On Rough Subgroup of a Group

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Summary. This article describes a rough subgroup with respect to a normal subgroup of a group, and some properties of the lower and the upper approximations in a group.

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The articles [2], [3], [1], [4], and [5] provide the terminology and notation for this paper.

For simplicity, we adopt the following rules: G denotes a group, A, B denote non empty subsets of G , N, H, H_1, H_2 denote subgroups of G , and x, a, b denote elements of G .

Next we state a number of propositions:

- (1) For every normal subgroup N of G and for all elements x_1, x_2 of G holds $x_1 \cdot N \cdot (x_2 \cdot N) = (x_1 \cdot x_2) \cdot N$.
- (2) For every group G and for every subgroup N of G and for all elements x, y of G such that $y \in x \cdot N$ holds $x \cdot N = y \cdot N$.
- (3) Let N be a subgroup of G , H be a subgroup of G , and x be an element of G . If $x \cdot N$ meets $\overline{H}$, then there exists an element y of G such that $y \in x \cdot N$ and $y \in H$.
- (4) For all elements x, y of G and for every normal subgroup N of G such that $y \in N$ holds $x \cdot y \cdot x^{-1} \in N$.
- (5) For every subgroup N of G such that for all elements x, y of G such that $y \in N$ holds $x \cdot y \cdot x^{-1} \in N$ holds N is normal.
- (6) $x \in H_1 \cdot H_2$ iff there exist a, b such that $x = a \cdot b$ and $a \in H_1$ and $b \in H_2$.
- (7) Let G be a group and N_1, N_2 be strict normal subgroups of G . Then there exists a strict subgroup M of G such that the carrier of $M = N_1 \cdot N_2$.

- (8) Let G be a group and N_1, N_2 be strict normal subgroups of G . Then there exists a strict normal subgroup M of G such that the carrier of $M = N_1 \cdot N_2$.
- (9) Let G be a group and N, N_1, N_2 be subgroups of G . Suppose the carrier of $N = N_1 \cdot N_2$. Then N_1 is a subgroup of N and N_2 is a subgroup of N .
- (10) Let N, N_1, N_2 be normal subgroups of G and a, b be elements of G . If the carrier of $N = N_1 \cdot N_2$, then $a \cdot N_1 \cdot (b \cdot N_2) = (a \cdot b) \cdot N$.
- (11) For every normal subgroup N of G and for every x holds $x \cdot N \cdot x^{-1} \subseteq \overline{N}$.

Let G be a group, let A be a subset of G , and let N be a subgroup of G . The functor $N^{\cdot}A$ yielding a subset of G is defined by:

(Def. 1) $N^{\cdot}A = \{x \in G: x \cdot N \subseteq A\}$.

The functor $N \sim A$ yielding a subset of G is defined as follows:

(Def. 2) $N \sim A = \{x \in G: x \cdot N \text{ meets } A\}$.

Next we state a number of propositions:

- (12) For every element x of G such that $x \in N^{\cdot}A$ holds $x \cdot N \subseteq A$.
- (13) For every element x of G such that $x \cdot N \subseteq A$ holds $x \in N^{\cdot}A$.
- (14) For every element x of G such that $x \in N \sim A$ holds $x \cdot N$ meets A .
- (15) For every element x of G such that $x \cdot N$ meets A holds $x \in N \sim A$.
- (16) $N^{\cdot}A \subseteq A$.
- (17) $A \subseteq N \sim A$.
- (18) $N^{\cdot}A \subseteq N \sim A$.
- (19) $N \sim A \cup B = (N \sim A) \cup (N \sim B)$.
- (20) $N^{\cdot}A \cap B = (N^{\cdot}A) \cap (N^{\cdot}B)$.
- (21) If $A \subseteq B$, then $N^{\cdot}A \subseteq N^{\cdot}B$.
- (22) If $A \subseteq B$, then $N \sim A \subseteq N \sim B$.
- (23) $(N^{\cdot}A) \cup (N^{\cdot}B) \subseteq N^{\cdot}(A \cup B)$.
- (24) $N \sim A \cup B = (N \sim A) \cup (N \sim B)$.
- (25) If N is a subgroup of H , then $H^{\cdot}A \subseteq N^{\cdot}A$.
- (26) If N is a subgroup of H , then $N \sim A \subseteq H \sim A$.
- (27) For every group G and for all non empty subsets A, B of G and for every normal subgroup N of G holds $(N^{\cdot}A) \cdot (N^{\cdot}B) \subseteq N^{\cdot}A \cdot B$.
- (28) For every element x of G such that $x \in N \sim A \cdot B$ holds $x \cdot N$ meets $A \cdot B$.
- (29) For every group G and for all non empty subsets A, B of G and for every normal subgroup N of G holds $(N \sim A) \cdot (N \sim B) = N \sim A \cdot B$.
- (30) For every element x of G such that $x \in N \sim N^{\cdot}(N \sim A)$ holds $x \cdot N$ meets $N^{\cdot}(N \sim A)$.
- (31) For every element x of G such that $x \in N^{\cdot}(N \sim A)$ holds $x \cdot N \subseteq N \sim A$.

