Forgotten topological index and reduced Zagreb index of four new operations of graphs ¹

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Abstract

Indulal and Balakrishnan (2016) have put forward the Indu-Bala product and based on this product four new operations are defined by the authors of this manuscript in the paper “Four new operations of graphs based on Indu-Bala product and the Zagreb indices”. In this paper we establish explicit formulas of the forgotten topological index and reduced second Zagreb index in connection with these new operations of graphs.

2010 Mathematics Subject Classification: 05C76, 05C07.

Key words and phrases: forgotten topological index, reduced Zagreb index, operations of graphs.

1 Introduction

Topological index, also called molecular structure descriptor, of a chemical compound is a numerical value associated with the chemical compounds which are calculated from its respective molecular graph. Over the years a huge number of graph invariants were introduced and among these Zagreb indices are the oldest vertex-degree based topological indices which are defined by *Gutman* and *Trinajstić* [11] in connection with π -orbital energy of molecules. A flood of publications and research papers were reported since its recognition as topological indices. Some of these studies may be found in [5, 7, 10, 14, 22].

The first Zagreb index, $M_1(G)$ and second Zagreb index, $M_2(G)$ of a graph G are defined as

$$M_1(G) = \sum_{u \in V(G)} d_G^2(u) = \sum_{uv \in E(G)} (d_G(u) + d_G(v)), \quad M_2(G) = \sum_{uv \in E(G)} d_G(u)d_G(v).$$

¹Received 27 August, 2018.

Accepted for publication (in revised form) 10 April, 2019

Historically, first and second Zagreb indices are known to be the first topological indices. Many modifications and generalizations of these indices are proposed and studied over the years [12, 17, 20]. Some of the studies of the generalization of the first Zagreb index denoted as M_1^α may be found in [15, 16]. For a connected graph G , M_1^α can be defined as

$$M_1^\alpha(G) = \sum_{u \in V(G)} d_G^\alpha(u).$$

This index is also studied under the name of zeroth order general Randić index [20]. For $\alpha = 3$, i.e., $M_1^3(G) = \sum_{u \in V(G)} d_G^3(u)$ is popularly known as forgotten topological index and is usually denoted as $M_3(G)$ [8].

In 2014, *Furtula et al.* proposed another topological index during their study on difference of Zagreb indices [9]. They named this index as Reduced second Zagreb index, which can be defined as

$$RM_2(G) = \sum_{uv \in E(G)} (d_G(u) - 1)(d_G(v) - 1).$$

In this communication we have considered only the simple connected graphs. Let G be a simple graph. We use $V(G)$ and $E(G)$ to denote the set of vertices and edges of G respectively. $d_G(u)$ denotes the degree of the vertex u in G . All other graph theoretic notations that are used in this paper are taken from [21].

The fact that along with the atom-atom interactions the intermolecular forces also exist between the atoms and bonds of a molecule leads to the four operations of graph a G viz., $S(G)$, $R(G)$, $Q(G)$ and $T(G)$ [3]. For a connected graph G , these four related graphs are given as:

- $S(G)$ is the graph obtained by replacing each edge of G by a path of length 2.
- $R(G)$ is obtained from G by adding a path of length 2 parallel to each of the edges of G .
- $Q(G)$ is obtained from G by replacing each edge by a path of length 2, then joining with edges those pairs of new vertices on adjacent edges of G .
- $T(G)$ has as its vertices the edges and vertices of G . Adjacency in $T(G)$ is defined as adjacency or incidence for the corresponding elements of G .

Different operations of graphs based on these four related graphs were defined and studied in connection with Wiener index [6], Zagreb indices [5, 18, 19] and forgotten index [1, 4].

In this paper, we establish formulae for the number of vertices and edges for graphs obtained by the four new operations of graphs. We also propose explicit expressions for the forgotten topological index, Reduced second Zagreb index of these new graph operations. The rest of the paper is organised as follows. In section 2 the new operations of graphs along with some results are discussed. In section 3 and 4 the expressions for forgotten topological index and reduced second Zagreb index are presented and in section 5 conclusions are made.

