General Mathematics Vol. 27, No. 2 (2019), 35–42

DOI:10.2478/gm-2019-0013

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$\mathbb{I}_{\mu}^{*}$ open sets in generalized topological spaces [1]

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Abstract

In this paper we have introduced two new types of sets termed as $\mathbb{I}_{\mu}^{*}$-open sets and strongly $\mathbb{I}_{\mu}^{*}$-open sets and discussed some of its properties. The relation between similar types of sets, characterizations and some basic properties of such sets have been studied.

2010 Mathematics Subject Classification: 54C10, 54C08.
Key words and phrases: μ-open set, ideal, $\mathbb{I}_{\mu}^{*}$-open set, strongly $\mathbb{I}_{\mu}$-open set

1 Introduction

The concept of ideals on topological spaces was studied by Kuratowski [18] and Vaidyanat-haswamy [23] which is one of the important area of research in the branch of mathematics. After then different mathematicians applied the concept of ideals in topological spaces (see [1, 13, 16, 17, 19, 23, 20]). In the past few years mathematicians turned their attention towards generalized open sets (see [2, 3, 6, 8, 9, 10, 11, 14, 15, 21] for details). Our aim in this paper is to use the concept of ideals in generalized topology introduced by A. Császár. We recall some notions defined in [3].

Let $expX$ denotes the power set of a non-empty set X. A class μ ($\subseteqq expX$) is called a generalized topology [3], (briefly, GT) if $\varnothing \in \mu$ and μ is closed under arbitrary union. The elements of μ are called μ-open sets and the complement of μ-open sets are known as μ-closed sets. A set X with a GT μ on it is known as a

[1] *Received 8 November, 2018*
Accepted for publication (in revised form) 15 December, 2019

generalized topological space (briefly, GTS) and is denoted by (X, μ). A GT μ is said to be a quasi topology (briefly QT) [5] if $M, M^{'} \in \mu$ implies $M \cap M^{'} \in \mu$. The pair (X, μ) is said to be a QTS if μ is a QT on X.

For any $A \subsetneqq X$, the generalized μ-closure of A is denoted by $c_{\mu}(A)$ and is defined by $c_{\mu}(A) = \cap\{F : F$ is μ-closed and $A \subsetneqq F\}$, similarly $i_{\mu}(A) = \cup\{U : U \subsetneqq A$ and $U \in \mu\}$ (see [3, 6]). Throughout the paper μ, λ will always mean GT on the respective sets.

An ideal [18] $\mathbb{I}$ on a generalized topological space (X, μ) is a non-empty collection of subsets of X with the following properties : (i) $A \subsetneqq B$ and $B \in \mathbb{I} \Rightarrow A \in \mathbb{I}$ (ii) $A \in \mathbb{I}$, $B \in \mathbb{I} \Rightarrow A \cup B \in \mathbb{I}$.

2 $\mathbb{I}^*_{\mu}$-open sets

Definition 1 *Let $\mathbb{I}$ be an ideal on a GTS (X, μ). A subset A of X is said to be an $\mathbb{I}^*_{\mu}$-open set if A is $\varnothing$ or there exists a non-empty μ-open G such that $G \setminus c_{\mu}(A) \in \mathbb{I}$.*

The complement of an $\mathbb{I}^*_{\mu}$-open set is known as an $\mathbb{I}^*_{\mu}$-closed set.

Theorem 1 *Let $\mathbb{I}$ be an ideal on a GTS (X, μ) and $A \subsetneqq X$. Then A is an $\mathbb{I}^*_{\mu}$-open set in X if and only if $A = \varnothing$ or there exists a set $I \in \mathbb{I}$ and a non-empty μ-open set G such that $G \setminus I \subsetneqq c_{\mu}(A)$.*

Proof. Let A be an $\mathbb{I}^*_{\mu}$-open set in X. If $A = \varnothing$ we have nothing to show. Suppose that $A \neq \varnothing$. Then there exists a non-empty μ-open set G such that $G \setminus c_{\mu}(A) \in \mathbb{I}$. Put $I = G \setminus c_{\mu}(A)$. Thus $I \in \mathbb{I}$ and $G \setminus I \subsetneqq c_{\mu}(A)$.

