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Some Properties of Line and Column Operations on Matrices

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Summary. This article describes definitions of elementary operations about matrix and their main properties.

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The articles [8], [13], [17], [11], [1], [18], [5], [6], [2], [7], [15], [16], [9], [10], [20], [4], [3], [21], [12], [14], and [19] provide the notation and terminology for this paper.

For simplicity, we adopt the following convention: j, k, l, n, m, i are natural numbers, K is a field, a is an element of K, M, M_1 are matrices over K of dimension $n \times m$, and A is a matrix over K of dimension n.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, and let l, k be natural numbers. The functor $\mathrm{InterchangeLine}(M, l, k)$ yielding a matrix over K of dimension $n \times m$ is defined by the conditions (Def. 1).

(Def. 1)(i) $\mathrm{len}\,\mathrm{InterchangeLine}(M, l, k) = \mathrm{len}\,M$, and

(ii) for all i, j such that $i \in \mathrm{dom}\,M$ and $j \in \mathrm{Seg}\,\mathrm{width}\,M$ holds if $i = l$, then $(\mathrm{InterchangeLine}(M, l, k))_{i,j} = M_{k,j}$ and if $i = k$, then $(\mathrm{InterchangeLine}(M, l, k))_{i,j} = M_{l,j}$ and if $i \neq l$ and $i \neq k$, then $(\mathrm{InterchangeLine}(M, l, k))_{i,j} = M_{i,j}$.

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The following three propositions are true:

(1) For all matrices M_1, M_2 over K of dimension $n \times m$ holds $\mathrm{width}\, M_1 = \mathrm{width}\, M_2$.

(2) Let given M, M_1, i such that $l \in \mathrm{dom}\, M$ and $k \in \mathrm{dom}\, M$ and $i \in \mathrm{dom}\, M$ and $M_1 = \mathrm{InterchangeLine}(M, l, k)$. Then

(i) if $i = l$, then $\mathrm{Line}(M_1, i) = \mathrm{Line}(M, k)$,

(ii) if $i = k$, then $\mathrm{Line}(M_1, i) = \mathrm{Line}(M, l)$, and

(iii) if $i \neq l$ and $i \neq k$, then $\mathrm{Line}(M_1, i) = \mathrm{Line}(M, i)$.

(3) For all a, i, j, M such that $i \in \mathrm{dom}\, M$ and $j \in \mathrm{Seg}\, \mathrm{width}\, M$ holds $(a \cdot \mathrm{Line}(M, i))(j) = a \cdot M_{i,j}$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, let l be a natural number, and let a be an element of K. The functor $\mathrm{ScalarXLine}(M, l, a)$ yields a matrix over K of dimension $n \times m$ and is defined by the conditions (Def. 2).

(Def. 2)(i) $\mathrm{len}\, \mathrm{ScalarXLine}(M, l, a) = \mathrm{len}\, M$, and

(ii) for all i, j such that $i \in \mathrm{dom}\, M$ and $j \in \mathrm{Seg}\, \mathrm{width}\, M$ holds if $i = l$, then $(\mathrm{ScalarXLine}(M, l, a))_{i,j} = a \cdot M_{l,j}$ and if $i \neq l$, then $(\mathrm{ScalarXLine}(M, l, a))_{i,j} = M_{i,j}$.

We now state the proposition

(4) If $l \in \mathrm{dom}\, M$ and $i \in \mathrm{dom}\, M$ and $a \neq 0_K$ and $M_1 = \mathrm{ScalarXLine}(M, l, a)$, then if $i = l$, then $\mathrm{Line}(M_1, i) = a \cdot \mathrm{Line}(M, l)$ and if $i \neq l$, then $\mathrm{Line}(M_1, i) = \mathrm{Line}(M, i)$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, let l, k be natural numbers, and let a be an element of K. Let us assume that $l \in \mathrm{dom}\, M$ and $k \in \mathrm{dom}\, M$. The functor $\mathrm{RlineXScalar}(M, l, k, a)$ yielding a matrix over K of dimension $n \times m$ is defined by the conditions (Def. 3).