2 The new F -sums of graphs

An example of the four new operations of graphs based on the Indu-Bala product is given in Figure 1. The new F -sum of G_1 and G_2 , denoted by $G_1 \nabla_F G_2$, is defined to be the graph with $V(G_1 \nabla_F G_2) = V(G_1) \cup E(G_1) \cup V(G_2)$ and $E(G_1 \nabla_F G_2) = E(F(G_1) \nabla G_2) \setminus E^*$, where ∇ is the Indu-Bala product of graphs [13] and $E^* = \{uv | u \in V(F(G_1)) \setminus V(G_1), v \in V(G_2)\}$.

Lemma 1. *Let G_1, G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then,*

(a)

$$d_{G_1 \nabla_S G_2}(u) = \begin{cases} d_{G_1}(u) + n_2 & \text{if } u \in V(G_1) \\ 2 & \text{if } u \in V(S(G_1)) \setminus V(G_1) \\ d_{G_2}(u) + n_1 + 1 & \text{if } u \in V(G_2). \end{cases}$$

(b)

$$d_{G_1 \nabla_R G_2}(u) = \begin{cases} 2d_{G_1}(u) + n_2 & \text{if } u \in V(G_1) \\ 2 & \text{if } u \in V(R(G_1)) \setminus V(G_1) \\ d_{G_2}(u) + n_1 + 1 & \text{if } u \in V(G_2). \end{cases}$$

(c)

$$d_{G_1 \nabla_Q G_2}(u) = \begin{cases} d_{G_1}(u) + n_2 & \text{if } u \in V(G_1) \\ d_{G_1}(w) + d_{G_1}(w') & \text{if } u \in V(Q(G_1)) \setminus V(G_1) \\ d_{G_2}(u) + n_1 + 1 & \text{if } u \in V(G_2), \end{cases}$$

where in the second case u is inserted into the edge $ww' \in E(G_1)$.

(d)

$$d_{G_1 \nabla_T G_2}(u) = \begin{cases} 2d_{G_1}(u) + n_2 & \text{if } u \in V(G_1) \\ d_{G_1}(w) + d_{G_1}(w') & \text{if } u \in V(T(G_1)) \setminus V(G_1) \\ d_{G_2}(u) + n_1 + 1 & \text{if } u \in V(G_2), \end{cases}$$

where in the second case u is inserted into the edge $ww' \in E(G_1)$.

Lemma 2. *Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then*

$$|V(G_1 \nabla_S G_2)| = |V(G_1 \nabla_R G_2)| = |V(G_1 \nabla_Q G_2)| = |V(G_1 \nabla_T G_2)| = 2[n_1 + m_1 + n_2].$$

Lemma 3. *Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then*

$$|E(G_1 \nabla_S G_2)| = 2[n_1 n_2 + 2m_1 + m_2] + n_2.$$

Figure 1: An example of the four new operations of graphs based on Indu-Bala product

Lemma 4. Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$|E(G_1 \nabla_R G_2)| = 2[n_1 n_2 + 3m_1 + m_2] + n_2.$$

Lemma 5. Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$|E(G_1 \nabla_Q G_2)| = 2[n_1 n_2 + 2m_1 + m_2 + \sum_{v \in V(G_1)} \binom{d_{G_1}(v)}{2}] + n_2.$$

Lemma 6. Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$|E(G_1 \nabla_T G_2)| = 2[n_1 n_2 + 3m_1 + m_2 + \sum_{v \in V(G_1)} \binom{d_{G_1}(v)}{2}] + n_2.$$

The first Zagreb index and the second Zagreb index of the new F -sums of graphs are given below.

Theorem 1. [2] Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$ where $i = 1, 2$. Then

$$\begin{aligned} M_1(G_1 \nabla_S G_2) = & 2 \left[M_1(G_1) + M_1(G_2) + n_1 n_2 (n_1 + n_2 + 2) \right. \\ & \left. + 4(m_1 + m_2) + 4n_1 m_2 + n_2 \right]. \end{aligned}$$

Theorem 2. [2] Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$\begin{aligned} M_2(G_1 \nabla_S G_2) = & M_1(G_2) + (n_1 + 1)\{n_2(n_1 + 1) + 4m_2\} + 2[2M_1(G_1) + (n_1 + 1)M_1(G_2) \\ & + M_2(G_2) + m_2(n_1 + 1)^2 + n_2(n_1 + 1)(2m_1 + n_1n_2) \\ & + 2m_2(2m_1 + n_1n_2) + 4m_1n_2]. \end{aligned}$$

