Analysis of some low- and high-dynamics errors of Low-Cost IMU

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Abstract: The paper expounds relevant results of some of the present author’s experiments defining the strapdown IMU sensors’ errors and their propagation into and within DGPS/IMU. In order to deal with this problem, the author conducted both the laboratory and field-based experiments.

In the landborne laboratory the stand-alone Low-Cost IMU MotionPak MKII was verified in terms of the accelerometer bias, scale factor, gyroscope rotation parameters and internal temperature cross-correlations.

The waterborne field-trials based on board dedicated research ships at the lake and at the busy small sea harbour were augmented by the landborne ones. These experiments conducted during the small, average, and high dynamics of movement provided comparative sole-GPS, stand-alone DGPS and integrated DGPS/IMU solution error analysis in terms of the accuracy and the smoothness of the solution. This error estimation was also carried on in the context of the purposely-erroneous incipient DGPS/IMU initialisation and alignment and further in the circumstances of on-flight alignment improvement in the absence of the signal outages.

Moreover, the lake-waterborne tests conducted during extremely low dynamics of movement informed about the deterioration of the correctly initialised DGPS/IMU solution with reference to the stand-alone DGPS solution and sole-GPS solution.

The above-mentioned field experiments have checked positively the DGPS/MKI research integrating software prepared during the Polish/German European Union Research Project and modified during the subsequent Project supported by the Polish Committee for Scientific Research.

Keywords: IMU, INS, MEMS, (D)GPS/IMU, Kalman filter, waterborne & landborne tests

1. Introduction

In the Space Research Centre of the Polish Academy of Sciences (SRC) the theoretical analyses and the experiments with the IMU and INS have been carried on since the early 1990’s. The applicability of this experience to the research on the space-vehicle became evident in 2007 when the SRC started cooperation with the Institute of Space Physics
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in Italy on the BepiColombo Mercury Mission and the Italian Space Accelerometer. The above defines the fundamental reasons for the report included below.

The paper strives to answer the question on the applicability of various experimental techniques for testing dynamics-dependent accuracy of IMU MEMS MKI, MKII, and \([D]GPS/MKI\).\(^1\)

In the first experimental section the paper provides a simple method of the preliminary laboratory testing of the MKII’s accelerometer’s errors using the gravity vector \(g\) as a reference, cf. (Lawrence, 1993; Titterton and Weston, 2004).

In the second experimental section the paper describes some of the IMU and INS error testing against the reference constituting the following simultaneous solutions: integrated \([D]GPS/MKI\), stand-alone \([D]GPS\), and sole-GPS, cf. procedures reported in (Grejner-Brzezinska, 2005).\(^2\)

The physical/chemical properties of MKI and MKII leading to the positive short-term qualities are widely known, cf. (Everett, 1995, p. 393; Wang, 1996, p. 54). The technical and mathematical-exactitude properties of the stand-alone MKI and MKII are broadly recognized throughout the researchers’ fraternity, cf. (AAS, 2005, pp. 343, 344).

In scientific applications since the beginning of this century MotionPak has been probably most frequently used in the research on manned vehicle-based-mapping and unmanned-robotics. For the first mentioned application cf. for example: (Valldorf and Gessner, 2005, p. 509). The last mentioned applications are usually land-/waterborne ones. For the landborne robots bearing MotionPak cf. (Caccia et al., 2008).

However, there is no reason why MotionPak-like IMU could/should not be mounted on board robots dedicated for the surface of Mars or Mercury, cf. (Fontaine et al., 2009; Novara, 2001).

The waterborne robots bearing MotionPak included Autonomous Underwater Vehicles (AUVs). Regarding AUVs, cf. (Wang, 1996, p. 37). MotionPak was also used in the seismic hardware setup, cf. (Teisseyre et al., 2006, p. 158). For the geodetic applications of the integrated satellite/Low-Cost-IMU, cf. (Grejner-Brzezinska, 2005).

Probably in most of the scientific projects mentioned the crucial factor of MotionPak applicability was its response to very-low and/or very-high dynamics. The paper strives to answer the question on the applicability of various experimental techniques for testing MKI’s and MKII’s dynamics-dependent accuracy. The dynamics dependence of the IMU errors’ behaviour has been very well explained in (Titterton and Weston, 2004). With regard to the testing of the dynamics-dependence of MKII and other MEMS Inertial Sensors ISs it is worthwhile to consult: (AAS, 2005, p. 343).

According to the contemporary literature the highest growth potential for the investigation on the MEMS accelerometers appears in the research for the automo-

\(^1\) MEMS – Micro-Electro-Mechanical Systems, cf. (Lobontiu and Garcia, 2005). For more information on the Systron Donner’s MotionPak\(^TM\) Mark I (MKI) and Mark II (MKII) cf. (Systron Donner Inertial, 2008).

\(^2\) The \([D]\) stands for the optionally Differential GPS. The sign / means “software-only Kalman filter integration”.

tive applications. Some of these require comparatively very small errors and small noise in the accelerometer’s zero-g dynamics and the immunity to vibrations and to the parasitic high frequency content present in the car’s chassis, for example for the Electrical Parking Brake (EPB), cf. (Reze and Hammond, 2005, pp. 461, 469, 470).

However, the integrating DGPS/MKI software and the testing techniques presented in the paper had in mind the scientific land- and waterborne geodetic (Grejner-Brzezinska, 2005), and hydrography and oceanography application.

The paper checks and explores the opportunities provided by the [D]GPS/MKI system with the open-code research Kalman filter driven software designed during the EU Research Programme.

2. Theoretical background for the error equations for IMU and [D]GPS/IMU

2.1. IMU sensor error compensation

Meticulous IMU laboratory testing is necessary for the determination of the coefficients of the error equations, which are prerequisite for the optimal IMU & INS error propagation tuning.

2.1.1. Fundamental error equations of the accelerometer and gyro


The observations and measurements provided by the Z-axis accelerometer, may be expressed in one of the simplest possible vector forms as follows:

pure inertial case:

\[ \mathbf{a}_{Z0} = S_{aZacc} \mathbf{a}_Z + B_{Zacc} + \mathbf{n}_{Zacc} \] (1a)

pure gravitational case:

\[ \mathbf{g}_{Z0} = S_{gZacc} \mathbf{g}_Z + B_{Zacc} + \mathbf{n}_{Zacc} \] (1b)

mixture of inertial and gravitational case:

\[ \mathbf{N}_{Z0} = S_{NZacc} \mathbf{N}_Z + B_{Zacc} + \mathbf{n}_{Zacc} \] (1c)

where above: we neglect the influence of the Earth’s rotation, and further: \( \mathbf{a}_{Z0}, \mathbf{g}_{Z0}, \mathbf{N}_{Z0} \) – output of the Z-axis accelerometer; \( \mathbf{a}_Z, \mathbf{g}_Z, \mathbf{N}_Z \) – input along Z-axis accelerometer;
\( S_{Z\text{acc}}, S_{g\text{acc}}, S_{\ell\text{acc}} \) – scale factor pertaining respectively to magnitudes of: \( a_Z, g_Z, \ell_Z \) (Input/Output (I/O) function is presumed to be symmetrical with reference to the Output axis); \( B_{Z\text{acc}} \) – gravitation \( g \) independent acceleration measurement bias, pertaining to the \( Z \)-axis accelerometer; \( n_{Z\text{acc}} \) – accelerometer zero-mean random bias.