- (32) For every element x of G such that $x \in N \sim N \sim A$ holds $x \cdot N$ meets $N \sim A$.
- (33) For every element x of G such that $x \in N \sim N' A$ holds $x \cdot N$ meets $N' A$.
- (34) $N'(N' A) = N' A$.
- (35) $N \sim A = N \sim N \sim A$.
- (36) $N'(N' A) \subseteq N \sim N \sim A$.
- (37) $N \sim N' A \subseteq A$.
- (38) $N'(N \sim N' A) = N' A$.
- (39) If $A \subseteq N'(N \sim A)$, then $N \sim A \subseteq N \sim N'(N \sim A)$.
- (40) $N \sim N'(N \sim A) = N \sim A$.
- (41) For every element x of G such that $x \in N'(N' A)$ holds $x \cdot N \subseteq N' A$.
- (42) $N'(N' A) = N \sim N' A$.
- (43) $N \sim N \sim A = N'(N \sim A)$.
- (44) For all subgroups N, N_1, N_2 of G such that $N = N_1 \cap N_2$ holds $N \sim A \subseteq (N_1 \sim A) \cap (N_2 \sim A)$.
- (45) For all subgroups N, N_1, N_2 of G such that $N = N_1 \cap N_2$ holds $(N_1' A) \cap (N_2' A) \subseteq N' A$.
- (46) Let N_1, N_2 be strict normal subgroups of G . Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $N' A \subseteq (N_1' A) \cap (N_2' A)$.
- (47) Let N_1, N_2 be strict normal subgroups of G . Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $(N_1 \sim A) \cup (N_2 \sim A) \subseteq N \sim A$.
- (48) Let N_1, N_2 be strict normal subgroups of G . Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $N \sim A \subseteq ((N_1 \sim A) \cdot N_2) \cap ((N_2 \sim A) \cdot N_1)$.

In the sequel N_1, N_2 are subgroups of G .

Let G be a group and let H, N be subgroups of G . The functor $N' H$ yielding a subset of G is defined by:

(Def. 3) $N' H = \{x \in G: x \cdot N \subseteq \overline{H}\}$.

The functor $N \sim H$ yields a subset of G and is defined as follows:

(Def. 4) $N \sim H = \{x \in G: x \cdot N \text{ meets } \overline{H}\}$.

We now state a number of propositions:

- (49) For every element x of G such that $x \in N' H$ holds $x \cdot N \subseteq \overline{H}$.
- (50) For every element x of G such that $x \cdot N \subseteq \overline{H}$ holds $x \in N' H$.
- (51) For every element x of G such that $x \in N \sim H$ holds $x \cdot N$ meets $\overline{H}$.
- (52) For every element x of G such that $x \cdot N$ meets $\overline{H}$ holds $x \in N \sim H$.
- (53) $N' H \subseteq \overline{H}$.