Theorem 3. [2] Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$\begin{aligned} M_1(G_1 \nabla_R G_2) = & 2\left[4M_1(G_1) + M_1(G_2) + n_1n_2(n_1 + n_2 + 2) \right. \\ & \left. + 4m_2(n_1 + 1) + n_2(8m_1 + 1) + 4m_1\right]. \end{aligned}$$

Theorem 4. [2] Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$\begin{aligned} M_2(G_1 \nabla_R G_2) = & M_1(G_2) + 4m_2(n_1 + 1) + n_2(n_1 + 1)^2 + 2\left[2(n_2 + 2)M_1(G_1) \right. \\ & + (n_1 + 1)M_1(G_2) + 4M_2(G_1) + M_2(G_2) + m_2(n_1 + 1)^2 \\ & \left. + n_2(n_1 + 1)(4m_1 + n_1n_2) + 2m_2(4m_1 + n_1n_2) + 4m_1n_2 + m_1n_2^2\right]. \end{aligned}$$

Theorem 5. [2] Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$\begin{aligned} M_1(G_1 \nabla_Q G_2) = & 2\left[M_1(G_1) + M_1(G_2) + 2M_2(G_1) + M_3(G_1) \right. \\ & \left. + n_1n_2(n_1 + n_2 + 2) + n_2(4m_1 + 1) + 4m_2(n_1 + 1)\right]. \end{aligned}$$

Theorem 6. [2] Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$\begin{aligned} M_2(G_1 \nabla_Q G_2) = & M_1(G_2) + 4m_2(n_1 + 1) + n_2(n_1 + 1)^2 + 2[2n_2M_1(G_1) + (n_1 + 1)M_1(G_2) \\ & + M_2(G_2) + \frac{1}{2}(M_3(G_1) + M_4(G_1)) + (n_1 + 1)\{m_2(n_1 + 1) \\ & + n_2(2m_1 + n_1n_2)\} + 2m_2(2m_1 + n_1n_2) + \sum_{u,w \in V(G_1)} \gamma_{uw}d_{G_1}(u)d_{G_1}(w) \\ & + \sum_{u \in V(G_1)} d_{G_1}^2(u) \sum_{\substack{v \in V(G_1), \\ uv \in E(G_1)}} d_{G_1}(v)], \end{aligned}$$

where γ_{uw} is the number of common neighbors of $u, w \in V(G_1)$ and M_4 is the generalized first Zagreb index for $\alpha = 4$.

Theorem 7. [2] Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$M_1(G_1 \nabla_T G_2) = 2 \left[4M_1(G_1) + M_1(G_2) + 2M_2(G_1) + M_3(G_1) \right. \\ \left. + n_1 n_2 (n_1 + n_2 + 2) + n_2 (8m_1 + 1) + 4m_2 (n_1 + 1) \right].$$

Theorem 8. [2] Let G_1 and G_2 be two graphs with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$, where $i = 1, 2$. Then

$$M_2(G_1 \nabla_T G_2) = M_1(G_2) + 4m_2(n_1 + 1) + n_2(n_1 + 1)^2 + 2 \left[4n_2 M_1(G_1) + (n_1 + 1)M_1(G_2) \right. \\ \left. + 6M_2(G_1) + M_2(G_2) + \frac{1}{2}(3M_3(G_1) + M_4(G_1)) + (n_1 + 1)\{m_2(n_1 + 1) \right. \\ \left. + n_2(2m_1 + n_1 n_2)\} + 2m_2(2m_1 + n_1 n_2) + n_2^2 m_1 \right. \\ \left. + \sum_{u, w \in V(G_1)} \gamma_{uw} d_{G_1}(u) d_{G_1}(w) + \sum_{u \in V(G_1)} d_{G_1}^2(u) \sum_{\substack{v \in V(G_1), \\ uv \in E(G_1)}} d_{G_1}(v) \right],$$

where γ_{uw} is the number of common neighbors of $u, w \in V(G_1)$ and M_4 is the generalized first Zagreb index for $\alpha = 4$.

Now we propose the following theorems.