In the 3D ISC or “i” the component \( \ell_{Z0} \) sensed by the \( Z \)-axis accelerometer will be as follows:

\[
\ell_{Z0} = a_{Z0i} + g_{Z0i}
\]  

Similarly, the measurements \( \hat{\omega}_{Z0} \) provided by a \( Z \)-axis gyroscope of the IMU triad may be expressed in rudimentary vector form as follows (allowance has been made for the Earth rate component):

\[
\hat{\omega}_{Z0} = S_{m_{Z\text{gyro}}} \hat{\omega}_{Z} + B_{Z_{\text{gyro}}} + n_{Z_{\text{gyro}}}
\] where: \( \hat{\omega}_{Z0} \) – output of the \( Z \)-axis gyroscope; \( \hat{\omega}_{Z} \) applied gyration components acting along gyro’s \( Z \)-rotation axis (input); \( \hat{\omega}_{Z} \) have magnitudes of the angular rotation, and have “senses” in terms of the clock-wise (CW, positive) or anti-clockwise (ACW, negative) rotation along the \( Z \)-rotation axis; \( S_{m_{Z\text{gyro}}} \) – scale factor pertaining to the \( Z \)-axis gyroscope and the magnitude \( \hat{\omega}_{Z} \); \( B_{Z_{\text{gyro}}} \) – gravitation \( g \) insensitive \( Z \)-gyro measurement bias; \( n_{Z_{\text{gyro}}} \) \( Z \)-axis gyro zero-mean random bias.

Further instead of \( S_{Z\text{acc}}, S_{g\text{acc}}, S_{\ell\text{acc}}, S_{m_{Z\text{gyro}}} \) we will here write \( S_{Z\text{acc}}, S_{Z_{\text{gyro}}} \) respectively. In (1a), (1b), (1c), (2), and (3) no allowance has been made for cross sampling coefficients, gravity dependent biases, and the drift errors, cf. (Lawrence, 1993; Jekeli, 2001; Vorbrich, 2003; Titterton and Weston, 2004). The most pronounced of the predictable and compensable errors are \( S_{Z\text{acc}}, S_{Z_{\text{gyro}}, B_{Z_{\text{acc}}}, B_{Z_{\text{gyro}}} \). Random biases \( n_{Z\text{acc}}, n_{Z_{\text{gyro}}} \) cannot be compensated accurately.

\( SIG_{\text{output}} \equiv K \cdot SIG_{\text{input}} + B \)  

where \( SIG_{\text{output}} \) – output signal, \( SIG_{\text{input}} \) – input signal, and \( B \) – bias. The term \( K \) in equation (4) is called here the scale factor (of IS – valid for any accelerometer and/or any gyro generally). Therefore, in particular for \( Z \)-axis IS the three terms: \( K \) and \( S_{Z\text{acc}}, S_{Z_{\text{gyro}}}, \) are related in terms of both the physical quantity and physical property and also the three terms: \( B \), and the magnitudes \( B_{Z\text{acc}} \) and \( B_{Z_{\text{gyro}}} \) are related in the
above-mentioned terms. Equation (4) describes certain general and ideal case in the absence of the additional random bias terms \( n_{Z_{\text{acc}}} \), \( n_{Z_{\text{gyro}}} \). The \( B \) factor could change from warm-up to warm-up, cf. (Kayton and Fried, 1997, p. 323). For any-axis rate gyro, its bias \( B_{\text{gyro}} \) when integrated, causes an angular error, which grows linearly with time. However, for any-axis accelerometer, its bias error \( B_{\text{acc}} \), when double integrated, causes an error in position which grows quadratically with time, cf. (Woodman, 1997, pp. 11, 15). Generally, the stability of the \( K \) factor for any IS (accelerometer or gyro) may depend on the I/O function, while specifically the stability of \( S_{\text{gyro}} \) is probably the single most important factor in the INS positioning accuracy deterioration with time, cf. (Kayton and Fried, 1997, p. 323). For a more detailed discussion of the above cf. (Lawrence, 1993; Jekeli, 2001; Titterton and Weston, 2004).

The following is applicable to all kinds of gyros: if a mission involves changing direction for a long time (circular routes at a rate \( \Omega \) over time \( t \)), than the \( Z \)-axis gyro scale factor error \( \delta S_{Z_{\text{gyro}}} \) influences the heading error \( \delta A_z \) in the following way:

\[
\delta A_z = \frac{\delta S_{Z_{\text{gyro}}}}{S_{Z_{\text{gyro}}}} \Omega t,
\]

cf. (Lawrence, 1993, pp. 26, 27).

The contemporary specialty literature shows the growing interest in tracing of the dynamics-wise dependence of the errors of the MEMS ISs. The subject of the errors of the MEMS accelerometers and gyros in the static mode is perused in some detail by Lobontiu and Garcia (2005, pp. 267, 269, 273, and passim). The literature points to the necessity of isolating the MEMS from the vibration induced noise. For example, Bose (1996, pp. 11–17) provides some numerical examples of the influence of the vibration-induced noise on the MAG output on the angle rate error.

Table 1 shows for the MKII the nominal values of the bias, scale factor and their dynamical “borders”.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>Bias (factory set)</th>
<th>Scale factor</th>
<th>Standard range</th>
<th>Threshold/ resolution</th>
<th>Full scale output</th>
<th>Input voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>([g]/[h])</td>
<td>([V])</td>
<td>([V/g])</td>
<td>([-2g] / [g/\pi])</td>
<td>([-2g] / [g/\pi])</td>
<td>([-2g] / [g/\pi])</td>
<td>([-2g] / [g/\pi])</td>
<td>([-2g] / [g/\pi])</td>
</tr>
<tr>
<td>±1</td>
<td>l</td>
<td>2.5</td>
<td>0.750</td>
<td>±1.5</td>
<td>±75</td>
<td>±4.8</td>
<td>≤ 10</td>
</tr>
<tr>
<td>±1</td>
<td>h</td>
<td>0.0</td>
<td>-6.680</td>
<td>±1.5</td>
<td>±75</td>
<td>±4.8</td>
<td>≤ 10</td>
</tr>
<tr>
<td>±3</td>
<td>h</td>
<td>0.0</td>
<td>-3.330</td>
<td>±2.7</td>
<td>±75</td>
<td>±4.8</td>
<td>≤ 10</td>
</tr>
</tbody>
</table>

Notes: cf. (Systron Donner Inertial, 2008, pp. 13–16); column 1 – “Version Standard Range \([g]/[h]\) Category” (see Standard range (saturation)); column 2 – mode \([V/g]\) – Mode Low/High; Full scale output for mode Low \(± 0.475 - 4.525 \text{ V} \); Volt. – Voltage; Input voltage: ±15 V is often cited as Standard Dual Supply (bipolar); the brackets [] signify physical units in SI and brackets {} signify non-SI physical units, in particular \([g] \equiv \hat{n}(\text{Earth}) \text{ [m/s}^2\text{]}\), \(\hat{n}(\text{Earth}) \approx 9.80665 \text{ for the nominal “average” value at the Earth’s surface, known as standard gravity}; in particular \([\hat{g}] \equiv \hat{n}(\text{Earth}) \text{ [m/s}^2\text{]}\), where \(\hat{n}(\text{Earth}) = 9.8 \) was adopted for the simplicity reasons.

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3 MAG – Micromachined Accelerometer Gyro.
2.1.3. Basic method for the laboratory determination of MKII accelerometer bias and scale factor

Let us adopt the equation (1b) above to a laboratory situation, assuming that the MKII ISs’ analogue output in [V] is digitalised by means of the A/D card and stored on a PC.