(Def. 3)(i) $\mathrm{len}\, \mathrm{RlineXScalar}(M, l, k, a) = \mathrm{len}\, M$, and

(ii) for all i, j such that $i \in \mathrm{dom}\, M$ and $j \in \mathrm{Seg}\, \mathrm{width}\, M$ holds if $i = l$, then $(\mathrm{RlineXScalar}(M, l, k, a))_{i,j} = a \cdot M_{k,j} + M_{l,j}$ and if $i \neq l$, then $(\mathrm{RlineXScalar}(M, l, k, a))_{i,j} = M_{i,j}$.

We now state the proposition

(5) If $l \in \mathrm{dom}\, M$ and $k \in \mathrm{dom}\, M$ and $i \in \mathrm{dom}\, M$ and $M_1 = \mathrm{RlineXScalar}(M, l, k, a)$, then if $i = l$, then $\mathrm{Line}(M_1, i) = a \cdot \mathrm{Line}(M, k) + \mathrm{Line}(M, l)$ and if $i \neq l$, then $\mathrm{Line}(M_1, i) = \mathrm{Line}(M, i)$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, and let l, k be natural numbers. We introduce $\mathrm{ILine}(M, l, k)$ as a synonym of $\mathrm{InterchangeLine}(M, l, k)$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, let l be a natural number, and let a be an element of K. We introduce $\mathrm{SXLine}(M, l, a)$ as a synonym of $\mathrm{ScalarXLine}(M, l, a)$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, let l, k be natural numbers, and let a be an element of K. We introduce $\mathrm{RLineXS}(M, l, k, a)$ as a synonym of $\mathrm{RlineXScalar}(M, l, k, a)$.

We now state several propositions:

(6) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n})$ and $k \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n})$, then $\mathrm{ILine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n}, l, k) \cdot A = \mathrm{ILine}(A, l, k)$.

(7) For all l, a, A such that $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n})$ and $a \neq 0_K$ holds $\mathrm{SXLine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n}, l, a) \cdot A = \mathrm{SXLine}(A, l, a)$.

(8) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n})$ and $k \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n})$, then $\mathrm{RLineXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n}, l, k, a) \cdot A = \mathrm{RLineXS}(A, l, k, a)$.

(9) $\mathrm{ILine}(M, k, k) = M$.

(10) $\mathrm{ILine}(M, l, k) = \mathrm{ILine}(M, k, l)$.

(11) If $l \in \mathrm{dom}\, M$ and $k \in \mathrm{dom}\, M$, then $\mathrm{ILine}(\mathrm{ILine}(M, l, k), l, k) = M$.

(12) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n})$ and $k \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n})$, then $\mathrm{ILine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n}, l, k)$ is invertible and $(\mathrm{ILine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n}, l, k))^{\smile} = \mathrm{ILine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n\times n}, l, k)$.

(13) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \neq l$, then $\mathrm{RLineXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, a)$ is invertible and $(\mathrm{RLineXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, a))^{\smile} = \mathrm{RLineXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n},$ $l, k, -a)$.

(14) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $a \neq 0_K$, then $\mathrm{SXLine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a)$ is invertible and $(\mathrm{SXLine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a))^{\smile} = \mathrm{SXLine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a^{-1})$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, and let l, k be natural numbers. Let us assume that $l \in \mathrm{Seg}\,\mathrm{width}\,M$ and $k \in \mathrm{Seg}\,\mathrm{width}\,M$ and $n > 0$ and $m > 0$. The functor $\mathrm{InterchangeCol}(M, l, k)$ yields a matrix over K of dimension $n \times m$ and is defined by the conditions (Def. 4).

(Def. 4)(i) $\mathrm{len}\,\mathrm{InterchangeCol}(M, l, k) = \mathrm{len}\,M$, and

(ii) for all i, j such that $i \in \mathrm{dom}\,M$ and $j \in \mathrm{Seg}\,\mathrm{width}\,M$ holds if $j = l$, then $(\mathrm{InterchangeCol}(M, l, k))_{i,j} = M_{i,k}$ and if $j = k$, then $(\mathrm{InterchangeCol}(M, l, k))_{i,j} = M_{i,l}$ and if $j \neq l$ and $j \neq k$, then $(\mathrm{InterchangeCol}(M, l, k))_{i,j} = M_{i,j}$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, let l be a natural number, and let a be an element of K. Let us assume that $l \in \mathrm{Seg}\,\mathrm{width}\,M$ and $n > 0$ and $m > 0$. The functor $\mathrm{ScalarXCol}(M, l, a)$ yielding a matrix over K of dimension $n \times m$ is defined by the conditions (Def. 5).

