Application of stand density indices for Chinese fir (*Cunninghamia lanceolata* (Lamb.) Hook) plantation management

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Abstract. The most important issues in Chinese fir (*Cunninghamia lanceolata* (Lamb.) Hook) management are the quantitative determination of stand density and the selection of appropriate density. Different stand density index models have advantages for special tree species, and this study aimed to estimate the carrying capacity of planted stands of Chinese fir and to select simple and reliable stand density indexes. Based on special experiment of different initial density, the maximum carrying capacity was estimated using Reineke's self-thinning rule, Nilson's sparsity theory, Beekhuis's relative-spacing hypothesis, Zhang's nutrient-competition rule, Curtis's maximum stand basal area and Hui's crowding degree based on mean tree distance. The restricted maximum likelihood method (REML) implemented with 'nlme' package in R software was used to refine the parameters of thinning age in Richard's growth model. The results showed that stand density index models can describe the trends of stand density in response to tree growth: the higher the plantation initial density, the earlier age and stronger self-thinning capacity of stands. Reineke's SDI and Zhang's Z model are the most stable and suitable to estimate changes in the density of Chinese fir plantations, competition intensification, and the thinning age. The RD model can also be used, except at low Chinese fir densities. K, which can be affected by the mean crown width of trees, directly reflects the forest area of photosynthesis; this index is easy and simple to apply, but more research is needed to optimize the equation to evaluate whether a forest requires management and to determine the appropriate time for the first thinning and its intensity.

Key words: stand density index, stand sparsity, relative spacing, ratio of dominant height to nutrient area, relative density index, stand crowding degree.

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Introduction

Chinese fir (*Cunninghamia lanceolata* (Lamb.) Hook) is a very important timber tree species (Zhang *et al.*, 2013). In order to satisfy the increase of timber demand, more silvicultural practices and long-term silvicultural trials are needed. In Chinese fir plantations, the spatial distribution of trees and relative density have a strong influence on tree height, diameter at breast height

(DBH), stand basal area and the use of nutrients and space (Nagaike *et al.*, 1999; Sun *et al.*, 2007). Therefore, in the management of Chinese fir, it is necessary to determine how tree height, DBH, and stand density vary and to investigate the maximum density of stands at different sites and initial densities. Multi-level linear models can simulate individual-tree diameter growth (Zhao *et al.*, 2013; Ma, 2015; Ma *et al.*, 2015), and the initial density will influence self-

pruning and nutrient competition (Sun *et al.*, 2014; Dong *et al.*, 2016).

Although Chinese fir management lags behind that of spruce (*Picea* sp.), radiata pine (*Pinus radiata* D.Don) and other major timber species in Europe (Xue *et al.*, 2015; Xiang *et al.*, 2015), sustainable forestry, as a paradigm of forest management, has proven that forest simulation is the best tool for forest management (Peng & Wen, 2006). Stand density can be regarded as the most important factor that affects the carrying capacity, growth rate, wood quality and forest ecological functions (Meng, 1996). Therefore, stand density can also be used as an important feature in the evaluation of forest quality. In a fully stocked forest, stand density depends on the biological environment (e.g., solar radiation, nutrient level, resource and sink functions) (Durham, 1991; Harms *et al.*, 1994), the physical environment (e.g., crown width, growing space), tree species, and age composition (Amateis & Burkhardt, 2012; Yang & Burkhardt, 2017). In Chinese fir plantation management, site index (SI) based on standard age = 20 depends on the stand density, and the carrying capacity depends on the stand density and site index. Therefore, stand density can be estimated using maximum size-density relationships (MSDRs) (Sun *et al.*, 1999; Gu *et al.*, 2001; Mitsuda *et al.*, 2003).