Let us then suppose for the simplicity reasons, that Z-axis accelerometer is (within sensitivity specified by $n_{Z_{\text{acc}}}$ in (1b)) to our purposes nearly vertical and (within this sensitivity) the $B_{Z_{\text{acc}}}$ and $S_{Z_{\text{acc}}}$ are constant, the I/O function is symmetrical. Let the IMU signal be PC stored in the IMU’s Up and Down (Dn) positions with reference to the Z-axis accelerometer. Further let the magnitudes of

$$B_{Z_{\text{acc}}} (\text{Up}) = B_{Z_{\text{acc}}} (\text{Dn}) = B_{Z_{\text{acc}}} \quad \text{and} \quad S_{Z_{\text{acc}}} (\text{Up}) = S_{Z_{\text{acc}}} (\text{Dn}) = S_{Z_{\text{acc}}}$$

within three hours time span between “L” experiment commencement and “U” experiment completion (cf. Table 4).

Then the sum of the readings in [V] of the output, that is magnitudes $g_{Z_{\text{acc}}}$ (Up), $g_{Z_{\text{acc}}}$ (Dn) of the readings of the Z-axis accelerometer facing in sequence Up and then Down respectively, should after the relevant transformation of (1b) lead to the determination of the bias magnitude $B_{Z_{\text{acc}}}$ [V] as follows:

$$B_{Z_{\text{acc}}} = \frac{[g_{Z_{\text{acc}}} (\text{Up}) - g_{Z_{\text{acc}}} (\text{Dn})]}{2.0} + g_{Z_{\text{acc}}} \quad (5)$$

where [V] contains the units of the accelerometer raw data digitized output in Volts.

In addition, similarly to the above, the difference between the reading in [V] of the output of the Z-axis accelerometer facing Up and Down respectively, should after the relevant transformation of (1b) lead to the determination of the scale factor magnitude $S_{Z_{\text{acc}}}$ [V/g(local)]

$$S_{Z_{\text{acc}}} = \frac{\left\{ \text{mod}[g_{Z_{\text{acc}}} (\text{Up}) - B_{Z_{\text{acc}}}] + \text{mod}[g_{Z_{\text{acc}}} (\text{Dn}) - B_{Z_{\text{acc}}}] \right\}}{2.0} \quad (6)$$

where g(local) is the magnitude of the gravity vector g(local), and the above coefficient 2.0 would be true if the Z axis were strictly vertical (i.e. strictly parallel to the direction of g(local)) and so when the statement written for the magnitudes $g \approx g_{Z_{\text{acc}}}$ (Up) $\approx g_{Z_{\text{acc}}}$ (Dn) would be true.

The equations (5) and (6) were used in the calculations cited in the experimental chapter, Table 5.

2.2. The EU JRP [D]GPS/IMU(MKI) integrated system in [D]GPS/INS(MKI) mode

The EU JRP strapdown integrated system [D]GPS/IMU(MKI) (further called system) was conceived during the European Union Polish-German Joint Research Project (further called EU JRP). The master computer and its software together with the DGPS...
and MKI hardware form the [DGPS/IMU]-AeroNav-H (or simply ANH) subsystem assembled for the EU JRP by the “AERODATA” Company in Braunschweig, Germany. The “research” software (further called [DGPS/IMU-R]), installed on slave-PC subordinated to ANH, has been prepared for the EU JRP by the Institute of Flight Guidance and Control at the Technical University in Braunschweig (TUBS.IFF) with the participation of the Space Research Centre of the Polish Academy of Sciences, in particular with the participation of the present author. Since its creation the system has undergone a series of maintenance and modification, cf. (Vorbrich, 2001b).

Table 2. Block diagram and configuration and description of the [DGPS/IMU-R system in [DGPS/INS(MKI) mode

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMU</td>
<td>DGPS to INS mechanism algorithm in [DGPS/INS(MKI) mode delivers the INS inertial navigation solution (position, velocity and attitude). In turn the DGPS delivers satellite navigation solution in terms of position, velocity, pseudo-ranges and pseudorange-rates. The reference trajectory immediately fed to the Kalman filter is provided by the combined effect of the satellite and inertial position and velocity errors thought of in terms of the pseudo-range and pseudorange-rate differences (satellite-derived “observed” minus INS—Least Squares calculated”). The closed loop leads to an extended Kalman filter. The system output is slaved to GPS time and is contained in a state vector, which for example has 17 state variables, cf. equation (7) below and in IMU (50 Hz) and [DGPS (1 Hz) raw data matrices.</td>
</tr>
<tr>
<td>INS software</td>
<td>DGPS hardware</td>
</tr>
<tr>
<td></td>
<td>DGPS software</td>
</tr>
<tr>
<td></td>
<td>LS</td>
</tr>
<tr>
<td></td>
<td>differences</td>
</tr>
<tr>
<td></td>
<td>Kalman filter</td>
</tr>
<tr>
<td></td>
<td>feedback</td>
</tr>
<tr>
<td></td>
<td>[DGPS / IMU solution]</td>
</tr>
</tbody>
</table>

Note: see (Wagner and Wieneke, 2003, p. 544) for a block diagram of an alternative DGPS/IMU mode where INS software is omitted altogether and the raw IMU data and DGPS ranges and range rates are input to the Kalman filter. In ibid. the feedback loop shown in Table 2 is also omitted.

The Kalman filter design in the EU JRP system is flexible. For example the state vector $x_{DGPS/IMU}$ in the Kalman filter mentioned in Table 2 can have 17 state variables and can be expressed as follows:

$$
x_{DGPS/IMU} = [\delta \text{POS}, \delta \text{VEL}, \delta \epsilon, \delta \mathbf{B}_{\text{acc}}, \delta \mathbf{B}_{\text{gyro}}, \delta t, \delta \dot{t}]^T \tag{7}
$$

where there are 9 vehicle or navigation error states (that is three position states $\delta \text{POS}$, three velocity states $\delta \text{VEL}$, and three attitude (Euler angles) error states $\delta \epsilon$ (where the heading orientation error is the most important one for waterborne or landborne navigation), 6 IMU sensor error states (3 corrections to predetermined accelerometer biases forming $\delta \mathbf{B}_{\text{acc}}$ sub-vector and 3 corrections to predetermined gyro- biases forming $\delta \mathbf{B}_{\text{gyro}}$ sub-vector), 2 GPS receiver states ($GPS$ clock error $\delta t$ and $GPS$ clock drift $\delta \dot{t}$). In one of its designs the Kalman filter in Table 2 can track in time domain the gyro drifting $\delta \mathbf{D}_{\text{gyro}}$ which is one of the key elements of such a filter design.
Also, cf. comparable solution in: (Wagner and Wieneke, 2003, p. 546) where 18 state variables state vector is very similar to (7), it solves for four quaternions instead of for three Euler angles, however.

For modern solution comparable to this presented in Table 2 see (Abdel-Hamid et al., 2006, p. 3). Also the similar method of calculating of the pseudorange and pseudorange-rate differences needed for the satellite – inertial o – c differences input into the extended Kalman filter (shown in Table 2) present (Wagner and Wieneke, 2003, p. 544; Kreye et al., 2005, p. 106, and passim). On the other hand Wagner and Wieneke (2003, p. 543, and passim) propose also serious modification of the above-mentioned method, albeit all based on (7) and Kalman filtering, cf. (Wagner and Wieneke, 2003, pp. 544, 546, and passim) and cf. Note to Table 2. According to Wagner and Wieneke (2003, p. 549) this new method which is working in the pure DGPS/IMU(MotionPak) mode instead of the DGPS/INS(MKI) mode proposed by the present paper, is not expected to improve the navigation solution accuracy during the standard dynamics of movement. However, it should slightly or moderately improve the system estimation quality during the very small dynamics of movement.