Conversely, suppose that $A = \varnothing$ or there exists a set $I \in \mathbb{I}$ and a non-empty μ-open set G such that $G \setminus I \subsetneqq c_{\mu}(A)$. Then $G \setminus c_{\mu}(A) \subsetneqq I$. Thus $G \setminus c_{\mu}(A) \in \mathbb{I}$ showing that A is an $\mathbb{I}^*_{\mu}$-open set. If $A = \varnothing$ then we have nothing to show.

Theorem 2 *Let $\mathbb{I}$ be an ideal on a GTS (X, μ) and $A \subsetneqq X$. Then A is an $\mathbb{I}^*_{\mu}$-open set in X if and only if either $A = \varnothing$ or there exists a non-empty μ-open G and $I \in \mathbb{I}$ such that $G \subsetneqq c_{\mu}(A) \cup I$.*

Proof. Follows from Theorem 1.

Definition 2 *[21] Let $\mathbb{I}$ be an ideal on a GTS (X, μ). A subset A of X is called weakly $\mathbb{I}_{\mu}$-open if $A = \varnothing$ or if $A \neq \varnothing$, there exists a non-empty μ-open set U such that $U \setminus A \in \mathbb{I}$. The complement of a weakly $\mathbb{I}_{\mu}$-open set is termed as a weakly $\mathbb{I}_{\mu}$-closed set.*

Remark 1 *Every weakly $\mathbb{I}_{\mu}$-open set in a GTS (X, μ) with an ideal $\mathbb{I}$ is an $\mathbb{I}^*_{\mu}$-open set but the converse is not true as shown in the next example. Also we note that every μ-open set is $\mathbb{I}^*_{\mu}$-open. In fact, μ-open set $\Rightarrow$ weakly $\mathbb{I}_{\mu}$-open set $\Rightarrow$ $\mathbb{I}^*_{\mu}$-open set.*

Example 1 *Let* $X = \{a, b, c\}$*,* $\mu = \{\varnothing, \{a\}, \{a, b\}, \{b, c\}, X\}$ *and* $\mathbb{I} = \{\varnothing, \{b\}\}$*. Then* $\mathbb{I}$ *is an ideal on the GTS* (X, μ)*. It is easy to check that* $\{b\}$ *is an* $\mathbb{I}_{\mu}^{*}$*-open set but not a weakly* $\mathbb{I}_{\mu}$*-open set (and hence not* μ*-open).*

Observation 1 *Let* $\mathbb{I}$ *be an ideal on a GTS* (X, μ)*. Then every* μ*-closed,* $\mathbb{I}_{\mu}^{*}$*-open subset is weakly* $\mathbb{I}_{\mu}^{*}$*-open. If* $\mathbb{I}$ *and* $\mathbb{J}$ *be two ideals on a GTS* (X, μ) *with* $\mathbb{I} \subseteqq \mathbb{J}$*, then every* $\mathbb{I}_{\mu}^{*}$*-open set is* $\mathbb{J}_{\mu}^{*}$*-open. If* $\mathbb{I}$ *is an ideal and* μ *and* λ *be two GT's on* X *with* $\mu \subseteqq \lambda$*, then every* $\mathbb{I}_{\mu}^{*}$*-open set is* $\mathbb{I}_{\lambda}^{*}$*-open.*

Example 2 *(a) Let* $X = \{a, b, c\}$*,* $\mu = \{\varnothing, \{a, b\}, \{b, c\}, X\}$ *and* $\mathbb{I} = \{\varnothing, \{a\}\}$*. Then* $\mathbb{I}$ *is an ideal on the GTS* (X, μ)*. It is easy to check that* $\{a, c\}$ *and* $\{b, c\}$ *are two* $\mathbb{I}_{\mu}^{*}$*-open sets but their intersection* $\{c\}$ *is not an* $\mathbb{I}_{\mu}^{*}$*-open set.*

(b) Let $X = \{a, b, c\}$*,* $\mu = \{\varnothing, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$*,* $\lambda = \{\varnothing, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$ *and* $\mathbb{I} = \{\varnothing, \{b\}\}$*. Then* $\mathbb{I}$ *be an ideal on the GTS's* (X, μ) *and* (X, λ)*. It is easy to check that the collection of* $\mathbb{I}_{\mu}^{*}$*-open sets and* $\mathbb{I}_{\lambda}^{*}$*-open sets are same though* μ *and* λ *are not comparable.*

