MODELLING THE EMPLOYMENT STRUCTURE IN PUBLIC COMPANIES FOR A SMALL SAMPLE

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Received 3 October 2008, Accepted 25 November 2008

Abstract

The analysis of employment structure is a very important issue in the companies and it is connected with many factors in the company and on the market. An interesting issue is whether labour efficiency affects economic and financial performance of the company and the whole market. In the case of any statistical regularities concerning this issue it is necessary to allow for the variables related to employment, e.g. in the analyses of listed companies. It is a particularly important matter in Poland where the ownership transformations and transition to market economy have resulted in major changes in employment in individual companies. The main goal of the paper is an analysis of the structure of companies by their group labour productivity in homogenous groups of listed companies, the so called megasectors, in relation to their economic and financial condition. The study refers to the period of 2000-2003 and includes 73 companies listed on the Warsaw Stock Exchange.

Key words: analysis of employment, market analysis, stock exchange, labour efficiency.

JEL classification: C10.

Introduction

The key success factor in an enterprise operating in the market economy is an optimum employment level and its productivity, e.g. measured with total revenues per employee. An interesting issue is whether labour efficiency affects the economic and financial performance of an individual company and the whole market. In the case of any statistical regularities concerning this issue it is necessary to allow for the variables related to employment, e.g. in the analyses of listed companies. Those variables can increase the efficiency of investment in terms of the expected rate of return on investment. This subject matter is of particular importance in Poland where the ownership transformations and transition to market economy have resulted in major changes in employment in individual companies. The paper provides an analysis of the structure of companies by their group labour productivity in homogenous groups of listed companies, the so called megasectors, in relation to their economic and financial condition.

The study covers the period of 2000-2003 and incorporates 73 companies listed on the Warsaw Stock Exchange that form the megasectors: industry (43) as well as trade and services (34). At present, plenty of research strives to find statistical methods which would avoid the limiting assumptions underlying classical methods. In this paper one of such methods is presented – the bootstrap method, which will be related to the estimation of group labour productivity in relation to financial and economic condition of listed companies where classical assumptions cannot be accepted. The authors are particularly interested in the use of the *TMAI*. It is a measure of multidimensional comparative analysis enabling an objective assessment of the stock company's quality with respect to its economic and financial condition. It may be said that it allows for determining the company's fundamental strength.

1. Research Methods

Description of the Synthetic Development Measure TMAI

The paper analyses the synthetic development measure *TMAI*¹:

$$TMAI_i = 1 - \frac{d_i}{d_0}, \quad (i = 1, 2, \dots, n), \quad (1)$$

where:

TMAI_i – synthetic development measure for the object *i*,

d_i – distance between the object i and the pattern object calculated according to the formula:

$$d_i = \sqrt{\sum_{j=1}^m w_j \cdot (z_{ij} - z_{0j})^2}, \quad (i = 1, 2, \dots, n),$$

z_{0j} – maximum value of z_{ij} for the object i ,

z_{ij} – standardized value of the attribute j for the object i ,

d_0 – norm assuring that $TMAI_i$ reaches values ranging from 0 to 1:

$$d_0 = \bar{d} + a \cdot S_d.$$

$\bar{d}$ – average value of d_i ,

S_d – standard deviation of d_i .

Using the relation (1) and the information that $0 \leq TMAI_i \leq 1$ and $d_i > 0$, we may determine the margin value for the constant a :

$$a \geq \frac{d_{i_{\max}} - \bar{d}}{S_d},$$

where $d_{i_{\max}}$ is the maximum value of d_i .

The weights w_i in the distance formula have been calculated according to the following formula based on the coefficient of variation of the variable:

$$w_j = \frac{V_j}{\sum_{j=1}^m V_j}, \quad (j = 1, 2, \dots, m).$$

The ratio, like $TMAI$ is standardised and reaches values ranging from 0 to 1. The closer the value of the measure to 1, the better the object in terms of the general criterion.

For the companies the following variables have been used: profitability ratios: return on equity (ROE), return on assets (ROA); liquidity ratio: current liquidity ratio; activity ratios: receivables turnover (in days), inventory turnover (in days), liabilities turnover (in days), assets turnover; debt ratio: debt margin.

Those variables are generally available and published, for instance by Notoria Service, by quarters of the year and years for all the companies, which should make it easy to carry out the analyses proposed in this paper in terms of data accessibility.

Among the variables presented above the turnover of receivables, the inventory and liabilities and the debt margin have been considered to be destimulants. The current liquidity ratio is a nominant, while all the other variables have been assumed as stimulants. *TMAI* defined in such a way enables the objective appraisal of the company's economic and financial condition which may be called its fundamental strength.