Modern literature sometimes proposes that the integrated satellite/Low-Cost IMU system based on the Kalman filtering for some applications could be replaced or aided by the other mathematical solutions such as the Multi-layer Feed-forward Neural Networks (MFNN), the Constructive Neural Network (CNN) (Huang and Chiang, 2008), the Artificial Neural Network (Kaygysyz et al., 2007), and the Fuzzy Modelling (Abdel-Hamid et al., 2006, p. 3). Nevertheless, Wagner and Wieneke (2003, p. 544) point out that the conventional methods based on Kalman filtering and similar to the one shown in Table 2 are widely accepted.

The direct indication of the accuracy of the state vector (7) estimate is given by the diagonal elements of the state covariance matrix $C_x$ (+) for the optimal estimate. Equations modelling the error propagation within the Kalman-filter-integrated DGPS/IMU system are given in more detail in (Vorbrich, 2001a).

The state vector in Kalman filter expressed by (7) describes one of the simplest possible cases of the inertial sensor’s errors determination. Clearly, Kalman filter state vector, which has 17 state variables described by (7) does not include information on the accelerometer and gyro scale factor, and gyro drifts. Prior to the mobile experiments, these last mentioned MKI’s parameters, together with the previously mentioned accelerometer biases and gyro-drift rates, were determined at the AERODATA’s laboratory both during the EU JRP and during the later modifications done within the framework of the Project financially supported by the Polish Committee for Scientific Research.

Inertial literature advocates elimination of the vertical channel from the surface waterborne and level landborne satellite/inertial solution, cf. (Chatfield, 1997, p. 13). It is due to the positive feedback of the altitude errors through the gravity model. It makes INS’s solution unstable for the vertical channel.
2.3. The observability of the [D]GPS/IMU and [D]GPS/INS

According to Wagner and Wiencke (2003, p. 547) who cited (Dambeck, 1995) the problem described by equation type (7) is weakly observable. The predicament connected with the observability of the [D]GPS/IMU and/or [D]GPS/INS has been described extensively by the modern inertial literature. Also the sensitivity of the said observability to the dynamics of movement of the platform bearing the [D]GPS/IMU and/or [D]GPS/INS has been well known and dealt in detail by this literature.

In particular, in the observation matrix expressed in the body system of coordinates the elements for the rows $\delta \epsilon$ and $\delta B_{gyro}$ (cf. (7)) are each governed by the product of the vector components from the angular rate vector and the lever arm vector $L$ (the last vector defines the displacement from the origin of the IMU’s “b” system to the GPS antenna electric centre). During the low dynamics of the platform’s movement, the above-mentioned rows do not lead to a full matrix rank, thus the system becomes unobservable. For the exact formulae necessary for the solving of the lever arm problem in the integrated satellite/INS system see (Wagner and Wiencke, 2003, p. 546). According to the formulae presented in ibid., theoretically the increasing of the lever arm to the full extent of such structure as the tip of the wing of a test aircraft should decrease the problem of the system unobservability due to the dramatically decreased weakness of the state vector $x_{DGPS/IMU}$ in (7) in the very low dynamics of movement. However, the present author was unable to confirm this assumption experimentally.

In general the problem of the initial determination of the sub-vectors $\delta \epsilon$ and $L$ belongs to the issue of the alignment. Mun et al. (2005, pp. 1257–1262) provide the mathematical formulae and the following conclusions on the observability properties of the GPS/IMU alignment under different system’s dynamics. If a platform experiences changes in angular velocity (moves with the angular acceleration which is not zero) with reference to a given direction, then the components of lever arm error that are perpendicular to the direction of the angular velocity become observable. Linear accelerations’ changes make observable the components of the relative attitude error that were perpendicular to the direction of the acceleration change.

2.4. Initialisation/alignment of [D]GPS/(Low-Cost IMU) System

By initiation we understand here all the procedures needed after the warm-up to prepare the system for the dynamic mission. The alignment procedure itself determines the initial Euler angles for the initial transformation matrix from “b” to the computations frame. Cf. GNSS/INS alignment procedure and the INS static tests’ procedure mentioned in (Kreye et al., 2005, p. 103). Also cf. the alternative method proposed by Hao et al. (2008, p. 117, and passim), which for GPS/Low-Cost IMU uses Kalman filter for the coarse alignment during the static phase of several minutes duration and the

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H (mini-max) filter for the fine tuning of the residual misalignment angles conducted under the moderate dynamics of app. 0.1 \( \ddot{g} \).

For \([D]\)GPS/MKI alignment to take place the \([D]\)GPS/IMU-R software requires the availability of an external heading sensor input, for example electronic magnetic compass, cf. (Nguyen et al., 2009, pp. 861, 864). The \([D]\)GPS/IMU-R software alignment procedures normally are based on \( g \) as a reference (cf. eq. (1b)) and the MKII laboratory stationary tests shown by the Figures 1a, 1b, 2a, and 2c. Therefore, during the alignment, the DGPS/IMU host-vehicle must be brought to a complete stop and it should not vibrate. During initialisation, the alignment of the ISs in terms of the estimation of inertial sensor mean biases (by means of a Kalman filter with state vector based on eq. (7) for zero velocity) takes place. The accuracy of the stand-alone IMU of the \([D]\)GPS/IMU in between the \([D]\)GPS updates during the mobile experiment depends on the \([D]\)GPS/IMU static alignment procedure accuracy.

In the \([D]\)GPS/IMU-R software the subsequent more accurate estimate of the heading (equivalent to Euler azimuth \( \Psi \) angle) can be obtained on-route from GPS-determined-velocity (components \( v_{\text{GPS,E}} \), \( v_{\text{GPS,N}} \)) as follows (iteratively, so-called “on-flight” alignment, cf. (Nguyen et al., 2009, p. 861)):

\[
\Psi = \arctan \left( \frac{v_{\text{GPS,E}}}{v_{\text{GPS,N}}} \right) \tag{8}
\]

3. Experiments with MKII and \([D]\)GPS/MKI

3.1. Laboratory-based analysis of stand-alone MKII output data

This section provides an example of a set of the experiments ascertaining the MKII’s accelerometer and gyro response in the stationary and rotational mode.

3.1.1. The experiments’ scenario

During his attachment to the Global Positioning Systems Centre at the Nanyang Technological University Singapore (NTU) the present author among-other conducted the laboratory experiments with the MKII, cf. (Vorbrich, 2002).

3.1.2. The pictorial representation of the experimental raw data

During presented MKII laboratory tests the MKII accelerometers’ bias and scale factor were initially calibrated on a horizontal test-bed by means of using the Earth’s gravity vector \( g \) as a reference, cf. (Kayton and Fried, 1997, p. 324; Titterton and Weston, 2004; Park and Gao, 2006, pp. 35, 38). All the data presented in this section below refer to the A/D card with the resolution of 24 bits, and a common warm-up time. For some information about the importance of the common warm-up time for a chain of inertial experiments see (Kayton and Fried, 1997, pp. 323, 381).
Table 3. Variable features of MKII laboratory experiments' sequence Mb24J31

<table>
<thead>
<tr>
<th>Figures</th>
<th>Experiment's name suffix</th>
<th>Dynamics mode</th>
<th>Reference or rotation axis</th>
<th>A/D sample rate [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a,1b,1c</td>
<td>L</td>
<td>stationary</td>
<td>Z facing up (Up)</td>
<td>10</td>
</tr>
<tr>
<td>2a,2b,2c</td>
<td>U</td>
<td>stationary</td>
<td>Z facing down (Dn)</td>
<td>10</td>
</tr>
<tr>
<td>3a,3b,3c,4a,4b</td>
<td>M</td>
<td>rotation</td>
<td>rot ACW: Z-Up/Z-Dn/ Z-Up via Y kept horizontal</td>
<td>100</td>
</tr>
<tr>
<td>4c</td>
<td>T</td>
<td>rotation</td>
<td>rot 90° CW/ 90° ACW via Z kept facing Up</td>
<td>100</td>
</tr>
</tbody>
</table>

Note: The ambient temperature was kept constant to within ±0.5°C.

