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EFFECTIVE PORTFOLIOS – AN APPLICATION OF MULTI-CRITERIA AND FUZZY APPROACH

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Abstract

Research background: When selecting effective portfolios, the portfolio risk is minimized at the given expected return rate or the expected return rate is maximized with a given risk level. However, it is also worth using additional information, such as fundamental and market indicators to examine the companies' economic and financial situation. Taking into account the chosen indicators, the initial selection of companies can be approached as a multi-criteria problem. Besides, the choice of the period from which data will be taken gives the opportunity to use non-standard tools.
Purpose: The main aim of the article is to compare the profitability of effective portfolios obtained on the basis of a multi-criteria grouping of companies.
Research methodology: In the study TOPSIS and FTOPSIS methods were used.
Results: The results showed that the fuzzy approach could be an effective tool in obtaining more beneficial effective portfolios. Moreover, in the research, two sets of criteria differing by one indicator were used: in one of the approaches the P/E ratio was used, in the second the P/E ratio was replaced by the value of net profit per share – the analyses showed that the portfolios built on the basis of the groups for which the P/E ratio was used, had recorded better results.
Novelty: The values of criterion evaluations from the considered years were treated as triangular fuzzy numbers – this enabled the use of the FTOPSIS method and a comparison of different approaches.

Keywords: TOPSIS method, FTOPSIS method, fuzzy modelling, portfolio selection, effective portfolios

JEL classification: C44, C61, G11

Introduction

The construction of the share portfolio is an important issue for many decision makers. The investor would like to build a portfolio of such shares that are characterized by high profit and low risk. However, most often high profits are accompanied by high risk. One solution is to designate effective portfolios – then the portfolio risk is minimized at a given expected rate of return or the expected rate of return of the portfolio is maximized at a given level of risk (Jajuga, Jajuga, 2015). Effective portfolios can be generated from a selected set of joint-stock companies. However, in the process of selecting it is important to be guided by additional information that will enable the choice of the best possible set. For this purpose, one can make an economic and financial evaluation of companies using fundamental and market indicators. This approach will enable the selection of companies that in terms of selected characteristics will be stable; this stability can guarantee attractive financial results. The evaluation of companies through the prism of many characteristics may suggest the application of a methodical approach in the form of the use of multi-criteria methods (Trzaskalik, 2006). In these methods, the variant is evaluated in terms of selected criteria. In the undertaken problem the fundamental and market indicators may represent the assessment criteria. Besides, an important thing in this issue is the choice of the period from which to get data for selected indicators. There are indications in the literature suggesting that this period should cover from three to five years (Tarczyński, 2002). Following this clue, the data from the selected period should be properly taken into account in the analyses. For this purpose, one can make the appropriate calculations of values or you can approach the issue using fuzzy numbers.

In the literature we may find many works in which multi-criteria methods taking into account the fuzzy approach are used to assess companies or to build a portfolio. Among many of them we can mention works:

- in which the applicability of FTOPSIS in optimal portfolio selection is researched (Ece, Uludag, 2017),
- with the proposal of a hybrid approach for asset allocation with simultaneous consideration of suitability and optimality – we can find there the application of a cluster analysis, the AHP method and a fuzzy multi-objective linear programming model for portfolio selection (Gupta, Inuiguchi, Mehlawat, 2011),
- where in the stock exchange market analyses a hybrid fuzzy decision making procedure was applied – among others, fuzzy AHP and fuzzy TOPSIS methods are used (Kazemi, Sarrafha, Bedel, 2014),

- which deals with multi-period portfolio selection problems in a fuzzy environment – where a TOPSIS-compromised programming approach is designed (Liu, Zhang, Xu, 2012),
- where for portfolio selection a behavioural construct of suitability is used to develop a multi-criteria decision making framework – with the use of the AHP method and fuzzy multiple criteria decision making methods (Mehlawat, 2013),
- where for portfolio optimizations, among others, trapezoidal fuzzy numbers, SAW and TOPSIS methods were used (Nguyen, Gordon-Brown, 2012),
- in which selected fuzzy multi-criteria methods (ANP, VIKOR, TOPSIS) are used for portfolio optimization (Raei, Bahrani Jahromi, 2012),
- which presents a fuzzy approach to evaluate the investment values of stocks, where the ratings of stocks are assessed in linguistic values represented by fuzzy numbers (Tsao, 2003).

However, in each of these works you can see a slightly different approach to the issue.

