BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A PERMEABLE SHRINKING CYLINDER WITH SURFACE MASS TRANSFER

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In the present paper, the axisymmetric boundary layer flow and heat transfer past a permeable shrinking cylinder subject to surface mass transfer is studied. The similarity transformations are adopted to convert the governing partial differential equations for the flow and heat transfer into the nonlinear self-similar ordinary differential equations and then solved by a finite difference method using the quasilinearization technique. From the current investigation, it is found that the velocity in the boundary layer region decreases with the curvature parameter and increases with suction mass transfer. Moreover, with the increase of the curvature parameter, the suction parameter and Prandtl number, the heat transfer is enhanced.

Key words: boundary layer, heat transfer, shrinking cylinder, mass suction.

1. Introduction

The steady hydrodynamic boundary layer flow of an incompressible viscous fluid over a stretching sheet has important applications in manufacturing industries. Crane (1970) first considered the steady laminar boundary layer flow of a Newtonian fluid caused due to linear stretching of a flat sheet and found an exact similarity solution in a closed analytical form. Gupta and Gupta (1977) discussed the heat and mass transfer for the Newtonian boundary layer flow over a stretching sheet with suction or blowing. Wang (1984) investigated the three-dimensional flow due to the stretching surface. The uniqueness of the solution obtained by Crane (1970) was established by McLeod and Rajagopal (1987). Furthermore, some important contributions in stretching sheet flow were made by Chakrabarti and Gupta (1979), Andersson (1992), Pop (1998), Ishak et al. (2008), Bhattacharyya and Layek (2010; 2011), Mukhopadhyay and Gorla (2012).

On the other hand, the day by day increasing applications of the flow of incompressible fluids due to a stretching cylinder attract the researchers to show their interest in this area. Crane (1975) investigated the boundary layer flow due to a stretching cylinder. Later, Wang (1988) also discussed the viscous flow over a stretching cylinder and obtained a similarity solution of the Navier-Stokes equations. Ishak et al. (2008) discussed the effects of mass suction/blowing on the flow and heat transfer due to a stretching cylinder. Ishak et al. (2008) also investigated the magnetohydrodynamic (MHD) flow and heat transfer outside a stretching cylinder. Ishak and Nazar (2009) explained the laminar boundary layer flow along a stretching cylinder

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taking variable surface temperature. Recently, Mukhopadhyay (2011; 2012) studied the chemically reactive solute transfer in boundary layer slip flow over a stretching cylinder and the boundary layer flow and heat transfer along a stretching cylinder in a porous medium.

An interesting character has been observed for the flow past a shrinking sheet. Normally, the steady flow due to shrinking is not possible. The physical reason behind this is that the generated vorticity due to shrinking is not confined within the boundary layer. So, to maintain the boundary layer structure of the flow one needs a certain amount of external opposite force at the sheet. Different aspects of the flow due to a shrinking sheet were discussed in the articles (Miklavčič and Wang, 2006; Hayat et al., 2007; Muhaimin et al., 2008; Fand and Zhang, 2009; Wang, 2008; Ishak et al., 2010; Bhattacharyya et al., 2011a; 2011b; Rosali et al., 2011; Bhattacharyya, 2011a; 2011b; 2011c; Yacob et al., 2011; Bhattacharyya and Pop, 2011; Bhattacharyya, 2011d; 2011e; Ishak et al., 2012; Bhattacharyya and Vajravelu, 2012; Bhattacharyya et al., 2012; Rosali et al., 2012; Bhattacharyya et al., 2012). Motivated by the nature of the shrinking flow in the present paper, the axisymmetric boundary layer flow and heat transfer over a permeable shrinking cylinder with mass suction are investigated. Using similarity transformation, the governing equations are transformed into a set of self-similar non-linear ordinary differential equations, which are then solved numerically by a finite difference method using the quasilinearization technique. The numerical results are plotted in some figures and the variations in velocity and temperature distributions for several physical parameters involved in the equations are discussed in detail.

