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Basic Properties of Periodic Functions

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Summary. In this article we present definitions, basic properties and some examples of periodic functions according to [5].

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The papers [2], [6], [3], [10], [11], [9], [8], [1], [4], and [7] provide the terminology and notation for this paper.

1. Basic Properties of a Period of a Function

We use the following convention: x, t, t_1, t_2, r, a, b are real numbers and F, G are partial functions from $\mathbb{R}$ to $\mathbb{R}$.

Let F be a partial function from $\mathbb{R}$ to $\mathbb{R}$ and let t be a real number. We say that t is a period of F if and only if:

(Def. 1) $t \neq 0$ and for every x holds $x \in \operatorname{dom} F$ iff $x+t \in \operatorname{dom} F$ and if $x \in \operatorname{dom} F$, then $F(x) = F(x+t)$.

Let F be a partial function from $\mathbb{R}$ to $\mathbb{R}$. We say that F is periodic if and only if:

(Def. 2) There exists t which is a period of F.

We now state a number of propositions:

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(1) t is a period of F iff $t \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t$, $x-t \in \operatorname{dom} F$ and $F(x) = F(x+t)$.

(2) If t is a period of F and a period of G, then t is a period of $F+G$.

(3) If t is a period of F and a period of G, then t is a period of $F-G$.

(4) If t is a period of F and a period of G, then t is a period of $F\,G$.

(5) If t is a period of F and a period of G, then t is a period of F/G.

(6) If t is a period of F, then t is a period of $-F$.

(7) If t is a period of F, then t is a period of $r\,F$.

(8) If t is a period of F, then t is a period of $r+F$.

(9) If t is a period of F, then t is a period of $F-r$.

(10) If t is a period of F, then t is a period of $|F|$.

(11) If t is a period of F, then t is a period of F^{-1}.

(12) If t is a period of F, then t is a period of $F^{\mathbf{2}}$.

(13) If t is a period of F, then for every x such that $x \in \operatorname{dom} F$ holds $F(x) = F(x-t)$.

(14) If t is a period of F, then $-t$ is a period of F.

(15) If t_1 is a period of F and t_2 is a period of F and $t_1+t_2 \neq 0$, then t_1+t_2 is a period of F.

(16) If t_1 is a period of F and t_2 is a period of F and $t_1-t_2 \neq 0$, then t_1-t_2 is a period of F.

(17) Suppose $t \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t$, $x-t \in \operatorname{dom} F$ and $F(x+t) = F(x-t)$. Then $2 \cdot t$ is a period of F and F is periodic.

(18) Suppose $t_1+t_2 \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t_1$, $x-t_1$, $x+t_2$, $x-t_2 \in \operatorname{dom} F$ and $F(x+t_1) = F(x-t_2)$. Then t_1+t_2 is a period of F and F is periodic.

(19) Suppose $t_1-t_2 \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t_1$, $x-t_1$, $x+t_2$, $x-t_2 \in \operatorname{dom} F$ and $F(x+t_1) = F(x+t_2)$. Then t_1-t_2 is a period of F and F is periodic.

(20) Suppose $t \neq 0$ and for every x such that $x \in \operatorname{dom} F$ holds $x+t$, $x-t \in \operatorname{dom} F$ and $F(x+t) = F(x)^{-1}$. Then $2 \cdot t$ is a period of F and F is periodic.

Let us observe that there exists a partial function from $\mathbb{R}$ to $\mathbb{R}$ which is periodic.

Let F be a periodic partial function from $\mathbb{R}$ to $\mathbb{R}$. One can check that $-F$ is periodic.

Let F be a periodic partial function from $\mathbb{R}$ to $\mathbb{R}$ and let r be a real number. One can check the following observations:

* $r\,F$ is periodic,
* $r+F$ is periodic, and

* $F - r$ is periodic.

Let F be a periodic partial function from $\mathbb{R}$ to $\mathbb{R}$. One can check the following observations:

* $|F|$ is periodic,
* F^{-1} is periodic, and
* $F^{\mathbf{2}}$ is periodic.

2. Some Examples

Let us note that the function sin is periodic and the function cos is periodic.

We now state two propositions:

(21) For every element k of $\mathbb{N}$ holds $2\cdot\pi\cdot(k+1)$ is a period of the function sin.

(22) For every element k of $\mathbb{N}$ holds $2\cdot\pi\cdot(k+1)$ is a period of the function cos.

Let us observe that the function cosec is periodic and the function sec is periodic.

We now state two propositions:

(23) For every element k of $\mathbb{N}$ holds $2\cdot\pi\cdot(k+1)$ is a period of the function sec.

(24) For every element k of $\mathbb{N}$ holds $2\cdot\pi\cdot(k+1)$ is a period of the function cosec.

Let us mention that the function tan is periodic and the function cot is periodic.

Next we state a number of propositions:

(25) For every element k of $\mathbb{N}$ holds $\pi\cdot(k+1)$ is a period of the function tan.

(26) For every element k of $\mathbb{N}$ holds $\pi\cdot(k+1)$ is a period of the function cot.

(27) For every element k of $\mathbb{N}$ holds $\pi\cdot(k+1)$ is a period of |the function sin|.

(28) For every element k of $\mathbb{N}$ holds $\pi\cdot(k+1)$ is a period of |the function cos|.

(29) For every element k of $\mathbb{N}$ holds $\frac{\pi}{2}\cdot(k+1)$ is a period of |the function sin| + |the function cos|.

(30) For every element k of $\mathbb{N}$ holds $\pi\cdot(k+1)$ is a period of (the function sin)$^{\mathbf{2}}$.

(31) For every element k of $\mathbb{N}$ holds $\pi\cdot(k+1)$ is a period of (the function cos)$^{\mathbf{2}}$.

(32) For every element k of $\mathbb{N}$ holds $\pi\cdot(k+1)$ is a period of (the function sin) (the function cos).

(33) For every element k of $\mathbb{N}$ holds $\pi\cdot(k+1)$ is a period of (the function cos) (the function sin).

(34) For every element k of $\mathbb{N}$ holds $2\cdot\pi\cdot(k+1)$ is a period of $b + a$ (the function sin).

(35) For every element k of $\mathbb{N}$ holds $2\cdot\pi\cdot(k+1)$ is a period of a (the function sin)$-b$.

(36) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of $b + a$ (the function cos).

(37) For every element k of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k+1)$ is a period of a (the function cos)$-b$.

(38) If $\operatorname{dom} F = \mathbb{R}$ and for every real number x holds $F(x) = a$, then for every element k of $\mathbb{N}$ holds $k+1$ is a period of F.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.
[2] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.
[3] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[4] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[5] Chuanzhang Chen. *Mathematical Analysis*. Higher Education Press, Beijing, 1978.
[6] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[7] Andrzej Trybulec. Binary operations applied to functions. *Formalized Mathematics*, 1(**2**):329–334, 1990.
[8] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[9] Peng Wang and Bo Li. Several differentiation formulas of special functions. Part V. *Formalized Mathematics*, 15(**3**):73–79, 2007, doi:10.2478/v10037-007-0009-4.
[10] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[11] Yuguang Yang and Yasunari Shidama. Trigonometric functions and existence of circle ratio. *Formalized Mathematics*, 7(**2**):255–263, 1998.

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