(c) Let $X = \{a, b, c\}$*,* $\mu = \{\varnothing, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$*,* $\mathbb{I} = \{\varnothing, \{b\}\}$ *and* $\mathbb{J} = \{\varnothing, \{c\}\}$*. Then* $\mathbb{I}$ *and* $\mathbb{J}$ *are two ideals on the GTS* (X, μ)*. It is easy to check that the collection of* $\mathbb{I}_{\mu}^{*}$*-open sets and* $\mathbb{J}_{\mu}^{*}$*-open sets are same though* $\mathbb{I}$ *and* $\mathbb{J}$ *are not comparable.*

Theorem 3 *Let* $\mathbb{I}$ *be an ideal on a GTS* (X, μ)*. Then the family of all* $\mathbb{I}_{\mu}^{*}$*-open sets of* (X, μ) *is a GT on* X*.*

Proof. Suppose that $\{A_{\alpha} : \alpha \in \Lambda\}$ be a family of $\mathbb{I}_{\mu}^{*}$-open sets in (X, μ). If $A_{\alpha} = \varnothing$ for all $\alpha \in \Lambda$, then we have nothing to show. Suppose that $A_{\alpha} \neq \varnothing$ for at least one $\alpha \in \Lambda$. Then there exists a non-empty μ-open set U such that $U \setminus c_{\mu}(A_{\alpha}) \in \mathbb{I}$. This implies that $U \setminus c_{\mu}(\cup\{A_{\alpha} : \alpha \in \Lambda\}) \subseteqq U \setminus c_{\mu}(A_{\alpha}) \in \mathbb{I}$. Thus $U \setminus c_{\mu}(\cup\{A_{\alpha} : \alpha \in \Lambda\}) \in \mathbb{I}$. Hence $\cup\{A_{\alpha} : \alpha \in \Lambda\})$ is also $\mathbb{I}_{\mu}^{*}$ open. Also note that by definition $\varnothing$ is an $\mathbb{I}_{\mu}^{*}$ open set.

Theorem 4 *Let* $\mathbb{I}$ *be an ideal on a topological space* (X, τ) *and* $A \subseteqq X$*. Then the following are equivalent:*
(i) A *is* $\mathbb{I}_{\mu}^{*}$*-closed in* X*,*
(ii) $A = X$ *or there exists an element* $I \in \mathbb{I}$ *and a* μ*-closed set* $K \neq X$ *such that* $i_{\mu}(A) \setminus I \subseteqq K$*,*
(iii) $A = X$ *or there exists a* μ*-closed set* $K \neq X$ *such that* $i_{\mu}(A) \setminus K \in \mathbb{I}$*.*

Proof. (i) $\Rightarrow$ (ii) : Let A be an $\mathbb{I}_{\mu}^{*}$-closed set. Then either $A = X$ or $A \neq X$. If $A \neq X$, then $X \setminus A \neq \varnothing$ and $X \setminus A$ is $\mathbb{I}_{\mu}^{*}$-open. Then there exists a non-empty μ-open set G such that $G \setminus c_{\mu}(X \setminus A) \in \mathbb{I}$. Put $I = G \setminus c_{\mu}(X \setminus A)$. Then $I \in \mathbb{I}$ and $G \subseteqq X \setminus i_{\mu}(A) \cup I$. This implies that intersection of $X \setminus X \setminus i_{\mu}(A)$ and $X \setminus I$ is contained in $X \setminus G$. Put $K = X \setminus G$. Then K is μ-closed and $K \neq X$. Hence $i_{\mu}(A) \cap (X \setminus I)$ is contained in K. Hence $i_{\mu}(A) \setminus I \subseteqq K$.

(ii) $\Rightarrow$ (i) : Suppose that $A = X$ or there exists an element $I \in \mathbb{I}$ and a μ-closed set $K \neq X$ such that $i_\mu(A) \setminus I \subsetneqq K$. If $A = X$, then A is $\mathbb{I}^*_\mu$-closed. Assume that there exists an element $I \in \mathbb{I}$ and a μ-closed set $K \neq X$ such that $i_\mu(A) \setminus I \subsetneqq K$. Then $X \setminus K \subsetneqq (X \setminus i_\mu(A)) \cup I$. Take $U = X \setminus K$. Then U is μ-open and $U \subsetneqq c_\mu(Y \setminus A) \cup I$. Thus by Theorem 2, $X \setminus A$ is $\mathbb{I}^*_\mu$-open and hence A is $\mathbb{I}^*_\mu$-closed.