In a fully stocked forest, the MSDR is the maximum “trade-off” between the average tree size and number of trees per unit area that can be supplied in a given stand (Arisman *et al.*, 2004; Arisman *et al.*, 2005; Inoue, 2009; Kurinobu & Miyaura, 2011). Measurements of MSDR have been carried out using different size variables, such as quadratic mean diameter (Reineke, 1933), relative space (Daniels & Burkhardt, 1988), nutrient area (Zhang *et al.*, 2011b), and stand crowding degree (Hui *et al.*, 2016). By comparing all MSDR measures with different size variables, Burkhardt (2013) argued that quadratic mean diameter was the most informative in theory if the differences among tree size variables were not

large. Zhang *et al.* (2011a) indicated that the ratio of dominant height (H_d) to nutrient area and the forest mean crown width reflect the density. Hierarchical Bayesian models are a useful method to predict the MSDR line (Cui *et al.*, 2016). Stand density models, i.e., Reineke’s stand density index (SDI), Nilson’s stand sparsity density (SD), Beekhuis’s relative spacing (RS), Zhang’s ratio of dominant height to nutrient area (Z), Curtis’s relative density index (RD), and Hui’s stand crowding degree (K), were compared and evaluated in this study.

Material and Methods

Data used in this study were from Chinese fir stands established using seedlings that were bare-root planted in 1982 at Weimin farm in Fujian, south of China. This region has a subtropical monsoon climate with an annual average temperature of 17.7 °C, average annual air relative humidity of 82%, sunshine duration of 1,740.7 h, a frost-free period of 270 d, and rainfall of 1,768 mm.

A total of 15 plots were planted in a randomized block arrangement with the following five densities (treatments): A (2 m × 3 m, 1,667 trees / ha), B (2 m × 1.5 m, 3,333 trees / ha), C (2 m × 1 m, 5,000 trees / ha), D (1 m × 1.5 m, 6,667 trees / ha) and E (1 m × 1 m, 10,000 trees / ha). Each treatment was replicated three times. Each plot was 0.06 ha in size and a buffer zone consisting of two lines of similarly treated trees surrounded each plot.

Diameter at breast height (DBH) was measured annually for ages 3–28. In addition, mean crown width (CW) and height to crown base were measured from the 4th year; tree height (H) was recorded annually for ages 1–28; each tree was measured annually for ages 1–10; and H , H_d , and CW of every 5th trees were measured biennially for ages 12–28. The descriptions of each plot are presented in Table 1.

Table 1. Descriptions of plots.

Plot	Site Index (m)	Initial density* (Trees/ha)	Height* (m)			Stand density** (Trees/ha)	Height** (m)			DBH** (cm)		
			Min.	Max.	Mean		Min.	Max.	Mean	Min.	Max.	Mean
A1	16	1,667	0.39	0.99	0.61	1,650	6.60	23.10	17.11	7.20	32.80	20.33
A2	20	1,667	0.26	0.89	0.53	1,567	8.20	26.90	20.51	7.30	33.30	22.60
A3	22	1,667	0.35	0.89	0.57	1,467	15.00	28.00	22.71	12.80	36.10	24.63
B1	14	3,333	0.27	69.00	0.91	3,250	4.50	21.20	15.43	4.30	26.30	16.14
B2	20	3,333	0.24	0.88	0.52	1,683	12.90	28.30	20.77	9.70	32.60	20.35
B3	18	3,333	0.21	0.89	0.55	2,150	9.60	28.30	19.60	2.00	41.30	18.98
C1	16	5,000	0.22	1.57	0.60	3,900	8.80	42.90	16.45	3.60	29.20	15.23
C2	22	5,000	0.18	1.46	0.58	2,617	11.30	113.60	18.90	8.80	33.70	17.28
C3	18	5,000	0.12	0.88	0.54	1,700	13.90	28.20	20.39	10.40	28.70	22.10
D1	14	6,667	0.19	1.00	0.60	4,183	9.80	25.20	16.59	5.70	31.00	14.79
D2	20	6,667	0.20	0.93	0.57	3,050	10.70	25.50	18.99	7.40	34.00	17.43
D3	14	6,667	0.22	0.87	0.51	1,683	13.10	24.40	19.50	11.40	31.10	19.67
E1	14	10,000	0.12	0.97	0.57	3,167	8.30	25.70	16.05	6.20	33.00	14.69
E2	20	10,000	0.25	6.00	0.58	1,667	11.40	28.70	20.30	7.90	36.10	19.80
E3	12	10,000	0.19	1.00	0.57	3,150	8.40	27.00	16.33	2.80	34.10	15.07

Note: *measured in 1982; **measured in 2010.