The article is a continuation of previous research, the aim of which was to use multi-criteria methods in the construction of the portfolio, including soft modelling (Pośpiech, 2017; Pośpiech, 2018; Pośpiech, Mastalerz-Kodzis, 2015; Pośpiech, Mastalerz-Kodzis, 2016). Previous studies often did not give unambiguous results. Hence, the constant search for new, non-standard solutions enabling effective application of the proposed approaches.

The main goal of the article is to compare the profitability of effective portfolios obtained on the basis of the selected multi-criteria grouping of companies and the confrontation of two approaches, including a fuzzy one, regarding the treatment of criterion assessments.

The research hypothesis states that the application of the proposed fuzzy approach to determining effective portfolios enables obtaining portfolios that are more attractive to the investor (more profitable).

1. Research methodology

The article uses the multi-criteria TOPSIS method. This method was considered in two versions: standard and fuzzy ones. The choice of this method was dictated by the fact that during previous analyses aimed at indicating a multi-criteria method supporting the selection of the share portfolio, this method proved to be one of the most effective (Pośpiech, Mastalerz-Kodzis, 2015). The TOPSIS method, both in the standard and fuzzy versions, enables obtaining the ranking of assessed decision variants on the basis of the decreasing value of the relevant indicator. Let the symbol $a_i^{(k)}$ denote the evaluation of the variant i, $i = 1, \ldots, m$, within the criterion k, $k = 1, \ldots, n$. The procedure for determining the ranking takes the following stages (Hwang, Yoon, 1981; Lai, Liu, Hwang, 1994; Roszkowska, Wachowicz, 2013; Trzaskalik, 2014):

– determination of the decision matrix of the form $\mathbf{X} = [\hat{x}_{ik}]$, where:

$$\hat{x}_{ik} = \frac{a_i^{(k)}}{\sqrt{\sum_{i=1}^{m}\left[a_i^{(k)}\right]^2}} \tag{1}$$

for $i = 1, 2, \ldots, m$, $k = 1, 2, \ldots, n$,

– determination of a weighted normalized decision matrix **Z**: $\mathbf{Z} = [w_k \hat{x}_{ik}] = [v_{ik}]$, where w_k are the weights of the criteria, $k = 1, 2, \ldots, n$,

– determining the evaluation of the weighted ideal v_k^+ and anti-ideal solutions v_k^- according to the formulas:

$$v_k^+ = \begin{cases} \max\limits_i v_{ik} & \text{when } k \text{ is maximized} \\ \min\limits_i v_{ik} & \text{when } k \text{ jest minimized} \end{cases} \tag{2}$$

$$v_k^- = \begin{cases} \max\limits_i v_{ik} & \text{when } k \text{ jest minimized} \\ \min\limits_i v_{ik} & \text{when } k \text{ jest maximized} \end{cases} \tag{3}$$

– calculating the distance of variants from solutions v_k^+ and v_k^- based on the formulas:

$$d_i^+ = \sqrt[p]{\sum_{k=1}^{n}\left|v_{ik} - v_k^+\right|^p}, \ i = 1, 2, \ldots, m \tag{4}$$

$$d_i^- = \sqrt[p]{\sum_{k=1}^{n}\left|v_{ik} - v_k^-\right|^p}, \ i = 1, 2, \ldots, m \tag{5}$$

where p specifies the type of metrics,

– determination (for each i object) of the relative distance S_i, $S_i \in [0, 1]$, on the basis of which the ranking is built:

$$S_i = \frac{d_i^-}{d_i^+ + d_i^-}, \ i = 1, 2, \ldots, m \tag{6}$$

In the case of a fuzzy TOPSIS method, a decision matrix is otherwise constructed – each of the criterion evaluations is treated as a triangular fuzzy number of the form:

$$\tilde{a}_i^{(k)} = (d_{ik}, s_{ik}, g_{ik}) \tag{7}$$

where:

d_{ik} – pessimistic assessment of the variant *i* within the criterion *k*,

s_{ik} – expected assessment of the variant *i* within the criterion *k*,

g_{ik} – optimistic assessment of the variant *i* within the criterion *k*.