2. Formulation of the problem

Let us consider the boundary layer flow of Newtonian fluids and heat transfer over a shrinking cylinder with wall mass suction. The governing equations of motion for the steady axisymmetric flow and the energy equation may be written in usual notation as Ishak and Nazar (2009), Mukhopadhyay (2012)

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0, \tag{2.1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = - \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right), \tag{2.2}
\]

and

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{\kappa}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right). \tag{2.3}
\]

where \(u\) and \(v\) are velocity components in the \(x\)- and \(r\)-directions, respectively, \(\nu\) is the kinematic fluid viscosity, \(T\) is the temperature and \(\kappa\) is the fluid thermal diffusivity. The appropriate boundary conditions for the velocity components and temperature are given by

\[
u = -v_w \quad \text{at} \quad r = R, \quad u \to 0 \quad \text{as} \quad r \to \infty, \tag{2.4}
\]

and

\[
T = T_w \quad \text{at} \quad r = R, \quad T \to T_\infty \quad \text{as} \quad r \to \infty. \tag{2.5}
\]

where \(c>0\) is the shrinking constant, \(L\) is the reference length, \(R\) is the radius of the cylinder, \(T_w\) is temperature of the surface of the cylinder and \(T_\infty\) is the free stream temperature with \(T_w>T_\infty\). Here \(v_w(>0)\) is a prescribed distribution of wall mass suction through the porous surface of the cylinder.

We now introduce the following similarity transformations (Mahmood and Merkin, 1988; Ishak, 2009; Ishak and Nazar, 2009)
\[ \psi = \sqrt{U \nu x} R f(\eta), \quad T = T_\infty + (T_w - T_\infty) \theta(\eta) \quad \text{and} \quad \eta = \left( \frac{r^2 - R^2}{2R} \right) \frac{U}{\nu x} \] (2.6)

where \( \psi \) is the stream function defined in the usual notation as \( u = \frac{1}{r} \frac{\partial \psi}{\partial r} \) and \( v = -\frac{1}{r} \frac{\partial \psi}{\partial \theta} \) and \( \eta \) is the similarity variable.

In view of Eqs (2.6), Eq.(2.1) is identically satisfied and Eqs (2.2) and (2.3) reduce to the following self-similar equations

\[ (1 + 2\gamma \eta) f'''' + 2\gamma f'' + \gamma f' - f'^2 = 0, \] (2.7)

and

\[ (1 + 2\gamma \eta) \theta'' + 2\gamma \theta' + \text{Pr} \theta' = 0 \] (2.8)

where \( \gamma = \frac{\nu L}{U_0 R^2} \) is the curvature parameter and \( \text{Pr} = \nu/\kappa \) is the Prandtl number.

The boundary conditions (2.4) and (2.5) reduce to the following forms

\[ f(\eta) = S, \quad f'(\eta) = -1 \quad \text{at} \quad \eta = 0, \quad f'(\eta) \to 0 \quad \text{as} \quad \eta \to \infty, \] (2.9)

and

\[ \theta(\eta) = 1 \quad \text{at} \quad \eta = 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \] (2.10)

where \( S = \nu_0 (cv/L)^{1/2} > 0 \) is the mass suction parameter.

3. Numerical method for solution

The nonlinear system of Eqs (2.7) and (2.8) along with the boundary conditions have been solved numerically by a finite difference method using the quasilinearization technique (Bellman and Kalaba, 1965).

The discretised version of Eqs (2.7) and (2.8) with the boundary conditions (2.9) and (2.10) are written as

\[ (1 + 2\gamma \eta) F''^{(i+1)} + 2\gamma F^{(i+1)} + F^{(i+1)} F^{(i+1)} = -F^{2(i)}, \] (3.1)

and

\[ (1 + 2\gamma \eta) \theta''^{(i+1)} + 2\gamma \theta^{(i+1)} + \text{Pr} F^{(i+1)} \theta^{(i+1)} = 0 \] (3.2)

where \( F = f' \).