Bootstrap Estimation

To analyse the relationship between the labour productivity (measured with total revenues per employee) and the condition of listed companies (measured with *TMAI*), bootstrap estimation of random variable distribution parameters was employed.

In the cases where random variable distributions are unknown, the bootstrap methods allow approximation of their number characteristics basing on the multitude of values generated according to the defined probability distribution². The procedure is as follows:

- the probability distribution from the sample is created,
- a random (bootstrap) sample for this distribution is generated,
- the distribution of the unknown statistic is approximated with the bootstrap distribution.

The bootstrap distribution can be defined by means of three methods³. In the first method, the distribution can be derived theoretically. The second method involves approximation of the expectation and the variance of that distribution by development into Taylor's series. The latter involves approximation with the *Monte Carlo* bootstrap distribution, and more precisely – the construction of a histogram for this distribution based on n -element sequences of the random sample bootstrap realisation generated N times on the basis of the sample probability distribution⁴. In our case, the third method of the bootstrap distribution estimation was employed – the *Monte Carlo* approximation. The parameters of the unknown random variable distribution can be found with the point and interval estimation.

Let θ be the estimated parameter of the distribution of the X random variable and R – the estimator of that parameter. To find the bootstrap estimate for θ , we generate N times the $x_1^*, \dots, x_n^*$ values according to the distribution:

$$P(X_B = x_k) = \frac{1}{n}, \quad \text{for } k = 1, \dots, n \quad (2)$$

where $x_1, \dots, x_n$ are the values observed in the sample of the independently sampled population.

For each $x_1^*, \dots, x_n^*$ sequence, we define the R^* statistics value and thus we obtain a sequence of the R^* random variable realisations: $r_1^*, \dots, r_N^*$. The estimator of the θ parameter is:

$$\bar{r}^* = \frac{1}{N} \sum_{k=1}^N r_k^* . \quad (3)$$

The confidence intervals for the θ parameter of the X random variable can be found, for instance, with the t -bootstrap method. We employ the following statistics here:

$$t^* = \frac{\hat{\theta}^* - \hat{\theta}}{\hat{D}(\hat{\theta}^*)} , \quad (4)$$

where $\hat{D}(\hat{\theta}^*)$ is the estimator of the standard deviation of $\hat{\theta}^*$ random variable for the defined bootstrap sample $(X_1^*, \dots, X_n^*)$.

We generate N times the values of the bootstrap sample according to the distribution (1) and we arrive at the sequence $(x_1^{*j}, \dots, x_n^{*j})_{j=1, \dots, N}$. For each j ($j = 1, \dots, N$) we calculate the $\hat{\theta}^*$ estimator value according to the formula:

$$\hat{\theta}^* = R^*(X_1, \dots, X_n) = R(X_1^*, \dots, X_n^*) , \quad (5)$$

and the $\hat{D}(\hat{\theta}^*)$ estimator value based on the $(x_1^{*j}, \dots, x_n^{*j})$ value, and then we calculate t_j^* . With the $(t_j^*)_{j=1, \dots, N}$ sequence we define the percentiles of α and $\alpha - 1$ rank, where $\alpha \in (0; 1)$.

The confidence interval for the θ parameter found with the t -bootstrap method at the $1 - 2\alpha$ significant level is defined as follows⁵:

$$P(\hat{\theta} - \hat{t}^{*(1-\alpha)} D(\hat{\theta}) < \theta < \hat{\theta} + \hat{t}^{*(\alpha)} D(\hat{\theta})) \approx 1 - 2\alpha , \quad (6)$$

where $D(\hat{\theta})$ is the standard deviation of the $\hat{\theta}$ estimator and $\hat{t}^{*(\alpha)}$ and $\hat{t}^{*(1-\alpha)}$ are α and $1 - \alpha$ percentiles of the empirical distribution of t^* statistics (cf.: formula (4)) based on the values generated with the distribution (2).

2. Empirical Research

The research focuses on the group labour productivity in the companies listed on the Warsaw Stock Exchange on the basic market at the end of 2003 since at least 2000, allowing for megasectors. At present three megasectors can be distinguished on the Warsaw Stock Exchange: 1) finance, 2) industry and 3) trade and services. Due to the comparability of data two megasectors were accepted for further research: industry (chemical, wood and paper, electro-mechanical, building and construction materials, light, metal, food, and other) and trade and services (building and construction, power industry, municipal services, trade, IT, media, telecommunication, and other services).