As in the case of the standard TOPSIS method, the multi-criteria procedure is carried out in stages (Jahanshahloo, Hosseinzadeh Lotfi, Izadikhah, 2006; Trzaskalik, 2014):

- determination of a normalized decision matrix $\tilde{\mathbf{X}} = [\tilde{x}_{ik}]$, whose elements are expressed for the maximized and minimized criteria, respectively, as follows:

$$\tilde{x}_{ik} = \left(\frac{d_{ik}}{\max\limits_j g_{jk}}, \frac{s_{ik}}{\max\limits_j g_{jk}}, \frac{g_{ik}}{\max\limits_j g_{jk}} \right) \tag{8}$$

$$\tilde{x}_{ik} = \left(\frac{\min\limits_j g_{jk}}{g_{ik}}, \frac{\min\limits_j g_{jk}}{s_{ik}}, \frac{\min\limits_j g_{jk}}{d_{ik}} \right), \tag{9}$$

- determination of a weighted normalized decision matrix $\tilde{\mathbf{Z}} = [\tilde{z}_{ik}] = [w_k \tilde{x}_{ik}]$, $i = 1, \ldots, m$, $k = 1, \ldots, n$,
- determination of the values of weighted fuzzy ideal and anti-ideal solutions:

$$\tilde{v}_k^+ = \max_i \tilde{z}_{ik} \tag{10}$$

$$\tilde{v}_k^- = \min_i \tilde{z}_{ik} \tag{11}$$

for $i = 1, 2, \ldots, m$, $k = 1, 2, \ldots, n$,

- calculation of the distances d_i^+ and d_i^- according to the formulas:

$$d_i^+ = \sum_{k=1}^{n} d(\tilde{z}_{ik}, \tilde{v}_k^+), \; i = 1, 2, \ldots, m \tag{12}$$

$$d_i^- = \sum_{k=1}^{n} d(\tilde{z}_{ik}, \tilde{v}_k^-), \; i = 1, 2, \ldots, m \tag{13}$$

where the distance between two triangular fuzzy numbers: $\tilde{a} = (d_a, s_a, g_a)$ and $\tilde{b} = (d_b, s_b, g_b)$, is defined as follows:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}\left[(d_a - d_b)^2 + (s_a - s_b)^2 + (g_a - g_b)^2 \right]} \tag{14}$$

– constructing a ranking based on the decreasing value of the S_i coefficient defined by the formula (6).

The use of the fuzzy TOPSIS method makes it possible to treat criterion assessments as triangular fuzzy numbers (l_k, m_k, u_k); the following assignment was made: l_k – the smallest value of the criterion assessment for a given variant (company) from the selected three years, m_k – the middle value of the criterion assessment from three years, u_k – the highest value of the criterion assessment from three years. In the standard version of the TOPSIS method, the criterion assessments were determined as an average from three years.

2. Empirical analysis – main results

The subject of the research was the companies that were included in the WIG20 index in December 2017. The period from which the data was taken was the years[1] 2015–2017. In the analyses, for the initial selection of companies, the multi-criteria TOPSIS and FTOPSIS methods were used. The following fundamental and market indicators, which are commonly used to assess the economic and financial condition of companies, have been accepted as the assessment criteria (Leszczyński, 2004; Łuniewska, Tarczyński, 2006; Tarczyński, 2001; Tarczyński, 2002):

– return of assets ROA (net income/average total assets) – criterion 1,
– return of equity ROE (net income/shareholder equity) – criterion 2,
– P/BV (price-book value) – criterion 3,
– P/E (price-earnings ratio) – criterion 4,
– earnings per share – criterion 5.

It was established that all criteria are just as important, so the same weight values were assigned to them. In addition, it was considered that the high criterion evaluation values of the selected criteria were desirable, therefore all criteria were maximized. Each company was evaluated through the prism of four criteria. Two approaches were considered: in the first one, criteria 1–4 were taken into account, while in the second one – criteria 1–3 and 5. This approach is justified by the fact that in certain situations (e.g. with very small profits) the P/E ratio can take high values, which is not the effect of positive changes. Due to the fact that the chosen multi-criteria method was applied in a standard and fuzzy version, four variants were considered:

– variant I – criteria 1–4, TOPSIS method,
– variant II – criteria 1–3, 5, TOPSIS method,

[1] The data was taken from sources shown in the references.

– variant III – criteria 1–4, FTOPSIS method,
– variant IV – criteria 1–3, 5, FTOPSIS method.

Taking into account the above variants, the values of the S_i indicator were determined and based on which the rankings were built – higher values of the indicator mean a higher position of the company in the ranking. In the deliberations, to calculate the distances given by the formulas (4) and (5), the Euclidean distance was used. The received S_i values are presented in Table 1.