Six different stand density index models were selected to measure the changes of stand density. In Reineke's self-thinning rule (Reineke, 1933), the maximum stand density (N) and quadratic mean diameter (D_q) in fully stocked even-aged stands have a linear relationship on a log-log scale; the equation is:

$$\ln N = a + b \times \ln D_q \quad (1)$$

where a and b are the equation coefficients.

Reineke defined a stand density index (SDI) to estimate the carrying capacity in even-aged stands. According to Reineke's theory, SDI is expressed in the form

$$SDI = N \left(\frac{D_q}{D_0} \right)^b \quad (2)$$

where the metric unit is number of trees per ha (N), cm (D_q and D_0). SDI describes the density of stands with D_q and N by calculating the number of stems per hectare in these stands at 10 inches index diame-

ter (25.4 cm). In China, an index diameter $D_0 = 20$ cm is used. Coefficient b , which was previously defined, is the self-thinning ratio.

Nilson proposed the 'sparsity index' model, using the number of trees and quadratic mean diameter of a stand (Nilson, 2006). In this model, stand sparsity (L) and quadratic mean diameter (D_q) are linearly related, that is,

$$L = \sqrt{\frac{10000}{N}} \quad \text{and} \quad D_q = a_2 + b_2 L = a_2 + 100b_2 N^{-1/2} \quad (3)$$

where a and b are coefficients. The model of Nilson's self-thinning model follows:

$$N = a \left(\frac{D_0 + k}{D_q + k} \right)^2 \quad (4)$$

where $a = 100b_2 / (D_0 - a_2)^2$, and $k = -a_2$ are constant parameters of a given tree species in a given region. According to Nilson's model of self-thinning, Nilson's stand

sparsity density (SD) can be described as (Zeide, 2010):

$$SD = N \left(\frac{D_q + k}{D_0 + k} \right)^2 \quad (5)$$

In Beekhuis's relative spacing (RS) equation (Beekhuis, 1965), the relative spacing value depends on the dominant tree height and the number of trees per ha, which cannot be reflected by stand site quality, diameter at breast height, and so on.

$$RS = \frac{100}{N^{1/2} \times H_0} = \sqrt{\frac{10000}{N \times H_0^2}} \quad (6)$$

H_0 is the dominant tree height of the stand (m).

Zhang *et al.* (2011a) focused on variation in Reineke's self-thinning ratio between different species. These authors defined nutrient area with the reciprocal of relative spacing, and the ratio of dominant height to nutrient area as a new density index (Z). Z is described as

$$Z = \frac{N \times H_0}{10000} \quad (7)$$

In forest management, the higher the plant density, the sooner the stand density is stable. Compared to Reineke's SDI, the Z index can be used across a wide age for plantation management. The Z index is a nonparametric density index, which increases with increasing planting density.

Similar to Reineke's SDI equation, when the stand is fully stocked, the stand has maximum stand basal area (G_L). Curtis used the ratio of observed stand basal area (G_{obs}) to G_L to calculate the change in stand basal area. Curtis's relative density index (RD) is expressed in the following form (Curtis, 2010),

$$RD = \frac{G_{obs}}{G_L} = \frac{G_{obs}}{k(D_q^b)} \quad (8)$$

where k and b are coefficients, and $b = 2 - \beta$ (β is the stand thinning ratio, β is the self-thinning ratio). Since k is a constant

multiplier, Curtis replaced k with 1.0; the value of RD and the relative rank of a stand remain unchanged, so the equation can be expressed as