- (54) $\overline{H} \subseteq N \sim H$.
- (55) $N'H \subseteq N \sim H$.
- (56) If H_1 is a subgroup of H_2 , then $N \sim H_1 \subseteq N \sim H_2$.
- (57) If N_1 is a subgroup of N_2 , then $N_1 \sim H \subseteq N_2 \sim H$.
- (58) If N_1 is a subgroup of N_2 , then $N_1 \sim N_1 \subseteq N_2 \sim N_2$.
- (59) If H_1 is a subgroup of H_2 , then $N'H_1 \subseteq N'H_2$.
- (60) If N_1 is a subgroup of N_2 , then $N_2'H \subseteq N_1'H$.
- (61) If N_1 is a subgroup of N_2 , then $N_2'N_1 \subseteq N_1'N_2$.
- (62) For every normal subgroup N of G holds $(N'H_1) \cdot (N'H_2) \subseteq N'H_1 \cdot H_2$.
- (63) For every normal subgroup N of G holds $(N \sim H_1) \cdot (N \sim H_2) = N \sim H_1 \cdot H_2$.
- (64) For all subgroups N, N_1, N_2 of G such that $N = N_1 \cap N_2$ holds $N \sim H \subseteq (N_1 \sim H) \cap (N_2 \sim H)$.
- (65) For all subgroups N, N_1, N_2 of G such that $N = N_1 \cap N_2$ holds $(N_1'H) \cap (N_2'H) \subseteq N'H$.
- (66) Let N_1, N_2 be strict normal subgroups of G . Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $N'H \subseteq (N_1'H) \cap (N_2'H)$.
- (67) Let N_1, N_2 be strict normal subgroups of G . Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $(N_1 \sim H) \cup (N_2 \sim H) \subseteq N \sim H$.
- (68) Let N_1, N_2 be strict normal subgroups of G . Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $(N_1'H) \cdot (N_2'H) \subseteq N'H$.
- (69) Let N_1, N_2 be strict normal subgroups of G . Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $(N_1 \sim H) \cdot (N_2 \sim H) \subseteq N \sim H$.
- (70) Let N_1, N_2 be strict normal subgroups of G . Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $N \sim H \subseteq ((N_1 \sim H) \cdot N_2) \cap ((N_2 \sim H) \cdot N_1)$.
- (71) Let H be a subgroup of G and N be a normal subgroup of G . Then there exists a strict subgroup M of G such that the carrier of $M = N \sim H$.
- (72) Let H be a subgroup of G and N be a normal subgroup of G . Suppose N is a subgroup of H . Then there exists a strict subgroup M of G such that the carrier of $M = N'H$.
- (73) For all normal subgroups H, N of G there exists a strict normal subgroup M of G such that the carrier of $M = N \sim H$.
- (74) Let H, N be normal subgroups of G . Suppose N is a subgroup of H . Then there exists a strict normal subgroup M of G such that the carrier of $M = N'H$.

- (75) Let N, N_1 be normal subgroups of G . Suppose N_1 is a subgroup of N . Then there exist strict normal subgroups N_2, N_3 of G such that the carrier of $N_2 = N_1 \sim N$ and the carrier of $N_3 = N_1'N$ and $N_2'N \subseteq N_3'N$.
- (76) Let N, N_1 be normal subgroups of G . Suppose N_1 is a subgroup of N . Then there exist strict normal subgroups N_2, N_3 of G such that the carrier of $N_2 = N_1 \sim N$ and the carrier of $N_3 = N_1'N$ and $N_3 \sim N \subseteq N_2 \sim N$.
- (77) Let N, N_1 be normal subgroups of G . Suppose N_1 is a subgroup of N . Then there exist strict normal subgroups N_2, N_3 of G such that the carrier of $N_2 = N_1 \sim N$ and the carrier of $N_3 = N_1'N$ and $N_2'N \subseteq N_3 \sim N$.
- (78) Let N, N_1 be normal subgroups of G . Suppose N_1 is a subgroup of N . Then there exist strict normal subgroups N_2, N_3 of G such that the carrier of $N_2 = N_1 \sim N$ and the carrier of $N_3 = N_1'N$ and $N_3'N \subseteq N_2 \sim N$.
- (79) Let N, N_1, N_2 be normal subgroups of G . Suppose N_1 is a subgroup of N_2 . Then there exist strict normal subgroups N_3, N_4 of G such that the carrier of $N_3 = N \sim N_1$ and the carrier of $N_4 = N \sim N_2$ and $N_3 \sim N_1 \subseteq N_4 \sim N_1$.
- (80) Let N, N_1 be normal subgroups of G . Then there exists a strict normal subgroup N_2 of G such that the carrier of $N_2 = N'N$ and $N'N_1 \subseteq N_2'N_1$.
- (81) Let N, N_1 be normal subgroups of G . Then there exists a strict normal subgroup N_2 of G such that the carrier of $N_2 = N \sim N$ and $N \sim N_1 \subseteq N_2 \sim N_1$.

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