The boundary conditions become

\[ f^{(i+1)}(\eta) = S, \quad F^{(i+1)} = -1 \quad \text{at} \quad \eta = 0, \quad F^{(i+1)} = 0 \quad \text{at} \quad \eta = \eta^*, \] (3.3)

and

\[ \theta^{(i+1)} = 1 \quad \text{at} \quad \eta = 0, \quad \theta^{(i+1)} = 0 \quad \text{at} \quad \eta = \eta^*. \] (3.4)

The functions with the iteration index \( (i) \) denote the \( i \)-th iteration level and the corresponding index \( ((i)+1) \) is the \( (i+1) \)-th level and \( \eta^* \) is a suitable dimensionless distance from the origin selected by considering the flow behaviour in the boundary layer region.
We divide the interval \([0, \eta^*]\) into \(N\) equal subintervals of length \(\Delta \eta = 0.001\) taking the non-dimensional distance \(\eta^* = 20\) for all cases under investigation. Applying the central finite difference formulae of the second and first orders derivatives of \(F\) as

\[
F_{j+1} = \frac{F_j + F_{j-1}}{2} \quad \text{and} \quad F' = \frac{F_{j+1} - F_{j-1}}{2 \Delta \eta},
\]

and similar for \(\theta\), the above system of Eqs (3.1) and (3.2) along with the boundary conditions (3.3) and (3.4) reduce to

\[
F_{j-1} a_j + F_j b_j + F_{j+1} c_j = d_j, \quad 1 \leq j \leq N, \tag{3.5}
\]

and

\[
\theta_{j-1}^{(i+1)} p_j + \theta_j^{(i+1)} q_j + \theta_{j+1}^{(i+1)} r_j = 0, \quad 1 \leq j \leq N, \tag{3.6}
\]

with

\[
f_0^{(i+1)} = S, \quad F_0^{(i+1)} = -1 \quad \text{and} \quad F_{N+1}^{(i+1)} = 0, \tag{3.7}
\]

and

\[
\theta_0^{(i+1)} = 1 \quad \text{and} \quad \theta_N^{(i+1)} = 0 \tag{3.8}
\]

where

\[
a_j = \frac{(1 + 2\gamma \eta)}{(\Delta \eta)^2} - \frac{2\gamma + f_j^{(i+1)}}{2(\Delta \eta)}, \quad b_j = \frac{-2(1 + 2\gamma \eta)(\Delta \eta)^2 - 2F_j^{(i)}}{(\Delta \eta)^2}, \quad c_j = \frac{(1 + 2\gamma \eta)}{(\Delta \eta)^2} + \frac{2\gamma + f_j^{(i)}}{2(\Delta \eta)},
\]

\[
d_j = -F_j^{(i+1)}; \quad 1 \leq j \leq N,
\]

\[
p_j = \frac{(1 + 2\gamma \eta)}{(\Delta \eta)^2} - \frac{2\gamma + \Pr f_j^{(i+1)}}{2(\Delta \eta)}, \quad q_j = \frac{-2(1 + 2\gamma \eta)}{(\Delta \eta)^2}, \quad r_j = \frac{(1 + 2\gamma \eta)}{(\Delta \eta)^2} + \frac{2\gamma + \Pr f_j^{(i+1)}}{2(\Delta \eta)}.
\]

\[1 \leq j \leq N.
\]

We solve the system of algebraic (tri-diagonal system) Eqs (3.5) with the conditions (3.7) by the standard Thomas algorithm. Using the newly obtained values of \(f_j^{(i+1)}\) the system (3.6), the discretised temperature equation with the conditions (3.8) are then solved by the same Thomas algorithm.

4. Results and discussion

Numerical computations are performed for various values of the physical parameters involved in the equations, viz., the curvature parameter \(\gamma\), the mass suction parameter \(S\) and the Prandtl number \(\Pr\). To ensure the occurrence of the steady flow near the shrinking cylinder and to confine the generated vorticity inside the boundary layer, the opposite force, i.e., the wall mass suction is taken quite strong by assigning large values of \(S\) in the investigation. The calculated results are presented in some figures to understand the effects of parameters on the flow and temperature field.