$$RD = \frac{G_{obs}}{D_q^b} \quad (9)$$

Hui *et al.* (2016) proposed a new index K (stand crowding degree) based on the forest mean crown width of trees (CW) and the mean tree distance (L). The reasonable interval of the K value ranges from 0.9 to 1.1; this index can be used to evaluate whether the forest requires management and to determine the appropriate time for the first thinning and its management intensity. If $K > 1.1$, this mean the forest density is low, and without thinning, if the K value is too high, the stand will need to be replanted. If $K < 0.9$, the stand needs to be thinned to reduce the crowding degree. The equation used to determine the stand crowding degree index, K, is

$$K = \frac{L}{CW} = \frac{\sqrt{10000 / N}}{CW} \quad (10)$$

where L is the average tree distance (m), and CW is the average stand crown width (m).

The changes in stand density index models, increasing with age (the crowding degree curve uses the $1/K$ - age relationship line), shown as an S-curve. Richards' equation is a widely used model for modelling the S-curve, which can be expressed as follows:

$$y = a (1 - e^{-b \times t})^c \quad (11)$$

where y denotes stand density index (e.g., SDI, SD, RS, RD, Z, K), t is stand age, and a , b , and c are equation coefficients.

In this study, a plot-level random-effects parameter was added to parameter a of the stand density index model. The random-effect parameter was defined as: $u \sim N(0, v)$, where v is the variance. All parameter

vectors can be estimated through the optimization of the likelihood function, that is, by applying the restricted maximum likelihood method (REML). The unstructured covariance structure was used for describing the variance-covariance structure of random effects. Parameter estimation was implemented with 'nlme' package in R software (Pinheiro *et al.*, 2017).

In the Richards equation, the age at the inflection point (t_r , equation 12) indicates that competition between trees has intensified, and the age where the increasing rate of the equation slows (t_m , equation 13) indicates that competition has approached a steady state.

$$t_r = \frac{\ln c}{b} \quad (12)$$

$$t_m = -\frac{\ln \frac{3c-1-\sqrt{5c^2-6c+1}}{2c^2}}{b} \quad (13)$$

Results and Discussion

Reineke's self-thinning rules were used to measure the relative density, and fully stocked stands with relative density greater than or equal to 0.7 were reported (Das & Singh, 2016, Ramanantoandro *et al.*, 2016; Dehane *et al.*, 2017). Tree growth increased the competition for resources such as solar radiation, water, and nutrients. Higher plantation density improved the competition and resulted in earlier full stocking and self-thinning age of stands. An estimate of relative density was obtained from $N/N_{\max}$ ($N_{\max}$: the maximum density under the self-thinning rule) in each stand; plot E with the highest density of $1 \times 1\text{m}$ was fully stocked at the 8th year, plots C and D at age 10, and the stand with the lowest plantation density reached full stocking by age 14. At the age of 28, as density increased, the relative density decreased from 0.94 to 0.27. Reineke (1933) reported that in fully stocked stands and even-aged forestry, the self-thinning

rate is approximately 1.605; in this study, as the plantation density decreased from $2 \times 3\text{m}$ to $1 \times 1\text{m}$, the self-thinning rate increased to 0.282, 1.213, 1.468, 1.406 and 1.507, and rate equaled 1.821 without considering the plantation density.

Site quality is considered as the capacity of nutrients, water and other resources to support tree growth. Nutto *et al.* (2007) used stand dominant height at base age as a site index to represent site quality. At the same plantation density, trees growth faster at a higher site index (Figure 1). If trees in stands are strongly crowded, plantation density determines the self-thinning rate and the survival rate rather than site quality (Figure 1 plot E).