Table 1. The values of the S_i indicator according to the considered variants

Co.	S_i			
	variant I	variant II	variant III	variant IV
ALR	0.577	0.343	0.503	0.498
ACP	0.607	0.354	0.506	0.491
BZW	0.627	0.389	0.531	0.528
CCC	0.968	0.587	0.792	0.764
CPS	0.644	0.386	0.538	0.519
ENG	0.600	0.349	0.497	0.479
EUR	0.724	0.424	0.586	0.567
JSW	0.473	0.199	0.381	0.273
KGH	0.416	0.035	0.290	0.270
LTS	0.630	0.384	0.506	0.492
LPP	0.856	0.854	0.686	0.811
MBK	0.602	0.373	0.509	0.512
OPL	0.476	0.188	0.377	0.370
PEO	0.619	0.362	0.523	0.510
PGE	0.576	0.290	0.443	0.428
PGN	0.643	0.390	0.534	0.515
PKN	0.723	0.498	0.617	0.603
PKO	0.604	0.338	0.510	0.493
PZU	0.663	0.407	0.574	0.554
TPE	0.241	0.257	0.417	0.402

Source: Pośpiech (2018).

On the basis of the obtained values of the S_i indicator, the groups of companies were selected, which constituted the basis for choosing the portfolio of shares. In earlier studies (Pośpiech, 2018), based on rankings, groups of companies were determined. This approach, however, did not give unambiguous results, which is why the groups were built using the values of the S_i indicator. Three threshold values of the indicator were adopted: 0.45, 0.5 and 0.55 – the individual groups included those companies for which the value of the indicator was not lower than the set threshold value. The results of grouping using this approach are presented in tables 2, 3 and 4.

Table 2. Results of grouping companies according to variants for $S_i = 0.45$

Variant/number of companies	Groups of companies
I – 18	ALR, ACP, BZW, CCC, CPS, ENG, EUR, JSW, LTS, LPP, MBK, OPL, PEO, PGE, PGN, PKN, PKO, PZU
II – 3	CCC, LPP, PKN
III, IV – 15	ALR, ACP, BZW, CCC, CPS, ENG, EUR, LTS, LPP, MBK, PEO, PGN, PKN, PKO, PZU

Source: own study.

Table 3. Results of grouping companies according to variants for $S_i = 0.5$

Variant/number of companies	Groups of companies
I – 16	ALR, ACP, BZW, CCC, CPS, ENG, EUR, LTS, LPP, MBK, PEO, PGE, PGN, PKN, PKO, PZU
II – 2	CCC, LPP
III – 14	ALR, ACP, BZW, CCC, CPS, EUR, LTS, LPP, MBK, PEO, PGN, PKN, PKO, PZU
IV – 10	BZW, CCC, CPS, EUR, LPP, MBK, PEO, PGN, PKN, PZU

Source: own study.

Table 4. Results of grouping companies according to variants for $S_i = 0.55$

Variant/number of companies	Groups of companies
I – 16	ALR, ACP, BZW, CCC, CPS, ENG, EUR, LTS, LPP, MBK, PEO, PGE, PGN, PKN, PKO, PZU
II – 2	CCC, LPP
III, IV – 5	CCC, EUR, LPP, PKN, PZU

Source: own study.

Depending on the adopted threshold value of the S_i indicator, different groups were obtained. It can be observed that at thresholds 0.45 and 0.55 the fuzzy approach generates the same sets regardless of the adopted criteria, while at the threshold of 0.5 each variant generates a different set. On the basis of the received groups of companies, effective portfolios were built, which were determined by solving the following optimization task (based on the approach by Markowitz (1952)):

$$
\begin{aligned}
& S_p^2 = \sum_{i=1}^{l} \sum_{j=1}^{l} x_i x_j \operatorname{cov}_{ij} \to \min \\
& R_p \geq R_0 \\
& \sum_{i=1}^{l} x_i = 1 \\
& x_i \geq 0, \qquad i = 1, \ldots, l
\end{aligned}
\tag{15}
$$

where:

S_p^2 – portfolio variance,

x_i, x_j – shares of the stocks i and j in the portfolio,

cov_{ij} – the covariance of return rates of i and j shares,

R_p – portfolio return rate,

R_0 – value of the portfolio return rate given by the decision maker (average of the positive return rates of the examined companies),

l – the size of the set on the basis of which the portfolio was built.

The tasks, in which the rates of return of companies from different periods were taken into account, were solved. Three-year (2015–2017), two-year (2016–2017), annual (2017) and semi-annual (second half of 2017) average rates of return were considered. Profitability of the portfolios for the included average rates of return is shown in tables 5, 6, 7 and 8.

3. Discussion

Several dozens of portfolios have been generated and their assessment has been made on the basis of profits or losses that would bring at the end of the next few months. The portfolios have been marked with the symbols Pt, t = 1, …, 4, where the number t is the variant number on the basis of which the portfolios have been determined. The P0 symbol denotes an effective portfolio, constructed in accordance with task (15), which was obtained on the basis of all of the 20 considered companies.