3 Forgotten topological index

Theorem 9. Let $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$ be the number of vertices and edges in G_i ; $i = 1, 2$. Then,

$$M_3(G_1 \nabla_S G_2) = 2 \left[3\{n_2 M_1(G_1) + (n_1 + 1)M_1(G_2)\} + M_3(G_1) + M_3(G_2) \right. \\ \left. + (n_1 + 1)^2 \{n_2(n_1 + 1) + 6m_2\} + n_2^2(n_1 n_2 + 6m_1) + 8m_1 \right].$$

Proof.

$$M_3(G_1 \nabla_S G_2) = 2 \left[\sum_{u \in V(G_1 \nabla_S G_2)} d^3(u) \right] \\ = 2 \left[\sum_{u \in V(G_1)} d^3(u) + \sum_{u \in V(G_2)} d^3(u) + \sum_{u \in V(S(G_1)) \setminus V(G_1)} d^3(u) \right]$$

Case I:

$$\sum_{u \in V(G_1)} d^3(u) = \sum_{u \in V(G_1)} \left[d_{G_1}(u) + n_2 \right]^3 \\ = \sum_{u \in V(G_1)} d_{G_1}^3(u) + \sum_{u \in V(G_1)} n_2^3 + 3n_2 \sum_{u \in V(G_1)} d_{G_1}^2(u) + 3n_2^2 \sum_{u \in V(G_1)} d_{G_1}(u) \\ = M_3(G_1) + n_1 n_2^3 + 3n_2 M_1(G_1) + 6n_2^2 m_1.$$

Case II:

$$\begin{aligned}
 \sum_{u \in V(G_2)} d^3(u) &= \sum_{u \in V(G_2)} \left[d_{G_2}(u) + n_1 + 1 \right]^3 \\
 &= \sum_{u \in V(G_2)} d_{G_2}^3(u) + \sum_{u \in V(G_2)} (n_1 + 1)^3 \\
 &\quad + 3(n_1 + 1) \sum_{u \in V(G_2)} d_{G_2}^2(u) + 3(n_1 + 1)^2 \sum_{u \in V(G_2)} d_{G_2}(u) \\
 &= M_3(G_2) + n_2(n_1 + 1)^3 + 3(n_1 + 1)M_1(G_2) + 6m_2(n_1 + 1)^2.
 \end{aligned}$$

Case III:

$$\sum_{u \in V(S(G_1)) \setminus V(G_1)} d^3(u) = \sum_{u \in V(S(G_1)) \setminus V(G_1)} (2)^3 = 8m_1.$$

Now combining all these three cases we get the desired result. $\square$

Theorem 10. Let $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$ be the number of vertices and edges in G_i ; $i = 1, 2$. Then,

$$\begin{aligned}
 M_3(G_1 \nabla_R G_2) &= 2 \left[3 \{ 4n_2 M_1(G_1) + (n_1 + 1) M_1(G_2) \} + 8M_3(G_1) + M_3(G_2) \right. \\
 &\quad \left. + (n_1 + 1)^2 \{ n_2(n_1 + 1) + 6m_2 \} + n_2^2(n_1 n_2 + 12m_1) + 8m_1 \right],
 \end{aligned}$$

Proof. Case I:

$$\begin{aligned}
 \sum_{u \in V(G_1)} d^3(u) &= \sum_{u \in V(G_1)} \left[2d_{G_1}(u) + n_2 \right]^3 \\
 &= 8M_3(G_1) + n_1 n_2^3 + 12n_2 M_1(G_1) + 12n_2^2 m_1.
 \end{aligned}$$

Case II:

$$\begin{aligned}
 \sum_{u \in V(G_2)} d^3(u) &= \sum_{u \in V(G_2)} \left[d_{G_2}(u) + n_1 + 1 \right]^3 \\
 &= M_3(G_2) + n_2(n_1 + 1)^3 + 3(n_1 + 1)M_1(G_2) + 6m_2(n_1 + 1)^2.
 \end{aligned}$$

Case III:

$$\sum_{u \in V(R(G_1)) \setminus V(G_1)} d^3(u) = \sum_{u \in V(R(G_1)) \setminus V(G_1)} (2)^3 = 8m_1.$$

Combining Case I, II and III we get the result. $\square$

Theorem 11. Let $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$ be the number of vertices and edges in $G_i ; i = 1, 2$. Then,

$$\begin{aligned} M_3(G_1 \nabla_Q G_2) = & 2 \left[3 \{ n_2 M_1(G_1) + (n_1 + 1) M_1(G_2) \} + M_3(G_1) + M_3(G_2) \right. \\ & + M_4(G_1) + (n_1 + 1)^2 \{ n_2(n_1 + 1) + 6m_2 \} + n_2^2(n_1 n_2 + 6m_1) \\ & \left. + 3 \sum_{ww' \in E(G_1)} d_{G_1}(w) d_{G_1}(w') \{ d_{G_1}(w) + d_{G_1}(w') \} \right], \end{aligned}$$

where M_4 is the generalized first Zagreb index for $\alpha = 4$.