Table 4. Stationary experiments’ time span

<table>
<thead>
<tr>
<th>Experiment name</th>
<th>L</th>
<th>U</th>
<th>Between L start and U end</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time span</td>
<td>820 s</td>
<td>app. 1000 s</td>
<td>app. 3 hours</td>
</tr>
</tbody>
</table>

3.1.2.1. The stationary experiments’ (selected) drawings

Fig. 1. Z-axis accelerometer experiment L a); Z-axis gyroscope experiment L b); MKII internal temperature experiment L c)

Fig. 2. Z-axis accelerometer experiment U a); MKII internal temperature experiment U b); Y-axis accelerometer experiment U c)
3.1.2.2. The rotational experiments’ (selected) drawings

Fig. 3. X-axis accelerometer experiment M a); Y-axis accelerometer experiment M b); Y-axis gyroscope experiment M c)

Fig. 4. Z-axis accelerometer experiment M a); MKII internal temperature experiment M b); Z-axis gyroscope experiment T c)

3.1.3. The tabularised representation of the data and the explanations

Table 5. Data for incipient determination of the bias and scale factor of the NTU MKII accelerometers

| Stationary experiments L and U (see Tables 3 and 4, and family of Figures 1 and 2) |
|--------|--------|-------|--------|
| Body “b” system axis | L (Z faces Up) minus U (Z faces Dn); mean [V] | Bias [V] | Scale factor $S_{Zacc}$ $[V/g (local)]$ |
| X(h) nearly horizontal | –0.015 | – | – |
| Y(h) nearly horizontal | –0.017 | – | – |
| Z(v) nearly vertical | 1.495 | 2.481 | 0.745 |

| Dynamic experiment M (see Table 3, and family of Figures 3 and Figures 4a, 4b) |
|--------|--------|-------|--------|
| Body “b” system axes | Z faces Up minus Z faces Dn for Z(v) X facing Up minus X facing Dn for X(v) Rotation via Y (ACW); mean [V] | Bias [V] | Scale factor $S_{Zacc}$ $[V/g (local)]$ |
| (h) nearly horizontal | (v) nearly vertical | |
| X(h) | 0.021 | – | – |
| Y(h) | 0.016 | – | – |
| Z(v) faces Up then Dn | 1.509 | 2.499 | 0.750 |
| X(v) facing Dn then Up | 1.500 | 2.500 | 0.750 |

Note: cf. Table 1. In Table 5 the bias and scale factor for Z(v)-axis were computed by means of the equations (5) and (6) respectively; these eq. were transformed for X(v)-axis, however. The observed digitalized values were depicted in the families of Figures 1, 2, and 3 and Figures 4a and 4b (i.e. accelerometers,
gyrosopes and temperature). The relevant values were in error amongst other due to the uncompensated:
a) Earth's rotation influence (surmising however, that MKII's output error level is too high to sense the
Earth's rotation); b) errors due to nonorthogonality and misalignment of sensitivity axes; in particular
non-perpendicularity (non-verticality) of the relevant Z(v)- or X(v)-axes and the non-horizontality of the
X(h)- and Y(h)-axes; c) cross-axes sensitivity (here considered negligible, cf. (Park and Gao, 2006, p. 38)).
The effect b) has been caused by the combined effect of the skewed X, Y, and Z axes of the “b” system,
of the tilt between the “b” system and the system of coordinates defined by the MKII enclosure walls
and by the non-parallelism of these walls. In particular, if the axes X(h) and Y(h) were strictly horizontal
(i.e. strictly perpendicular to the local gravity vector g(local) direction), then the values for delta for X(h)
and Y(h) should have oscillated around the zero value [V] (within the boundaries of the measurement
error). The above would be true – assuming, that all the scale factors and biases are constant (within the
time span of the experiment). The actual measurement for Z(v) was long-term-static and for X(v) was
dynamic. In contrast, during the actual measurement used for computation of data illustrating dynamic
experiment M, the values X(h) and Y(h) were static only.

3.1.4. Determining the direction of the rotation

The “senses” of the gyration vector in (3)-like equation can be determined by means of
comparison of the Figures 3a, 3b, 3c, and 4a (accelerometers and gyros) and the Figure
4c (gyro). It shows that a) ACW rotation lowers the magnitude of the output voltage
of the gyros for Z-and Y-axes; b) the above mentioned fact # a) allows to control the
history of the direction of the rotation (ACW versus the reverse); c) it follows that
the MKII’s body “b” horizontal X, Y system of axes is left-handed. In other words
when the MKII is rotated ACW 90° in the X, Y plane about Z-axis, then after the
90° rotation had been completed the X-axis coincided with the previous direction and
pointing (i.e. sense or arrow) of the Y-axis.

3.2. Comparative GPS, DGPS and [D]GPS/IMU solution error analysis

3.2.1. The test lake and the test-boat

Fig. 5. Łukomie (Charzykowskie) lake and some experiments’ traces traversed by M/B Bartek
between the unmooring and mooring event (for red trace cf. Figure 7, for violet-pink trace cf. Figure 12)
and M/B Bartek
3.2.2. DGPS/IMU hardware mount on board the research M/B Bartek on Lukomie lake and on the jetty

M/B Bartek was built in the Ustka Shipyard and has a “Conrad 900” type of hull. It is a hybrid Diesel/electric powered research boat dedicated for the biological and zoological exploration of the Lukomie lake. The small jitter of the electric engines was important for the \([D]\)GPS/IMU experiments scheduled for the very low dynamics sailing (see below). The \([D]\)GPS/IMU system itself was mounted in the forward-cabin, the GPS antenna on top of the cabin roof (Fig. 5), and the DGPS reference station RS telemetry antenna mast was fixed at the Lukomie lake’s jetty of the Ecological/Biological Research Centre at Funka.

Table 6. The dynamics parameters of M/B Bartek powered by the Diesel engine

<table>
<thead>
<tr>
<th>Maximum speed (v_{max}) [knots]</th>
<th>Mean acceleration from 0 [knots] to (v_{max}) [(a/\hat{g})]</th>
<th>Mean deceleration from (v_{max}) to 0.5 [knots] [(a/\hat{g})]</th>
<th>Circulation minimal diameter at 25 [knots] [(\Delta t)] [m]</th>
<th>CW [m]</th>
<th>ACW [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>48.15</td>
<td>14</td>
<td>3.44</td>
<td>0.35</td>
<td>120</td>
</tr>
</tbody>
</table>

Note: experimentally determined by the present author as a mean from several runs in calm weather at 0 \(\circ B\); 1 [knot] \(\equiv\) 1.852 [m per hour] \(\equiv\) [Nautical Mile per hour]; [km/h] – [km per hour]; \(v\) – boat’s speed relative to water, \(\Delta t\) – mean time span; \(a = \Delta v/\Delta t\).