(ii) $\Rightarrow$ (iii) : Suppose that there exists an element $I \in \mathbb{I}$ and a μ-closed set $K \neq X$ such that $i_\mu(A) \setminus I \subsetneqq K$. Then $i_\mu(A) \setminus K \subsetneqq I$ and hence $i_\mu(A) \setminus K \in \mathbb{I}$.

(iii) $\Rightarrow$ (ii) : Suppose that there exists a μ-closed set $K \neq X$ such that $i_\mu(A) \setminus K \in \mathbb{I}$. Let $I = i_\mu(A) \setminus K$. Then I is an element of $\mathbb{I}$ and $i_\mu(A) \setminus I \subsetneqq K$.

Definition 3 *Let (X, μ) be a GTS and $A \subsetneqq X$. Then A is called μ-dense [12, 7] if $c_\mu(A) = X$.*

Theorem 5 *Let $\mathbb{I}$ be an ideal and $\mu(\neq \{\varnothing\})$ be a GT on X. If A is μ-dense, then A is $\mathbb{I}^*_\mu$-open.*

Proof. Let A be a μ-dense subset of X. Since $\mu \neq \{\varnothing\}$, there exists a non-empty set μ-open subset U of X. Then $U \setminus c_\mu(A) = U \setminus X = \varnothing \in \mathbb{I}$. Thus A is $\mathbb{I}^*_\mu$-open.

Theorem 6 *Let $\mathbb{I}$ be an ideal on a GTS (X, μ). If $A \in \mathbb{I} \cap \mu \setminus \{\varnothing\}$, then every subset B of X is $\mathbb{I}^*_\mu$-open.*

Proof. Let B be any subset of X. Then $A \setminus c_\mu(B) \in \mathbb{I}$ (as $A \setminus c_\mu(B) \subseteq A$). Hence B is $\mathbb{I}^*_\mu$-open.

Example 3 *$X = \{a, b, c\}$, $\mu = \{\varnothing, \{a, b\}, \{a, c\}, X\}$ and $\mathbb{I} = \{\varnothing, \{a\}\}$. Then $\mathbb{I}$ is an ideal on the GTS (X, μ). It can be checked easily that there does not exist any A such that $A \in \mathbb{I} \cap \mu \setminus \{\varnothing\}$ but every subset of X is $\mathbb{I}^*_\mu$-open.*

Definition 4 *Let (X, μ) be a GTS and $A \subsetneqq X$. A is said to be μ-nowhere dense [12, 7] in (X, μ) if $i_\mu(c_\mu(A)) = \varnothing$.*

Theorem 7 *Let $\mathbb{I}$ be an ideal on a GTS (X, μ) and $A(\neq \varnothing) \subsetneqq X$. If A is not a μ-nowhere dense set, then A is an $\mathbb{I}^*_\mu$-open set.*

Proof. Let $A \neq \varnothing$ be not a μ-nowhere dense set. Then $i_\mu(c_\mu(A)) \neq \varnothing$. Also $i_\mu(c_\mu(A)) \subsetneqq c_\mu(A))$. Put $U = i_\mu(c_\mu(A))$. Then U is a non-empty μ-open set such that $U \setminus c_\mu(A) \in \mathbb{I}$. Thus A is an $\mathbb{I}^*_\mu$-open set.

Theorem 8 *Let $\mathbb{I}$ be an ideal on a GTS (X, μ). Let $A \neq \varnothing$ be an $\mathbb{I}^*_\mu$-open set and $A \subsetneqq B$. Then B is an $\mathbb{I}^*_\mu$-open set.*

Proof. Let $A \neq \varnothing$ be an $\mathbb{I}^*_\mu$-open set. Then there exists a non-empty μ-open set U such that $U \setminus c_\mu(A) \in \mathbb{I}$. Then $U \setminus c_\mu(B) \subsetneqq U \setminus c_\mu(A) \in \mathbb{I}$. Thus B is an $\mathbb{I}^*_\mu$-open set.

We recall that a subset A of a GTS (X, μ) is called μ-semiopen [4] if $A \subsetneqq i_\mu(c_\mu(A))$ and the complement of a μ-semiopen set is known as μ-semiclosed.