Proof.

$$\begin{aligned} M_3(G_1 \nabla_Q G_2) = & 2 \sum_{u \in V(G_1 \nabla_Q G_2)} d_{(G_1 \nabla_Q G_2)}^3(u) \\ = & 2 \left[\sum_{u \in V(G_1)} + \sum_{u \in V(G_2)} + \sum_{u \in V(Q(G_1)) \setminus V(G_1)} \right] d_{(G_1 \nabla_Q G_2)}^3(u) \\ = & 2 \left[\{ M_3(G_1) + n_1 n_2^3 + 3n_2 M_1(G_1) + 6n_2^2 m_1 \} + \{ M_3(G_2) + n_2(n_1 + 1)^3 \right. \\ & + 3(n_1 + 1) M_1(G_2) + 6m_2(n_1 + 1)^2 \} + \{ M_4(G_1) \\ & \left. + 3 \sum_{ww' \in E(G_1)} d_{G_1}(w) d_{G_1}(w') (d_{G_1}(w) + d_{G_1}(w')) \} \right] \end{aligned}$$

Hence the theorem. $\square$

Theorem 12. Let $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$ be the number of vertices and edges in $G_i ; i = 1, 2$. Then,

$$\begin{aligned} M_3(G_1 \nabla_T G_2) = & 2 \left[3 \{ 4n_2 M_1(G_1) + (n_1 + 1) M_1(G_2) \} + 8M_3(G_1) + M_3(G_2) + M_4(G_1) \right. \\ & + (n_1 + 1)^2 \{ n_2(n_1 + 1) + 6m_2 \} + n_2^2(n_1 n_2 + 12m_1) \\ & \left. + 3 \sum_{ww' \in E(G_1)} d_{G_1}(w) d_{G_1}(w') (d_{G_1}(w) + d_{G_1}(w')) \right]. \end{aligned}$$

where M_4 is the generalized first Zagreb index for $\alpha = 4$.

Proof.

$$\begin{aligned}
M_3(G_1 \nabla_T G_2) &= 2 \sum_{u \in V(G_1 \nabla_T G_2)} d_{(G_1 \nabla_T G_2)}^3(u) \\
&= 2 \left[\sum_{u \in V(G_1)} + \sum_{u \in V(G_2)} + \sum_{u \in V(T(G_1)) \setminus V(G_1)} \right] d_{(G_1 \nabla_T G_2)}^3(u) \\
&= 2 \left[\sum_{u \in V(G_1)} \left(2d_{G_1}(u) + n_2 \right)^3 + \sum_{u \in V(G_2)} \left(d_{G_1}(u) + n_1 + 1 \right)^3 \right. \\
&\quad \left. + \sum_{u \in V(T(G_1)) \setminus V(G_1)} \left(d_{G_1}(w) + d_{G_1}(w') \right)^3 \right], \\
&\quad u \text{ is assume to be inserted in } ww' \in E(G_1) \\
&= 2 \left[\{8M_3(G_1) + n_1 n_2^3 + 12n_2^2 m_1 + 12n_2 M_1(G_1)\} + \{M_3(G_2) \right. \\
&\quad \left. + n_2(n_1 + 1)^3 + 3(n_1 + 1)M_1(G_2) + 6m_2(n_1 + 1)^2\} + \{M_4(G_1) \right. \\
&\quad \left. + 3 \sum_{ww' \in E(G_1)} d_{G_1}(w)d_{G_1}(w')(d_{G_1}(w) + d_{G_1}(w'))\} \right]
\end{aligned}$$

Hence the theorem. $\square$

4 Reduced second Zagreb index

Theorem 13. Let $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$ be the number of vertices and edges in G_i ; $i = 1, 2$. Then,

$$\begin{aligned}
RM_2(G_1 \nabla_S G_2) &= (1 + 2n_1)M_1(G_2) + 2M_2(G_2) + n_1^2(1 + 2m_2 + 2n_2^2 - 2n_2) \\
&\quad + n_1(2 - n_2 + 4m_1 + 4n_2 m_1 + 4n_2 m_2 - 8m_2) \\
&\quad + m_1(12n_2 + 8m_2 - 4) + 4m_2 n_2 + 1.
\end{aligned}$$