The impacts of the curvature parameter \(\gamma\) on the velocity and temperature profiles are very much significant in the flow dynamics. In Fig.1 and Fig.2, the variations in velocity field and temperature
distribution for several values of $\gamma$ are depicted. The dimensionless velocity $f(\eta)$ decreases with increasing values of $\gamma$. This is due to an increase of the momentum boundary layer thickness with $\gamma$. Actually, the increase of the curvature parameter $\gamma$ decreases the skin friction (in a shrinking case, but in a stretching case the effect is opposite (Ishak and Nazar, 2009)) and consequently the momentum boundary layer thickness is increased. On the other hand, from Fig. 2, the converse effect of the curvature parameter $\gamma$ on temperature at a point can be observed. The dimensionless temperature $\theta(\eta)$ at a point increases with $\gamma$, but ultimately similar to velocity distribution the thermal boundary layer thickness becomes thicker.

Next we consider the effects of the mass suction parameter $S$ on the velocity and temperature profiles. The mass suction is essential for the steady flow. Due to the application of suction, the fluid mass is
removed through the permeable surface of the cylinder from the flow region, which controls the generated vorticity due to shrinking and ultimately a steady boundary layer is found in an immediate neighbourhood of the surface of the cylinder. The velocity profiles for several values of the mass suction parameter $S$ are demonstrated in Fig.3. From the figure it is seen that for a fixed value of $\eta$, the velocity increases as mass suction increases, which makes the momentum boundary layer thickness thinner. The temperature profiles for various values of $S$ are plotted in Fig.4. From the figure, it is observed that with increasing mass suction the temperature $\theta(\eta)$ for fixed $\eta$ decreases and consequently, the thickness of the thermal boundary layer reduces.

![Fig.3. Velocity profiles $f(\eta)$ for several values of $S$.](image)

![Fig.4. Temperature profiles $\theta(\eta)$ for several values of $S$.](image)
The temperature profiles for various values of the Prandtl number Pr are illustrated in Fig. 5. With an increasing Pr, the dimensionless temperature profiles as well as the thermal boundary layer thickness quickly decrease. An increase in the Prandtl number means a decrease of fluid thermal conductivity which causes the reduction of the thermal boundary thickness and the fluids with a lower Prandtl number have higher thermal conductivity. Since the momentum equation is independent of $\theta$, so no effect of Pr on the velocity field is observed.

Finally, the values of the skin friction coefficient $f'(0)$ and the temperature gradient at the sheet $-\theta'(0)$ which is proportional to the rate of heat transfer from the surface are presented in Tab. 1. It can be easily found that due to an increase of the curvature parameter $\gamma$ the value of $f'(0)$ decreases and $f'(0)$ increases with the mass suction parameter $S$. While the value of $-\theta'(0)$ increases with the increasing curvature parameter, suction parameter and Prandtl number and so the increase of these parameters enhances the heat transfer from the surface of the cylinder.

Table 1. Values of $f'(0)$ and $-\theta'(0)$ for different values of $\gamma$, $S$ and Pr.

<table>
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<th>$S$</th>
<th>Pr</th>
<th>$f'(0)$</th>
<th>$-\theta'(0)$</th>
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5. Conclusions

The objective of this investigation is to study the axisymmetric boundary layer flow and heat transfer over a permeable shrinking cylinder subject to strong mass suction. Using similarity transformations the
nonlinear self-similar equations are obtained from the governing equations. The self-similar equations are linearised by the quasilinearization technique and are then solved by the finite difference method. This analysis reveals that the increase of the curvature parameter broadens the momentum boundary layer thickness as well as the thermal boundary layer thickness. Velocity inside the boundary layer region increases with mass suction, but the temperature decreases. The temperature as well as the thermal boundary layer thickness decrease with increasing values of the Prandtl number. The heat transfer is enhanced for an increase of the curvature parameter, suction parameter and the Prandtl number.

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Nomenclature

- $c$ – shrinking constant
- $f$ – dimensionless stream function
- $L$ – reference length
- $Pr$ – Prandtl number
- $R$ – radius of the cylinder
- $S$ – mass suction parameter
- $T$ – temperature
- $T_w$ – temperature of the surface of the cylinder
- $T_\infty$ – free stream temperature
- $u, v$ – velocity components
- $v_w$ – distribution of wall mass suction
- $\gamma$ – curvature parameter
- $\eta$ – similarity variable
- $\theta$ – dimensionless temperature
- $\kappa$ – fluid thermal conductivity
- $\nu$ – kinematic fluid viscosity
- $\psi$ – stream function

References


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