Hui *et al.* (2016) reported that after site quality, stand density is the second most important factor that influences stand growth, the carrying capacity, and forest quality and stand stability. Different stand density index models have different advantages to describe the stage of survival during the competition process throughout forest formation and development. Figure 2 shows that during the earlier periods of stand growth, as stand age increases, the stand density index models SDI, SD, Z and RD increased, and the change became steady or declined slightly during the later stage. This is consistent with stand development and the principles of stand density index models, such as SDI, SD, Z, and RD, i.e., the greater the density index, the more crowded the forest. The changes in RS among different initial density values decreased with stand growth and became steady at later forest ages, which is in line with the following: the smaller the RS, the more crowded the forest. When we use a density index to measure stand crowding, the best density index model can reflect the degree of tree crowding as well as distinguish between different initial densities. From the 10th year, RS differs little between different plantation densities (Figure 2); consequently, this density index is not suitable to assess the degree of tree crowding.

Figure 1. Relationship between relative density and planting density across different site index values.

Figure 2. Relationship between stand age and stand density index models for different planting densities.

A similar result was obtained in the study of Zhang (2011b) with respect to the first thinning age of a *Pinus massoniana* Lamb. plantation.

Changes in K decreased initially and then increased with forest growth. Stands with a higher initial density were the first to reach the lowest point of K ; this indicates that higher plantation density and crowding can increase stand competition. Figure 2 shows that in plot B from 3–12 years and in stands with density C, D, and E from 3–28 years, the value of K was less than 0.9 (except in plot C, where the value was slightly higher than 0.9 at 20–24 a). By contrast, the K of plot A was higher than 0.9 for almost the whole growth period and was even higher than 1.1 from 14–24 a. Li (2012) reported that the stand crowding degree (K) can even be used in studies of natural forest management.

The values from these density index models (SDI, SD, RD and Z) increase with forest growth. In order to distinguish these indexes, it is necessary to establish relationship equations between stand age and density indexes.

The results (Table 2) shows that, except for density index model SD, as plantation density increased, t_r and t_m , two important characteristic parameters, decreased, which indicates that at high stand density, competition and self-thinning occur earlier. This is consistent with the process of stand growth,

and the density index model SD is not suitable for explaining the growth of Chinese fir plantations. According to Hui's definition of K and the results presented in Figure 2, the changes of $1/K$ between the 3rd–10th year conform to Richard's equation, and the value of t_r is 1.5–2.2, and that of t_m is 2.4–3.2. These values are obviously lower than those of SDI, Z and RD, especially when thinning starts at 2.0–2.6 years old; this result does not conform to Chinese fir plantation management, so the stand density index of K is not suitable.

A smaller value of t_m indicates the density index has high sensitivity, and the mean values of t_r and t_m are usually used as the beginning age of stand thinning. In addition, SDI depends on tree species, site quality, fully stocked status, and the selection of D_q , therefore, RD is more simple than SDI for application in forest management.

A comparison of the values of SDI, Z and RD with the observed values using a paired samples T test indicated that the differences between observed values and the SDI model were not significant (0.059–0.604), and the correlation coefficient ranged from 0.985–0.999 (Table 3). The same trend of model Z was observed, with a difference of 0.270–0.984, and correlation of 0.801–0.987. RD is another excellent model, except for the higher initial density of E: significant differences (0.010) and a lower correlation (0.251) were observed.

Table 2. t_r and t_m of different stand density index models for different initial density values.

Initial density	SDI			SD			Z			RD			K		
	t_r	t_m	mean	t_r	t_m	mean	t_r	t_m	mean	t_r	t_m	mean	t_r	t_m	mean
A	5.3	10.7	8.0	8.5	15.2	11.9	3.5	5.8	4.7	4.5	9.4	7.0	2.2	3.0	2.6
B	4.7	8.8	6.8	8.4	14.1	11.2	1.7	5.1	3.4	4.1	7.3	5.7	1.7	2.5	2.1
C	4.6	8.4	6.5	9.8	16.6	13.2	2.2	5.0	3.6	4.1	7.3	5.7	2.2	3.2	2.7
D	4.8	8.5	6.7	10.8	17.0	13.9	2.1	4.7	3.4	4.1	7.4	5.8	1.8	2.8	2.3
E	4.0	6.8	5.4	11.7	17.9	14.8	2.1	3.6	2.9	3.5	5.8	4.7	1.5	2.4	2.0

Note: SDI, stand density index; SD, stand sparsity density; Z, ratio of dominant height to nutrient area; RD, relative density index; K, stand crowding degree.