Theorem 14. Let $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$ be the number of vertices and edges in G_i ; $i = 1, 2$. Then,

$$\begin{aligned}
RM_2(G_1 \nabla_R G_2) &= 4n_2 M_1(G_1) + n_1 M_1(G_2) + 4M_2(G_1) + M_2(G_2) \\
&\quad + (n_1 + 1)^2(2m_2 + n_2) + (n_1 + 1)(8m_1 n_2 + 2n_1 n_2^3 - 4m_2) \\
&\quad + 8m_1(2m_2 - n_2) + 2n_1 n_2(2m_2 - n_1 - n_2 - 1) + m_1 n_2^2 + 2m_1 \\
&\quad + 2m_2 - n_2.
\end{aligned}$$

Theorem 15. Let $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$ be the number of vertices and

edges in G_i ; $i = 1, 2$. Then,

$$\begin{aligned} RM_2(G_1 \nabla_Q G_2) = & (4n_2 + 2)M_1(G_1) + (2n_1 + 1)M_1(G_2) - 4M_2(G_1) + 2M_2(G_2) \\ & - M_3(G_1) + M_4(G_1) + (2m_2 + n_2)(n_1 + 1)^2 + 4m_1(m_2 - 2n_2 + 1) \\ & + (n_1 + 1)(4m_1n_2 + 2n_1n_2^2 - 4m_2) - 2n_1n_2(n_1 + n_2) \\ & + n_2(2m_2n_1 - 2n_1 - 1) + 2m_2 + 2 \left[\sum_{u,w \in V(G_1)} \gamma_{uw} d_{G_1}(u) d_{G_1}(w) \right. \\ & \left. + \sum_{u \in V(G_1)} d_{G_1}^2(u) \sum_{\substack{u \in V(G_1), \\ uv \in E(G_1)}} d_{G_1}(v) \right] + 2 \sum_{u \in V(G_1)} \binom{d_{G_1}(u)}{2}, \end{aligned}$$

where γ_{uw} is the number of common neighbors of $u, w \in V(G_1)$ and M_4 is the generalized first Zagreb index for $\alpha = 4$.

Theorem 16. Let $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$ be the number of vertices and edges in G_i ; $i = 1, 2$. Then,

$$\begin{aligned} RM_2(G_1 \nabla_T G_2) = & 8(n_2 - 1)M_1(G_1) + (2n_1 + 1)M_1(G_2) + 8M_2(G_1) + 2M_2(G_2) \\ & + M_3(G_1) + M_4(G_1) + (n_1 + 1)^2(n_2 + 2m_2) \\ & + (n_1 + 1)(4m_2 + 4m_1n_2 + 2n_1n_2^2) - 2n_1n_2(n_1 + n_2) \\ & + (4m_2n_1 - 2n_1 + m_1n_2 - 16n_1 - 1)n_2 + m_2(8m_1 - 8n_1 - 6) \\ & + 6m_1 + 2 \left[\sum_{u,w \in V(G_1)} \gamma_{uw} d_{G_1}(u) d_{G_1}(w) \right. \\ & \left. + \sum_{u \in V(G_1)} d_{G_1}^2(u) \sum_{\substack{u \in V(G_1), \\ uv \in E(G_1)}} d_{G_1}(v) \right] + 2 \sum_{u \in V(G_1)} \binom{d_{G_1}(u)}{2}, \end{aligned}$$

where γ_{uw} is the number of common neighbors of $u, w \in V(G_1)$ and M_4 is the generalized first Zagreb index for $\alpha = 4$.

Theorem 13 to theorem 16 can be proved using the relation $RM_2(G) = M_2(G) - M_1(G) + |E(G)|$ and the Lemmas 2-6

5 Conclusion

The authors of this paper proposed four new operations based on Indu-Bala product and computed the first and second Zagreb indices of these operations [2]. In this communication, we establish explicit formulas of the forgotten index and reduced second Zagreb index for the four new operations of graphs.

Acknowledgement

The authors are thankful to the anonymous reviewers for carefully reading our manuscript and for their valuable comments.

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