(Def. 5)(i) $\mathrm{len}\,\mathrm{ScalarXCol}(M, l, a) = \mathrm{len}\,M$, and

(ii) for all i, j such that $i \in \mathrm{dom}\,M$ and $j \in \mathrm{Seg}\,\mathrm{width}\,M$ holds if $j = l$, then $(\mathrm{ScalarXCol}(M, l, a))_{i,j} = a \cdot M_{i,l}$ and if $j \neq l$, then $(\mathrm{ScalarXCol}(M, l, a))_{i,j} = M_{i,j}$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, let l, k be natural numbers, and let a be an element of K. Let us assume that $l \in \operatorname{Seg} \operatorname{width} M$ and $k \in \operatorname{Seg} \operatorname{width} M$ and $n > 0$ and $m > 0$. The functor $\operatorname{RcolXScalar}(M, l, k, a)$ yielding a matrix over K of dimension $n \times m$ is defined by the conditions (Def. 6).

(Def. 6)(i) $\operatorname{len} \operatorname{RcolXScalar}(M, l, k, a) = \operatorname{len} M$, and

(ii) for all i, j such that $i \in \operatorname{dom} M$ and $j \in \operatorname{Seg} \operatorname{width} M$ holds if $j = l$, then $(\operatorname{RcolXScalar}(M, l, k, a))_{i,j} = a \cdot M_{i,k} + M_{i,l}$ and if $j \neq l$, then $(\operatorname{RcolXScalar}(M, l, k, a))_{i,j} = M_{i,j}$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, and let l, k be natural numbers. We introduce $\operatorname{ICol}(M, l, k)$ as a synonym of $\operatorname{InterchangeCol}(M, l, k)$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, let l be a natural number, and let a be an element of K. We introduce $\operatorname{SXCol}(M, l, a)$ as a synonym of $\operatorname{ScalarXCol}(M, l, a)$.

Let us consider n, m, let us consider K, let M be a matrix over K of dimension $n \times m$, let l, k be natural numbers, and let a be an element of K. We introduce $\operatorname{RColXS}(M, l, k, a)$ as a synonym of $\operatorname{RcolXScalar}(M, l, k, a)$.

We now state several propositions:

(15) If $l \in \operatorname{Seg} \operatorname{width} M$ and $k \in \operatorname{Seg} \operatorname{width} M$ and $n > 0$ and $m > 0$ and $M_1 = M^{\mathrm{T}}$, then $(\operatorname{ILine}(M_1, l, k))^{\mathrm{T}} = \operatorname{ICol}(M, l, k)$.

(16) If $l \in \operatorname{Seg} \operatorname{width} M$ and $a \neq 0_K$ and $n > 0$ and $m > 0$ and $M_1 = M^{\mathrm{T}}$, then $(\operatorname{SXLine}(M_1, l, a))^{\mathrm{T}} = \operatorname{SXCol}(M, l, a)$.

(17) If $l \in \operatorname{Seg} \operatorname{width} M$ and $k \in \operatorname{Seg} \operatorname{width} M$ and $n > 0$ and $m > 0$ and $M_1 = M^{\mathrm{T}}$, then $(\operatorname{RLineXS}(M_1, l, k, a))^{\mathrm{T}} = \operatorname{RColXS}(M, l, k, a)$.

(18) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $n > 0$, then $A \cdot \operatorname{ICol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k) = \operatorname{ICol}(A, l, k)$.

(19) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $a \neq 0_K$ and $n > 0$, then $A \cdot \operatorname{SXCol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a) = \operatorname{SXCol}(A, l, a)$.

(20) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $n > 0$, then $A \cdot \operatorname{RColXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, a) = \operatorname{RColXS}(A, l, k, a)$.

(21) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $n > 0$, then $(\operatorname{ICol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k))^{\smile} = \operatorname{ICol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k)$.

(22) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \neq l$ and $n > 0$, then $(\operatorname{RColXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, a))^{\smile} = \operatorname{RColXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, -a)$.

(23) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $a \neq 0_K$ and $n > 0$, then $(\operatorname{SXCol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a))^{\smile} = \operatorname{SXCol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a^{-1})$.

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