3.2.3. DGPS/IMU hardware mount on board the research M/B Imoros in the Wladyslawowo harbour

The paper also presents the \([D]\)GPS/MKI experiments on board the research M/B Imoros (powered by the Diesel engines only) in the Wladyslawowo harbour. M/B Imoros and M/B Bartek have comparable dynamics characteristics, cf. (Vorbrich, 2001a).

3.2.4. DGPS/IMU hardware mount on board the research van

The DGPS/MKI-research-van was the present author’s main platform, cf. (Vorbrich, 2001a). It offered easiest way to control both the reference route and the dynamics of movement.

3.2.5. Alignment/mount of DGPS/(Low-Cost IMU) System

In the stationary vehicle/boat environment (no wind, no waves, no traffic, engine not running) the optimal value of the alignment time of the DGPS/MKI system was found to be equal to app. 5 min. This period is consistent with the period of at least few minutes reported by the modern literature. However, Nguyen et al. (2009, p. 864) report the successful static alignment period as short as 10 s.
3.2.6. Comparative GPS, DGPS and DGPS/IMU solution error analysis during medium to high dynamics of movement on water and on land

The methodology of the tests described below in the paper to a considerable extent agrees with the comparable satellite/Low-Cost IMU tests described in the contemporary literature, in particular cf. (Abdel-Hamid et al., 2006, p. 7, and passim; Kaygysyz et al., 2007, p. 175; Huang and Chiang, 2008, p. 140, and passim; Nguyen et al., 2009, p. 866). Also see the comparable airborne satellite/MotionPak test methodology in: (Wagner and Wieneke, 2003, p. 547, and passim).

3.2.6.1. Calibrating on water the system’s on-flight initialisation and alignment

The experiment 13aa reported in this paper involved: purposefully-incorrect GPS/IMU “stop” phase initialisation and alignment followed by incorrect GPS/IMU on-flight alignment during low-dynamics movement further followed by successful GPS/IMU on-flight alignment during high-dynamics circulation.

To avoid the anticipated problems with the intermittent telemetry outage and its impact on the Kalman filter, the RS telemetry transmitter was turned off completely.

In Figure 6, apart of the initial erratic manoeuvres discernible at the bottom right-hand-side-corner of the latitude/longitude LAT/LON trace; there are visible three distinctive legs: 1) outward, good visibility facilitated semi-straight course app. NWbW, the sole-GPS solution is smooth and can serve as a reference, while the GPS/IMU solution, following the “stop phase’s” incorrect initialisation and alignment, did not converge, was wavy, not smooth; the straight-course-low-dynamics on-flight alignment is not effective; 2) high dynamics circulation 270° CW plus 90° ACW at the upper left-hand-side-corner of the drawing; 3) return leg, course app. SE to SEbE when the GPS/IMU solution is smooth following the correct on-flight alignment effected by the high dynamics circulation mentioned in point 2) above.
Table 7. Experiment 15aa GPS/IMU on-flight alignment following the deliberately erroneous “stop phase” alignment

<table>
<thead>
<tr>
<th>#</th>
<th>Phase</th>
<th>$t_{[\text{h:min}]}$ or $\Delta t_{[\text{min}]}$</th>
<th>Notes</th>
<th>System</th>
<th>$\sigma_{[\text{m}]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>stop</td>
<td>$\Delta t \approx 2$</td>
<td>Diesel engine on, moored boat rolling on waves (wash)</td>
<td>(I+A)</td>
<td>$\pm 10$</td>
</tr>
<tr>
<td>2</td>
<td>OFA</td>
<td>17:11</td>
<td>low-dynamics manoeuvres nearby the Funka jetty</td>
<td>NOK</td>
<td>$\pm 7$</td>
</tr>
<tr>
<td>3</td>
<td>out OFA</td>
<td>$\Delta t \approx 6$</td>
<td>low-dynamics semi-straight course app. NW, at 17:12 velocity GPS – 6.1 knots, wind app. S, steady 2-3 (°B)</td>
<td>NOK</td>
<td>$\pm 10$</td>
</tr>
<tr>
<td>4</td>
<td>OFA</td>
<td>17:17</td>
<td>high-dynamics tight circulation 270° CW + 90° ACW</td>
<td>OK</td>
<td>$\pm 7$</td>
</tr>
<tr>
<td>5</td>
<td>return</td>
<td>$\Delta t \approx 8$</td>
<td>low-dynamics semi-straight course app. SE</td>
<td>OK</td>
<td>$\pm 7$</td>
</tr>
<tr>
<td>6</td>
<td>mooring</td>
<td>17:26</td>
<td>mobile experiment duration: 17:11 – 17:26, $\Delta t = 15$ min</td>
<td>OK</td>
<td>$\pm 7$</td>
</tr>
</tbody>
</table>

Note: sailing mission powered by Diesel; $t$ – time; $\Delta t$ – phase duration; $\{\text{h:min}\}$ – {hour:minute GPS time local zone}; (I+A) – “stop phase” initiation and alignment; OFA – on-flight alignment; system – system state; NOK – GPS/MKI Kalman filter LAT/LON solution not converging; OK – successful OFA and Kalman filter LAT/LON convergence; after the tight circular movement (# 4) had been completed, the alignment oscillations definitely ceased, oscillations in the LAT/LON navigation solution GPS/MKI was smooth (# 5); $\sigma$ – Sig POS from Kalman filter variance/covariance matrix.

3.2.6.2. GPS signal outage problems in the sole-GPS solution smoothed by the integrated DGPS/MKI solution during high dynamics rotation on the lake

Cf. Figure 5 (left-hand-side-apex of the red trace) and Figure 7. After the correct system initialisation the boat was cruising at full speed app. WNW-half-W course. Then at the apex of this course, the helm was put hard left and the boat turned almost instantly app. 180°. During this rotation, the signal from some of the satellites was blocked by the nearby woods and the IMU was operating in the stand-alone mode. Then the boat resumed her app. ESE-half-E course. Integrated DGPS/IMU solution serves as a reference for studying the the short-term stand-alone IMU solution.
Analysis of some low- and high-dynamics errors of Low-Cost IMU

Fig. 7. Sole-GPS a); DGPS/MKI b)
Note: According to the manufacturer the CEP – Circular Error Probable of a very well tuned MotionPak I IMU in a stand-alone mode roving with the rather high dynamics below the saturation band level of the I/O function obeys the following equation: $\text{CEP}[m] = \sqrt{k_0^2 + (k_2 \Delta t^2)^2 + (k_3 \Delta t^3)^2}$; where $k_2 < \text{app.} 1 \cdot 10^{-2}$; $k_3 < \text{app} 1 \cdot 10^{-4}$ where the constants $k_0$ [m], $k_2$ [m/s$^2$], and $k_3$ [m/s$^3$] were given by the manufacturer as approximate 1 $\sigma$ (one sigma) values.

3.2.6.3. GPS signal outage problems in the stand-alone DGPS solution smoothed by the integrated DGPS/MKI solution during average waterborne dynamics movement in the Wladyslawowo harbour

As mentioned in section 3.2.3, the DGPS/MKI mobile experiments were conducted on board M/B Imoros in Wladyslawowo (DGPS RS at Rozewie lighthouse). The boat sailed in the vicinity of water-walls and jetties so the sailing path was controlled within a metre. DGPS/MKI system retained the $\pm 1$ m to $\pm 2$ m one sigma (Kalman filter) accuracy and effectively bridged DGPS solution LAT/LON gaps caused by the satellite signal shadowing and multipath (Figures 8a and 8b). The landborne reference was created by the van-DGPS/MKI solution. The van at night followed the details of the port (no GPS gaps), cf. Figure 8c.