In order to optimize the stand density index, explain the process of stand density change and reduce the influence of uncertainty, the NLME method with the R software was used to fit the density model (Table 4). The correlation coefficients showed that the SDI, SD and RD models had the highest fitting degree (0.9044–0.9935), and the model Z showed the worst performance at the lowest and highest initial densities of the A and E plots ($R^2 = 0.0000$ –0.0700). The 1/K model did not exhibit a good fitting effect under different initial densities ($R^2 = 0.1354$ –0.7328). The parameters *a*, *b* and *c* of the SDI and RD models showed an increasing trend with increasing initial density, which accorded with the definitions of the parameters. The relationship between the parameters of models SD and Z and the initial planting density remains uncertain. The performance of these two models in the density management and regulation of Chi-

nese fir plantation was worse than that of the SDI model and RD model.

According to the definition and changes in *K*, a 1/*K* value between 3–10 years was used to calculate the coefficients of Richard's growth equation (Equation 11). When the stand age was > 10 years, stand density decreased obviously, and the stand crowding degree increased. In addition, some of the trees died due to competition for light and nutrients, and tree survival depends on density (Zhang *et al.*, 2009). According to the results of Hui's study, if the value of *K* is lower than 0.9, the trees in a stand are crowded; the results in Figure 2 show that the *K* value of plots B–E was lower than 0.9 from the beginning to age 28 years, which is consistent with the conclusion drawn from table 3 but different from the results of the RD and SDI models and the actual crown closure. However, based on the value of R^2 (Table 4) using Richard's growth equation

Table 3. Paired samples T test of different models.

Model	Initial density	Mean	SD	SE	95% confidence interval		Sig.	R^2
					Lower	Higher		
SDI	A	-9.706	36.204	8.781	-28.320	8.908	0.285	0.999
	B	-18.177	35.139	8.523	-36.244	-0.109	0.059	0.999
	C	22.941	57.542	13.956	-6.644	52.526	0.120	0.998
	D	22.294	52.644	12.768	-4.773	49.361	0.100	0.998
	E	15.706	122.413	29.689	-47.233	78.645	0.604	0.985
Z	A	-2.284	0.941	0.228	-2.767	-1.800	0.270	0.895
	B	0.005	0.252	0.061	-0.125	0.134	0.941	0.987
	C	0.011	0.511	0.124	-0.252	0.274	0.930	0.965
	D	0.020	0.602	0.146	-0.289	0.330	0.892	0.959
	E	0.007	1.419	0.344	-0.723	0.737	0.984	0.801
RD	A	-45.497	217.626	52.782	-157.390	66.396	0.401	0.998
	B	-892.307	395.369	95.891	-1095.587	-689.027	0.074	0.996
	C	128.430	308.294	74.772	-30.080	286.941	0.105	0.998
	D	134.078	419.077	101.641	-81.392	349.547	0.206	0.996
	E	2788.701	3905.110	947.128	780.879	4796.523	0.010	0.251

Note: SDI, stand density index; Z, ratio of dominant height to nutrient area; RD, relative density index.

Table 4. Parameter estimation and R^2 of density models.