In the first part of the research, an attempt was made to evaluate the strength and direction of the relationship between labour productivity (total revenues per employee) and economic and financial performance of the listed companies. The economic and financial performance of companies was measured with the synthetic measure *TMAI*. Statistical data provided information on those variables for the listed companies at the end of the years 2000-2003. The observations for each year were treated as if they concerned different populations due to qualitative changes in individual companies over the analysed period. The availability of the data and the presence on the basic market of the Warsaw Stock Exchange since 2000 limited the sample to 77 companies (34 representing the trade and service megasector, and 43 companies from the industrial megasector). Between *TMAI*, measuring economic and financial condition of companies, and the logarithm of labour productivity⁶ for all the analysed companies and companies in megasectors, the Pearson linear correlation coefficients were calculated. The analysis of correlation graphs indicates there is no need to extend the research to non-linear relationship. The values of coefficients in individual years are presented in Table 1.

Table 1. Pearson linear correlation coefficients between the logarithm of labour productivity and *TMAI* measure in the analysed companies and by megasectors in 2000-2003

Years			
2000	2001	2002	2003
TOTAL COMPANIES			
-0.2290 p=0.045	-0.1079 p=0.350	-0.0552 p=0.633	0.1993 p=0.082
INDUSTRY			
-0.4524 p=0.020	-0.1562 p=0.317	-0.0850 p=0.588	0.2828 p=0.066

TRADE AND SERVICES			
-0.1700 p=0.336	-0.1046 p=0.556	-0.0677 p=0.703	0.1455 p=0.411

Source: own calculations.

Statistical analysis bases on the 0.05 significance level. According to the data in Table 1, there is no linear relationship between the logarithm of labour productivity and the *TMAI* measure, except for the significant relationship between *TMAI* and the logarithm of labour productivity among all the companies and in industrial companies in 2000. We may thus conclude that in the period 2000-2003 labour productivity had no statistically significant impact on the economic and financial condition of the analysed listed companies in total and by megasectors.

It proves indirectly that there is little development in Polish companies, where the issues related to employment productivity will undoubtedly be subject to major transformations in the nearest future.

Additionally, the labour productivity distribution and the *TMAI* measure in selected megasectors were analysed. The histograms were constructed on the basis of the Gauss kernel estimator bandwidth⁷. The labour productivity histograms and the *TMAI* measure histograms in contrast to the density function for normal distribution⁸ for the years 2000-2003 in “industry” megasector are presented in Figures 1-8, while in the “trade and services” megasector – in Figures 9-16.

Fig. 1. Histogram of group labour productivity logarithms in contrast to the density function of the normal distribution (2000) – industry

Source: own research.

Fig. 2. Histogram of *TMAI* coefficients in contrast to the density function of the normal distribution (2000) – industry

Source: own research.

Fig. 3. Histogram of group labour productivity logarithms in contrast to the density function of the normal distribution (2001) – industry

Source: own research.

Fig. 4. Histogram of *TMAI* coefficients in contrast to the density function of the normal distribution (2001) – industry

Source: own research.

Fig. 5. Histogram of group labour productivity logarithms in contrast to the density function of the normal distribution (2002) – industry

Source: own research.

Fig. 6. Histogram of *TMAI* coefficients in contrast to the density function of the normal distribution (2002) – industry

Source: own research.

Fig. 7. Histogram of group labour productivity logarithms in contrast to the density function of the normal distribution (2003) – industry

Source: own research.

Fig. 8. Histogram of *TMAI* coefficients in contrast to the density function of the normal distribution (2003) – industry

Source: own research.

Fig. 9. Histogram of group labour productivity logarithms in contrast to the density function of the normal distribution (2000) – trade and services

Source: own research.

Fig. 10. Histogram of *TMAI* coefficients in contrast to the density function of the normal distribution (2000) – trade and services

Source: own research.

Fig. 11. Histogram of group labour productivity logarithms in contrast to the density function of the normal distribution (2001) – trade and services

Source: own research.

Fig. 12. Histogram of *TMAI* coefficients in contrast to the density function of the normal distribution (2001) – trade and services

Source: own research.

Fig. 13. Histogram of group labour productivity logarithms in contrast to the density function of the normal distribution (2002) – trade and services

Source: own research.

Fig. 14. Histogram of *TMAI* coefficients in contrast to the density function of the normal distribution (2002) – trade and services

Source: own research.

Fig. 15. Histogram of group labour productivity logarithms in contrast to the density function of the normal distribution (2003) – trade and services

Source: own research.

Fig. 16. Histogram of *TMAI* coefficients in contrast to the density function of the normal distribution (2003) – trade and services

Source: own research.

All the group labour productivity distributions and *TMAI* distributions in the analysed periods are left- and right-tailed. The least fit observations are those with the highest and lowest values, thus affecting the goodness of fit⁹.