Fig. 8. WLA, water, DGPS solution a); WLA, water, DGPS/MKI b); WLA, van, DGPS/MKI c)
3.2.6.4. GPS signal outage problems (overhead bridge) in the stand-alone GPS solution smoothed by the GPS/MKI solution during average landborne dynamics movement proceeded by high dynamics, PRCT

Results from sample experiment when the van travelled five routes in ACW direction on the Poznan Racing Car Track PRCT – see Figure 9. The greatest dynamics of movement – 0.5 \(\bar{g}\) – were experienced in the northernmost part of the track, the van first accelerating to 100 km/h and then braking rapidly with the deceleration up to 0.5 \(\bar{g}\) while approaching the NE turn corner and then the overhead bridge. Figure 9a shows the sole-GPS solution LAT/LON positions and the gap in these solutions caused by GPS signal obstruction under the bridge. Figure 9b presents the uninterrupted GPS/MKI solution.

![Fig. 9. PRCT, sole-GPS, ACW connected points, 1 Hz a); PRCT, DGPS/MKI, ACW, 50 Hz b)](image)

3.2.6.5. GPS signal outage problems (buildings, lorries traffic) in the stand-alone DGPS solution smoothed by the integrated DGPS/MKI solution during average to low landborne dynamics movement

The Cracow AGH University of Science and Technology DGPS/MKI test-bed presents map Fig. 10a and semi-rectangle in red ink to the left. The DGPS RS was located at the roof of the AGH building (right). The van completed two circulations CW, no-stop, with an average speed and dynamics of movement, apart of some of the distances on the Armii Krajowej St. when both the speed and dynamics were very low due to very dense traffic. Figure 10b shows the stand-alone DGPS solution LAT/LON positions and the gaps in this solution caused by the GPS signal obstruction effected by the hotels (left) and the high-lorries passing on the Armii Krajowej St. In contrast, Figure 10c presents the completely uninterrupted DGPS/MKI solution. Cf. the response of the short-term stand-alone IMU solution of the DGPS/IMU to different signal outages and different dynamics of movement scenario in (Huang and Chiang, 2008, pp. 141, 143).
3.2.6.6. Telemetry outage problems in the stand-alone DGPS smoothed by integrated DGPS/IMU solution

Telemetry signal outage problems in the stand-alone DGPS solution were caused most probably by the saturation of the telemetry signal by the strong-signal military radar, mounted relatively nearby.

<table>
<thead>
<tr>
<th>#</th>
<th>Phase /leg</th>
<th>τ [h:min] or Δt [min]</th>
<th>Notes</th>
<th>GPS</th>
<th>DGPS</th>
<th>DGPS/MKI</th>
<th>σ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>stop</td>
<td>Δt ≈ 5</td>
<td>I+A OK</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
<td>±1</td>
</tr>
<tr>
<td>2</td>
<td>out</td>
<td>10:41 – 10:50/1</td>
<td>Funka RS – Kopernica landing stage. Poor visibility: first 2/3 part of leg steering compass course (Kopernica not visible), last 1/3 part steering to visible Kopernica</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
<td>±1</td>
</tr>
<tr>
<td>3</td>
<td>turn back</td>
<td>10:51 Δt ≈ 1</td>
<td>Kopernica landing stage dist. app. 10 m stop; then turn going astern portside CW 90° then forward CW 90°</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
<td>±2</td>
</tr>
<tr>
<td>4</td>
<td>return</td>
<td>10:51 – 11:01/1</td>
<td>From #3 steering towards the objects in-line at Funka slipway (well defined and moderately visible). DGPS deteriorates 3 times when telemetry outage observed</td>
<td>OK</td>
<td>NOK</td>
<td>OK</td>
<td>±1</td>
</tr>
<tr>
<td>5</td>
<td>final appr.</td>
<td>11:01 – 11:02/2</td>
<td>After approaching Funka slipway turning abruptly 90° ACW portside and steering along the Funka shore then turning sharply CW towards Funka RS and mooring</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
<td>±1</td>
</tr>
<tr>
<td>6</td>
<td>stop</td>
<td>Δt ≈ 5</td>
<td>Misclosure comp.; mobile exp. duration: Δt ≈ 21 min</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
<td>±1</td>
</tr>
</tbody>
</table>
3.2.7. Deterioration of the DGPS/IMU solution with reference to the DGPS solution and the GPS solution during extremely low dynamics of movement on water

The experiment time-wise description: at first the boat was sailing under her electric engine app. NWbN pilot course (cf. Figure 5, violet-pink trace). Then at the apex of this cruising the engine was stopped and the boat was drifting (see upper part of the drawings LAT/LON in Figure 12). Then the electric engine was started and engaged again and the boat sailed first app. SW and then finally took app. SE course back to her moorings. During the drift phase, the stand-alone DGPS solution serves as a reference for studying the IMU accelerometers’ behaviour in the case when both the linear accelerations and turn-rates are so low as to approach the “dead-band” of the IMU I/O function.

![Fig. 12. Stand-alone DGPS solution a); DGPS/MKI solution b); sole-GPS solution c)](image)

Table 9. Computational Conclusions Lake-Borne [D]GPS/IMU – Drift. Voyage # 01J15cc

<table>
<thead>
<tr>
<th>Boundaries/conditions</th>
<th>Wind direction and [°B]</th>
<th>Lake – water [°P]</th>
<th>Drift speed over water (knots)</th>
<th>Drift speed over bottom (knots)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drift time span [min]</td>
<td>West 1÷2</td>
<td>2</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Accuracy one sigma [m] POS from Kalman filter variance/covariance matrix during the drift phase</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Drift time span [min]</th>
<th>stand-alone DGPS σ [m]</th>
<th>[D]GPS/IMU σ [m]</th>
<th>sole-GPS σ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>±2</td>
<td>±2 to ±8</td>
<td>±6 to ±10</td>
</tr>
</tbody>
</table>

Note: electric engine in transit, during the drift phase engine stopped, cf. Figure 12.

---

5 In 1927 P. Petersen, a German sea captain, published a scale from 0 to 12 for the state-of-the-sea.
4. Conclusions

4.1. The conclusions from the MKII laboratory experiments

Table 5 shows that the values of the MKII accelerometer bias and scale factors determined separately during long-term (3 hours) stationary tests and separately during rotation are remarkably comparable for the Z-axis and X-axis and comparable to the specification provided by Table 1. Figures 1 to 4a show that under constant laboratory conditions the inertial and gravitational specific forces could well be separated phenomenologically. Therefore, according to both above worded conclusions the gravity \( \mathbf{g} \) calibrated MKII accelerometer well. This in turn could be utilized during the \([D]GPS/IMU\) field experiments combining the forces according to \((1c)\)-like and \((2)\)-like equations.

The laboratory experiments in general confirmed the assertions of the inertial literature of the necessity of maintaining the MKII’s internal temperature reasonably constant. Nevertheless, the cross-correlation in time domain between the MKII internal temperature output and gyro output was negligible, cf. Figures 1b and 1c. Interestingly, in the marked contrast the cross-correlation in time domain between the internal temperature output and the accelerometer output was notably high. However, the accelerometer’s bias and scale factor computed from various experiments and axes with the common warm-up time were reasonably constant when the ambient temperature was kept constant to within \( \pm 0.5^\circ C \).

In addition, the interpretation of Figures 3 and 4 for the accelerometer and gyro shows that the MKII accelerometers can (albeit not directly) provide valuable pieces of information (in terms of direction, sense and turn rate) about the rotation vector \( \mathbf{\omega} \) of the “body” system of coordinates with reference to the local vertical. These tests comply with the latest trends to seek accelerometer’s design measuring also rotations or a gyro measuring acceleration as well, cf. (O’Brien et al., 1995, p. 9, and passim; Kayton and Fried, 1997, p. 323; Titterton and Weston, 2004, p. 153, and passim).