Model	Initial density	Parameter			R^2
		a	b	c	
SDI	A	2009.083	0.126	1.948	0.992
	B	2071.422	0.181	2.363	0.992
	C	2142.677	0.198	2.498	0.964
	D	2318.751	0.204	2.649	0.977
	E	2085.886	0.295	3.265	0.931
SD	A	137.506	0.116	2.687	0.992
	B	115.122	0.142	3.278	0.985
	C	124.109	0.120	3.232	0.979
	D	123.073	0.137	4.396	0.994
	E	104.427	0.138	5.011	0.952
Z	A	5.905	0.361	3.592	0.000
	B	5.866	0.137	1.268	0.908
	C	7.164	0.218	1.627	0.807
	D	8.044	0.246	1.688	0.813
	E	9.124	0.515	2.940	0.070
RD	A	11107.907	0.136	1.850	0.992
	B	12933.349	0.232	2.564	0.953
	C	12933.349	0.232	2.564	0.953
	D	14238.486	0.236	2.648	0.964
	E	13365.905	0.361	3.592	0.904
1/K	A	1.270	1.086	10.650	0.647
	B	1.429	0.976	5.015	0.317
	C	1.750	0.871	6.724	0.733
	D	1.786	0.852	4.546	0.360
	E	1.710	0.923	4.083	0.135

Note: SDI, stand density index; SD, stand sparsity density; Z, ratio of dominant height to nutrient area; RD, relative density index; K, stand crowding degree. A, 1,667 trees/ha; B, 3,333 trees/ha; C, 5,000 trees/ha; D, 6667 trees/ha; E, 10,000 trees/ha; a, b, c, equation coefficients; R^2 , goodness of fit.

for ages 3–10 years, the accuracy of the K and 1/K models is lower than that of other models.

The value of RD was determined using the stand basal area and self-thinning ratio (β); these two indicators are affected by the growth of trees, while light intensity can also affect Chinese fir (Cheng *et al.*, 2011). Reineke's equation also uses a self-thinning

ratio (β) to calculate SDI and the maximum size density, such as in the management of Japanese cypress and red pine (Inoue *et al.*, 2004), but another important coefficient D_q has different standard values for different tree species, operation methods and objectives (Burkhart & Tomé, 2012). The growth and self-thinning of Chinese fir plantations can be influenced by site quality (such as

site index, climate factors) (Zhang *et al.*, 2011; Charru *et al.*, 2012; Brunet-Navarro *et al.*, 2016). Therefore, when using the R procedure to estimate the parameters of Richards equation, the random effects of parameters a and b between plots affect the accuracy of model estimation. The random effects increased with increasing initial density. The R^2 value of model RD ranged from 0.9044–0.9915 under different initial densities.

The stand crowding degree (K) not only considers the relative space but can be used in natural forestry management, compared with RD and SDI, i.e., three different stand density models that do not consider the effect of site, climate, forest management and other factors.

Reineke (1933) stated that SDI was a stable index that cannot be influenced by site quality and initial density when stands are fully stocked. However, mortality models can be improved by selecting variables such as regional climate (Qiu *et al.*, 2015), and site index plays an important role in self-thinning in Chinese fir plantations (Figure 1, Table 3). Therefore, in order to increase the accuracy of model prediction, whether climate factors and site index will be included in the self-thinning rule and Richards equation requires more research and reflection because more parameters will increase the complexity of models.

Conclusions

Our study evaluated approaches to estimate the stand carrying capacity of tree numbers per unit area in Chinese fir management and enhanced the understanding of competition and forest stand density dynamics. These results can provide information on plantation density, thinning schedules and thinning ratios.

As a classic stand density index model, SDI is also suitable for the management of Chinese fir plantations, but its self-thinning rate and D_q need to be determined based on

tree species and a considerable amount of data. According to Richard's growth equation, the first thinning age under the SDI model was calculated as 5.4 to 8.0 years, which is similar to actual Chinese fir management in China. RD can be regarded as the deformation of Reineke's SDI model; although the predicted thinning age is not significantly different from that of SDI, it is not suitable for management at higher initial density, and the self-thinning rate and D_q also need to be calculated.

Z and K , used as stand density indexes, have appeared in recent years. Compared with SDI and RD, Z is an excellent model with fewer statistics and easy measurements, and its predicted results of t_r and t_m are not significantly different from those of SDI. Although the results showed that K was not suitable, Hui provided a range of K values (0.9, 1.1) for plantations and natural forest management. This index has few parameters that can be easily and accurately measured; therefore, its further optimization is worthwhile.

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