Theorem 9 *Let $\mathbb{I}$ be an ideal on a GTS (X,μ). Then $\{x\}$ is either μ-semiclosed or an $\mathbb{I}^*_\mu$-open set.*

Proof. If $\{x\}$ is μ-semiclosed, then we have nothing to show. If $\{x\}$ is not μ-semiclosed, then $i_\mu(c_\mu(\{x\})) \nsubseteq \{x\}$. Then $i_\mu(c_\mu(\{x\})) \setminus c_\mu(\{x\}) \in \mathbb{I}$.

Definition 5 *A GTS (X,μ) is said to be μ-locally indiscrete if every μ-open set is μ-closed.*

Theorem 10 *Let $\mathbb{I}$ be an ideal on a GTS (X,μ). If (X,μ) is μ-locally indiscrete, then every subset of X is $\mathbb{I}^*_\mu$-open.*

Proof. Suppose $A \subseteqq X$. Then if $A = \varnothing$ we have nothing to show. Thus without loss of generality we may assume that $A \neq \varnothing$. Since X is μ-locally indiscrete, $c_\mu(A)$ is μ-open. Also $c_\mu(A) \neq \varnothing$. Let $U = c_\mu(A)$. Then $U \setminus c_\mu(A) \in \mathbb{I}$. Thus A is an $\mathbb{I}^*_\mu$-open set.

3 Strongly $\mathbb{I}^*_\mu$-open sets

Definition 6 *Let $\mathbb{I}$ be an ideal in a GTS (X,μ). A subset A of X is said to be a strongly $\mathbb{I}^*_\mu$-open set if A is $\varnothing$ or there exists a non-empty μ-open G such that $G \setminus A \in \mathbb{I}$ and $A \setminus c_\mu(G) \in \mathbb{I}$.*

We note from Definitions 1 and 6 that every μ-open set is a strongly $\mathbb{I}^*_\mu$-open set but the converse is not true. Also, every strongly $\mathbb{I}^*_\mu$-open set is an $\mathbb{I}^*_\mu$-open set.

Example 4 *Let $X = \{a,b,c\}$, $\mu = \{\varnothing, \{a,b\}, \{a,c\}, X\}$ and $\mathbb{I} = \{\{\varnothing\}, \{a\}\}$. It is easy to observe that $\{b\}$ is a strongly $\mathbb{I}^*_\mu$-open set which is not μ-open. $\{a\}$ is an $\mathbb{I}^*_\mu$-open set but not a strongly $\mathbb{I}^*_\mu$-open set. Also note that $\{a,b\}$ and $\{a,c\}$ are two strongly $\mathbb{I}^*_\mu$-open sets but their intersection is not so.*

Theorem 11 *Let $\mathbb{I}$ be an ideal in a GTS (X,μ) and suppose that there exists a μ-open μ-dense set $A \in \mathbb{I}$. Then every subset B of X is strongly $\mathbb{I}^*_\mu$-open.*

Proof. Let B be any subset of X. Note that $A \setminus B \in \mathbb{I}$ (as $A \setminus B \subseteqq A \in \mathbb{I}$). Furthermore, $B \setminus c_\mu(A) = B \setminus X = \varnothing \in \mathbb{I}$.

Proposition 1 *Let $\mathbb{I}$ be an ideal in a GTS (X,μ) and suppose that $A \subseteqq B \subseteqq c_\mu(A)$ for some μ-open subset A of X. Then B is a strongly $\mathbb{I}^*_\mu$-open set.*

Proof. Obvious.

Definition 7 *A GTS (X,μ) is said to be μ-irreducible [22] if for any two non-empty μ-open sets U and V of X, $U \cap V \neq \varnothing$.*

Definition 8 *A GTS (X,μ) is called hyperconnected [12] if every non-empty μ-open subset of X is μ-dense.*

Remark 2 *A GTS (X,μ) is hyperconnected [12] if and only if (X,μ) is μ-irreducible.*

Proposition 2 *Let $\mathbb{I}$ be an ideal in a GTS (X,μ) and suppose that every non-empty μ-open subset of X is μ-dense and let A be a strongly $\mathbb{I}^*_\mu$-open set.*
(i) If $A \subsetneqq B$, then B is a strongly $\mathbb{I}^*_\mu$-open set.
(ii) For any subset B of X, $A \cup B$ is a strongly $\mathbb{I}^*_\mu$-open set.
(iii) Moreover if μ is closed under finite intersection and (X,μ) is hyperconnected, then intersection of two strongly $\mathbb{I}^*_\mu$-open sets is strongly $\mathbb{I}^*_\mu$-open.