We cannot therefore assume that the variables productivity and *TMAI* represent two-dimensional normal distribution. In such a case, the application of classical statistical methods provides a distorted view of the analysed regularity. Therefore, the bootstrap estimation of the sample distribution parameters of the correlation coefficient between the group labour productivity and the *TMAI* measure in megasectors was conducted. First, the standard error of the

correlation coefficient from the population of distribution other than the multidimensional normal distribution was estimated by means of the maximum likelihood method. The estimates of the correlation coefficient, standard error and t statistics by megasectors are presented in Table 2.

Table 2. Correlation coefficients between productivity and $TMAI$ measure with their estimates by megasectors in 2000-2003

YEARS				
	2000	2001	2002	2003
INDUSTRY				
Correlation coefficient	-0.4524	-0.1562	-0.0850	0.2828
Standard error	0.1227	0.1505	0.1532	0.1420
t statistics	-3.6868	-1.0373	-0.5549	1.9923
Probability level	0.0002	0.2996	0.5790	0.0463
TRADE AND SERVICES				
Correlation coefficient	-0.1700	-0.1046	-0.0677	0.1455
Standard error	0.1690	0.1722	0.1733	0.1704
t statistics	-1.0059	-0.6072	-0.3909	0.8542
Probability level	0.3145	0.5437	0.6959	0.3930

Source: own research.

Then, the estimate of the sample variation of the correlation coefficient based on the *Monte Carlo* method was estimated with the bootstrap method based on the n -element ($n = 1000$) sequences of the random sample bootstrap realisation, generated N -times ($N = 100$) on the basis of the sample probability distribution. The percentiles of the rank $\alpha = 0.05$ and $1 - \alpha = 0.95$ were defined, and the correlation coefficients from the bootstrap sample were estimated along with the confidence intervals at the $1 - 2\alpha = 0.9$ confidence level. The results are presented in Table 3.

Table 3. Correlation coefficients between the productivity and the $TMAI$ measure with confidence intervals by megasectors in 2000-2003 in the bootstrap sample

YEARS				
	2000	2001	2002	2003
INDUSTRY				
Correlation coefficient	-0,4560	-0,1520	-0,0902	0,2812
Standard deviation	0,0250	0,0309	0,0313	0,0291
Confidence interval	(-0,5127; -0,4509)	(-0,2368; -0,1505)	(-0,1974; -0,0869)	(0,2258; 0,2833)
TRADE AND SERVICES				
Correlation coefficient	-0,1697	-0,1054	-0,0640	0,1442
Standard deviation	0,0307	0,0313	0,0315	0,0310
Confidence interval	(-0,2495; -0,1680)	(-0,1757; -0,1039)	(-0,1229; -0,0624)	(0,0811; 0,1458)

Source: own research.

We may therefore conclude that the correlation between productivity and *TMAI* by megasectors in the analysed period shows significant variation – it increases from highly negative values in 2000 (significant in the “industry” megasector) to reach positive values in 2003.

The confidence level for a high quality and statistically significant correlation coefficient between productivity and *TMAI* in 2003 in the industry megasector at 90% confidence level reached the following values: (0.2258; 0.2833) while in the trade and services megasector: (0.0811; 0.1458), yet it was not statistically significant.

Conclusions

In the first three years of the analysis (2000-2003), the correlation between the group labour productivity and *TMAI* is negative, being significant only in 2000. It means that we cannot reach the conclusion that in the analysed companies an increase in the group labour productivity leads to improvements in an economic and financial condition of the company measured with the synthetic development measure. It is not a good situation and it is rather a symptom of the economy weakness in that period. On the other hand, in 2003 the correlation is positive, classically speaking – insignificant, whereas in the bootstrap analysis it takes the values from the interval given in the Table 3. The tendencies of changes in that area are proper but still far from the ideal. Nevertheless, we can conclude that the economy is still improving and there is an increasingly stronger relationship between the labour productivity and the efficiency of enterprise operations in Poland.

Notes

¹ See: Tarczyński (2002).

² Those methods were proposed in 1979 by Efron (1979), see also Domański, Pruska (2000), pp.260-274.

³ Cf. Suchecka, Nieszporska (2000), p.37.

⁴ More on bootstrap methods in Domański, Pruska (2000), pp.261-264.

⁵ Cf. Efron, Tibshirani (1993).

⁶ Preliminary analysis has proved a strong right asymmetry of the group labour productivity in the whole period of analysis.

⁷ For more on the construction of histograms with the use of the kernel estimator see Gajek, Kałuszka (2000), pp.98-100, Domański, Pruska (2000), pp.132-135, Mojsiewicz (2002), p.85.

⁸ Normal distribution parameters were estimated as mean and standard deviation of the natural logarithm of the group labour productivity in a given year.

⁹ For more on the labour productivity modelling see Batóg, Gazińska, Mojsiewicz (2002).

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