4.2. The conclusions from the \([D]GPS/MKI\) lake-borne and land-borne experiments

In the absence of the S/A the short-term telemetry outages (not coupled with the GPS signal outages), which by their nature did not degrade \([D]GPS/MKI\) solution to the clear case sole-GPS/MKI nor to the stand-alone MKI, were smoothed satisfactorily by \([D]GPS/MKI\) solution which uninterruptedly maintained \( \sigma = \pm 1 \text{ m} \) accuracy during relatively low dynamics of movement on water, see Figure 11.\(^6\)

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\(^6\) S/A – Selective Availability, off, cf. tests with the S/A-on (Vorbrich, 2001a, 2001b).
However, according to the theory, overall the performance of the $[D]GPS$/MKI system under certain conditions should depend on the dynamics of motion. It seems that none of the waterborne and landborne tests was conducted by the author with the dynamics of movement, where the MKI I/O function passed into the “saturation band”. On the contrary, very high dynamics of waterborne manoeuvres in tight circulation were necessary for the successful on-flight $GPS$/MKI alignment following purposefully incorrect $GPS$/MKI “stop phase” initialisation and alignment (cf. Fig. 6a and Fig. 6b). Also, boat’s tight and speedy circulation facilitated the smoothing out of the $GPS$ signal outages, which led to the short-term stand-alone $IMU$ MKI mode of navigation (cf. Fig. 7a and Fig. 7b).

Likewise the van’s maximum achievable dynamics of motions realised by the crush braking (app. 0.5 ($g$)) and by the van’s centrifugal accelerations in bends, facilitated the short-time calibration of the MKI’s errors by means of the $DGPS$/MKI system. This in turn enhanced the short-time stand-alone MKI performance during consequent driving under the bridge with the average dynamics – Figures 9a and 9b. This kind of examination relating the $IMU$ output signal to the input (in terms of the maximum and minimum linear/angular accelerations which the $INS$ calibrated by $[D]GPS$/IMU can resolve) is consistent with the investigation trends reported by the inertial literature, cf. (Titterton and Weston, 2004, pp. 264, 266, and passim). However, the procedures demonstrated in the present paper are expected to be applicable in situations where body rates may approach the MKI’s sensor maximum angular rate capability only transiently. Nevertheless, the inertial literature suggests that in many applications where high turn rates coupled with the high amplitude vibrations are likely to be sustained, the procedure described in the present paper may not be adequate for the Low Cost $IMU$, cf. (Kayton and Fried, 1997, p. 324). According to the literature the $IMU$ should be scaled to sense/pick up considerably more than the maximum expected short-time acceleration, in order to prevent saturation in the presence of the sustained acceleration combined with vibration, cf. ibid. Nevertheless, the inertial literature proposes that then the Low-Cost $IMU$ performance should be compared for example with the ring laser gyro, which offers superior dynamic stability, cf. (Titterton and Weston, 2004).

Still, for the present author it has been impossible to ascertain the lowest applicable borders of the horizontal dynamics movement. We are writing here about the dynamic conditions, which are too low for the correct performance of the MKI during the short-term forced stand-alone mode (short term satellite signal outage). MKI in this mode performed satisfactorily during the average/low dynamics movement in the harbour when sometimes the $GPS$ was cut off (Figures 8a and 8b). Likewise MKI in this mode bridged $GPS$ signal outages during experiment when the van was driven slowly and jerkily along the quasi-jammed road while the $GPS$ antenna was sometimes sheltered by passing lorries (Figures 10b and 10c). Similarly the $DGPS$/MKI system’s solution in general mirrored closely the true pass when the van was driven comparatively slowly and jerkingly in order to follow all the details of the Wladyslawowo harbour and its jetties, cf. Figure 8c.
On the other hand, extremely low dynamics of the boat’s progress over the bottom degraded to a great extend the performance of the DGPS/MKI with reference to the stand-alone DGPS solution. Summary of IMU MKI error testing by means of the comparison of the results of the DGPS and DGPS/MKI solutions for the drift phase (set of Figures 12a – 12c) is as follows: a) LAT/LON solutions for DGPS are evidently smoother than for the DGPS/MKI; b) During the drift phase the accuracy sigma of the 2D position for DGPS is on average up to four times better than for the DGPS/MKI; c) The above mentioned superior performance of the stand-alone DGPS solution with reference to the DGPS/MKI solution has been most probably caused by the MKI accelerometers measuring in the “dead-band” of the I/O function.

The key issue is that in the “dead-band” the “not-white-noise” and not-realistic signals of the IMU were wrongly interpreted by the Kalman filter, which did not converge fully satisfactorily. Wagner and Wieneke (2003, p. 548) reported comparable results of the satellite/MotionPak airborne tests. According to ibid., because of the weak satellite/MotionPak observability during platform’s very low dynamics, the error variance (of the state vector equation equivalent to eq. (7) above) rises. In turn this could lead to stability problems making worthless the Kalman solution for state vector type (7).

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Analiza pewnych błędów „mało dokładnej” inercjalnej jednostki pomiarowej przy niskiej i wysokiej dynamiczce ruchu

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Streszczenie
Część zasadnicza pracy przedstawia pewne rezultaty doświadczeń przeprowadzanych w laboratorium Uniwersytetu w Singapore i wyznaczających błędy stosunkowo „mało-dokładnej” (ag. Low-Cost lub „Low-Grade”) inercjalnej jednostki pomiarowej MotionPak™ typu MKII w modzie statycznym i dynamicznym. Ten ostatni był ograniczony do obrotów wokół osi poziomej układu współrzędnych wyznaczanych przez obudowę MKII. Współczynnik skali i stałe przesunięcia skalowane były wartością wektora przyspieszenia ziemskiego.

Testy polowe przeprowadzano na pokładzie statków badawczych na jeziorze i na morzu we Władysławowie oraz na lądzie na samochodzie. Praca prezentuje niektóre wyniki testów mobilnych (porównanie ciągłości rozwiązań GPS, DGPS, DGPS/MKI), m.in. przy dużej, średniej oraz bardzo małej (dryf statku) dynamiczce ruchu. Oprogramowanie umożliwiało pracę jednocześnie w modach o notacji: [D]GPS/MKI oraz [D]GPS/INS(MKI). Dla pierwszego z tych modów filtr Kalmana wykorzystywał surowe dane dla inercjalnych przyspieszeń i kątów Eulera. Dla drugiego z tych modów do filtra Kalmana wprowadzono inercjalne pseudoodległości i ich zmiany w czasie. Praca prezentuje rezultaty rozwiązania dla modu drugiego. Rozwiązanie ruchu dla GPS, DGPS, oraz [D]GPS/INS(MKI) służyło jako wzorzec dla badania ciągłości rozwiązania samo-stojącego MKI w czasie nieciągłości odbioru sygnału satelity lub telemetrii. Dodatkowo, testy wykonane na jeziorze przy prawie zerowej dynamiczce ruchu wykazały zmniejszenie dokładności rozwiązania [D]GPS/INS(MKI) w porównaniu do rozwiązania DGPS. Doświadczenia polowe miały m.in. na celu wykazanie prawidłowości działania oprogramowania integrującego wykonanego w ramach polsko-niemieckiego projektu Unii Europejskiej i zmodyfikowanego w czasie grantu finansowanego przez Komitet Badań Naukowych.