Proof. (i) Suppose that A is a strongly $\mathbb{I}^*_\mu$-open set and $A \subsetneqq B$. Then there exists a non-empty μ-open set G such that $G \setminus A \in \mathbb{I}$ and $A \setminus c_\mu(G) \in \mathbb{I}$. Since $A \subsetneqq B$, $G \setminus B \subsetneqq G \setminus A \in \mathbb{I}$ and $B \setminus c_\mu(G) = B \setminus X = \varnothing \in \mathbb{I}$.

(ii) Follows from (i) as $A \subsetneqq A \cup B$.

(iii) Let A and B be two strongly $\mathbb{I}^*_\mu$-open sets. If $A \cap B = \varnothing$, then there is nothing to show. Suppose that $A \cap B \neq \varnothing$. Then there exist non-empty μ-open sets G and H such that $G \setminus A \in \mathbb{I}$, $A \setminus c_\mu(G) \in \mathbb{I}$, $H \setminus B \in \mathbb{I}$ and $B \setminus c_\mu(H) \in \mathbb{I}$. Consider the μ-open set $G \cap H$ which is non-empty (as (X,μ) is hyperconnected). Since $G \cap H \setminus (A \cap B) = [(G \setminus A) \cap H] \cup [(H \setminus B) \cap G] \in \mathbb{I}$, $A \cap B \setminus c_\mu(G \cap H) = (A \cap B) \setminus X = \varnothing \in \mathbb{I}$. Thus $A \cap B$ is strongly $\mathbb{I}^*_\mu$-open.

Proposition 3 *Let $\mathbb{I}$ be an ideal in a QTS (X,μ) and suppose that every non-empty μ-open set is μ-dense. Then a non μ-dense subset A is strongly $\mathbb{I}^*_\mu$-open if and only if $c_\mu(A)$ is strongly $\mathbb{I}^*_\mu$-open.*

Proof. Let A be a strongly $\mathbb{I}^*_\mu$-open set. Then as $A \subsetneqq c_\mu(A)$, by Proposition 2(i), $c_\mu(A)$ is also strongly $\mathbb{I}^*_\mu$-open. Conversely, suppose that $c_\mu(A)$ is strongly $\mathbb{I}^*_\mu$-open which is not μ-dense. Then there exists a non-empty μ-open set G such that $G \setminus c_\mu(A)$ and $c_\mu(A) \setminus c_\mu(G)$ are both members of $\mathbb{I}$. Consider the μ-open set $H = G \setminus c_\mu(A) = G \cap (X \setminus c_\mu(A)) \in \mathbb{I}$. We observe that $H \neq \varnothing$. For otherwise, $G \subsetneqq c_\mu(A) \Rightarrow c_\mu(A) = X$. Again $H \setminus A = [G \cap (X \setminus c_\mu(A))] \cap (X \setminus A) \subsetneqq G \cap (X \setminus c_\mu(A)) \in \mathbb{I}$. Thus $H \setminus A \in \mathbb{I}$. Again $A \setminus c_\mu(H) = A \setminus c_\mu(G \cap (X \setminus c_\mu(A))) = A \setminus X = \varnothing \in \mathbb{I}$.

Theorem 12 *Let $\mathbb{I}$ be an ideal in a GTS (X,μ). Then $X \setminus A$ is strongly $\mathbb{I}^*_\mu$-open if and only if there exists a μ-closed set F such that $i_\mu(F) \setminus A \in \mathbb{I}$ and $A \setminus F \in \mathbb{I}$.*

Proof. Suppose that $X \setminus A$ is strongly $\mathbb{I}^*_\mu$-open. Then there exists a μ-open set G such that $(G \setminus (X \setminus A)) = A \setminus (X \setminus G) \in \mathbb{I}$ and $(X \setminus A) \setminus c_\mu(G) = i_\mu(X \setminus G) \setminus A \in \mathbb{I}$. Let $F = X \setminus G$. Then F is μ-closed and the rest follows. The converse part can be done in a similar manner.

Acknowledgement: The authors are thankful to the referees for some comments for the improvement of the paper.

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