_{x} \) have been determined in the transform domain and these enable us to find the displacements, microrotation, temperature distribution field, stresses, scalar microstretch and microstress.
4. Method for the inversion of transforms

The transformed solutions are functions of the form \( \tilde{f}(\xi, z, p) \) and to get the function \( f(r, z, p) \), first we invert the Hankel transform by using

\[
\tilde{f}(\xi, z, p) = \int_{0}^{\infty} \xi \tilde{f}(\xi, z, p) J_{\nu}(\xi r) d\xi.
\]

The expression Eq.(4.1) gives us the Laplace transform \( \tilde{f}(\xi, z, p) \) of the function \( f(r, z, p) \). Now for the fixed values of \( r \) and \( z \) the function \( \tilde{f}(\xi, z, p) \) can be considered as the Laplace transform \( \tilde{g}(p) \) of some function \( g(t) \). Following Honig and Hirdes (1984), the Laplace transformed function \( \tilde{g}(p) \) can be inverted numerically as given below.

The function \( g(t) \) can be obtained from \( \tilde{g}(p) \) by using the inversion formula

\[
g(t) = \frac{1}{2\pi i} \int_{c+\rho i}^{c+i\infty} e^{pt} \tilde{g}(p) d\xi
\]

where \( C \) is an arbitrary real number greater than all the real parts of the singularities of \( \tilde{g}(p) \). The actual procedure to invert the Laplace transform consists of Eq.(4.2) together with the \( \varepsilon \)-algorithm. The values of \( C \) and \( L \) are chosen according to the criteria outlined by Honig and Hirdes (1984).

The last step is to calculate the integral in Eq.(4.1). The method for evaluating this integral is described by Press et al. (1986), which involves the use of Romberg’s integration with an adaptive step size. It also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

5. Numerical results and discussion

Following Eringen (1984), we take the following values of relevant parameters for a magnesium crystal as

\[
\lambda = 9.4 \times 10^{10} N/m^2, \quad \mu = 4 \times 10^{10} N/m^2, \quad K = 1 \times 10^{10} N/m^2,
\]

\[
\rho = 1.74 \times 10^{3} kg/m^3, \quad \gamma = 0.779 \times 10^{-9} N, \quad j = 0.2 \times 10^{-19} m^2,
\]

\[
K^* = 1.1753 \times 10^{-19} m^2, \quad \omega^* = 0.0787 \times 10^{-1} N sec/m^2, \quad \tau_0 = 6.131 \times 10^{13} s,
\]

\[
\tau_1 = 8.765 \times 10^{13} s, \quad \varepsilon = 0.073, \quad T_0 = 296 K, \quad \lambda_0 = 0.5 \times 10^{10} N/m^2,
\]

\[
\lambda_1 = 0.5 \times 10^{10} N/m^2, \quad \alpha_0 = 0.779 \times 10^{-9} N, \quad C^* = 3.525 J Kg^{-1}K^{-1}.
\]

The variations of the non-dimensional normal displacement \( U_z (= 4\pi u_z / F_0) \), non-dimensional normal stress \( T_{zz} (= 4\pi u_{zz} / F_0) \), non-dimensional tangential couple stress \( M_{z\theta} (= 4\pi m_{z\theta} / F_0) \), non-
dimensional microstress $\lambda_z^*(=4\pi\dot{\lambda}_z/F_0)$ and non-dimensional temperature distribution $T^*(=4\pi T/F_0)$ with the non-dimensional radial distance $'r'$ at the plane $z=1$, $h=10^{-10}$ m and the coupling coefficient $\varepsilon=0.073$ have been shown in Figs 2-6 for (a) generalized thermo microstretch elastic (GTMSE) medium (b) generalized thermo micropolar elastic (GTME) medium; (c) generalized thermoelastic (GTE) for time $t=0.1$, $0.125$ and $0.5$.

The behaviour of displacement for both theories (L-S and G-L) in all three media (GTMSE, GTMS, GTE) is similar, whereas due to the stretch effect, the value of normal displacement in the GTMSE medium is slightly different as compared to those in the GTME medium for L-S and G-L theories as shown in Fig.2.

The value of normal stress in the GTMSE medium is smaller as compared to those in GTME in the ranges $0<r<1.8$ and $2.8<r<4.6$ but it is larger in the ranges $1.8<r<2.8$ and $4.6<r<6$. The value of normal stress in the GTE is very small as compared to those in the GTMSE and GTME medium in the ranges $0<r<1.5$ and $3.6<r<4.8$, whereas the reverse happens in other ranges. The distribution of normal stress for both the theories in all the three media has been shown in Fig.3.

The stretch effect on tangential couple stress can be observed in Fig.4, where the value of the tangential couple stress in the GTMSE medium is large in the ranges $0<r<1.5$ and $4<r<5.5$; small in the ranges $1.5<r<4$ and $5.5<r<6$ as compared to those in the GTME medium for both the theories.

The behaviour of microstress in the GTMSE medium is similar for both theories, as shown in Fig.5, whereas the value for the G-L theory is large in comparison with those of the L-S theory in the ranges $0<r<1.75$ and $3<r<5$, but is small in the range $1.75<r<3$, while the values are same for both the theories for $r>5$.

The value of the temperature field is large in the GTMSE medium as compared to those in the GTME and GTE media for both theories as depicted in Fig.6, where the value of the temperature field in the GTME medium are multiplied by $10^2$ and $10^4$ for L-S and G-L theories, respectively and in the GTE medium by $10^2$ for both the theories, to show the behavior simultaneously.
Fig. 3. Variations of normal stress $T_{zz} = 4\pi n_{zz}/F_0$.

Fig. 4. Variations of tangential couple stress $M_{z0} (= 4\pi n_{z0}/F_0)$.
Fig. 5. Variations of microstress $\lambda^{*}_z (= 4\pi \lambda_z / F_0)$.

Fig. 6. Variations of temperature distribution $T^* (= 4\pi T / F_0)$. 
6. Conclusion

Hence we conclude that the effect of microstretch on displacement, normal stress, tangential couple stress, microstress and temperature distribution depends upon the radial distance \( r \). Also, for a mechanical source this effect is inversely proportional to the radial distance. Using these results, it is possible to investigate the disturbance caused by a more general source for practical applications.

Nomenclature

\[ C^* \] – specific heat at constant strain
\[ j \] – micro-inertia
\[ K^* \] – coefficient of thermal conductivity
\[ m_{ij} \] – couple stress tensor
\[ T \] – temperature change
\[ t_{ij} \] – force stress tensor
\[ u \] – displacement vector
\[ v, \ v_j \] – mechanical and thermal constant
\[ \alpha, \beta, \gamma, K \] – micropolar material constants
\[ \alpha_{ij}, \alpha_{12} \] – coefficient of linear expansion
\[ \Delta \] – gradient operator
\[ \delta_{ij} \] – Kronecker delta
\[ \epsilon_{ij} \] – alternating tensor
\[ \lambda, \mu \] – Lame’s constants
\[ \lambda_z \] – microstress component
\[ \rho \] – density
\[ \tau_0 \] – thermal relaxation time
\[ \tau_1 \] – thermal relaxation time
\[ \psi \] – microrotation vector
\[ \phi^* \] – scalar microstretch

References


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