Table 5. Profits/losses of effective portfolios (%) – three-year average rates of return

Rate of profit of portfolio compared to 3.01.2018	S_i = 0.45			S_i = 0.5				S_i = 0.55			P0
	P1	P2	P3 = P4	P1	P2	P3	P4	P1	P2	P3 = P4	
31.01.2018	4.07	2.84	4.19	4.11	–0.96	4.74	2.37	4.11	–0.96	3.36	4.00
28.02.2018	–1.88	–8.51	–1.46	–1.82	–13.28	–0.51	–6.12	–1.82	–13.28	–6.06	–2.22
29.03.2018	–5.69	–20.27	–5.79	–6.02	–19.75	–4.67	–9.45	–6.02	–19.75	–12.08	–6.02
30.04.2018	–5.54	–14.98	–4.59	–4.74	–10.87	–3.69	–5.43	–4.74	–10.87	–7.29	–6.07
30.05.2018	–8.39	–22.75	–7.35	–7.53	–6.13	–6.32	–10.45	–7.53	–6.13	–16.04	–8.84
29.06.2018	–14.09	–20.32	–12.94	–13.11	–28.77	–11.80	–15.06	–13.11	–28.77	–17.12	–14.33
31.07.2018	–8.05	–12.19	–7.11	–7.26	–22.57	–5.78	–11.84	–7.26	–22.57	–9.96	–8.53

Source: own calculations.

Based on the analysis of profits and losses presented in Table 5, it can be concluded that for thresholds set at 0.45 and 0.5, portfolios obtained from groups based on the fuzzy approach at the end of the next few months are characterized by not lower profits or not higher losses compared to portfolios obtained according to the standard approach (the exception is the end of May at threshold 0.5). At the same time, it can be concluded that the approach in which the fourth criterion was used instead of the fifth one, enabled selecting portfolios with more favourable results. Against this background, P0 portfolio looks less favourable than the portfolio generated on the basis of the standard (P1) and fuzzy (P3) variant with criterion 4. After replacing the fourth criterion with the fifth one, P0 would be more preferred by an investor than P2 (at 0.45 and 0.5) and P4 (at threshold 0.5). In the case of the threshold for the S_i indicator at 0.55, it can be observed that the portfolio generated on the basis of the standard approach with criterion 4 (variant I) has better properties than the others (P0, P2, P3 = P4). In addition, the portfolio generated in fuzzy terms with criterion 5 (variant IV) gives mostly higher gains or lower losses than in the standard approach with criterion 5 (variant II).

Table 6. Profits/losses of effective portfolios (%) – two-year average rates of return

Rate of profit of portfolio compared to 3.01.2018	S_i = 0.45			S_i = 0.5				S_i = 0.55			P0
	P1	P2	P3 = P4	P1	P2	P3	P4	P1	P2	P3 = P4	
31.01.2018	3.93	2.84	3.89	3.80	–0.96	4.30	1.61	3.80	–0.96	3.46	3.87
28.02.2018	–1.41	–8.51	–1.43	–1.90	–13.28	–0.74	–6.85	–1.90	–13.28	–5.46	–1.76
29.03.2018	–5.01	–20.27	–5.25	–5.45	–19.75	–4.38	–10.04	–5.45	–19.75	–9.93	–5.39
30.04.2018	–4.24	–14.98	–3.83	–3.99	–10.87	–3.12	–4.37	–3.99	–10.87	–5.23	–4.87
30.05.2018	–6.10	–22.75	–6.42	–6.59	–6.13	–5.62	–8.05	–6.59	–6.13	–14.14	–6.66
29.06.2018	–12.49	–20.32	–12.54	–12.68	–28.77	–11.63	–15.80	–12.68	–28.77	–16.35	–12.73
31.07.2018	–5.97	–12.19	–6.64	–6.73	–22.57	–5.63	–11.17	–6.73	–22.57	–9.47	–6.46

Source: own calculations.

Taking into account the two-year average rates of return at the threshold of 0.45, the “fuzzy” P3 portfolio had comparable or slightly worse results than the “standard” P1 portfolio, while the “fuzzy” P4 portfolio had better results than the “standard” P2 portfolio. Again, taking into account the set of criteria 1–4, instead of 1–3 and 5, we get more profitable (or less lossy) portfolios. At the threshold of 0.5, “fuzzy” portfolios and those obtained using criteria 1–4 have better results. For threshold 0.55, this rule is confirmed for the standard approach; in the fuzzy approach, the same portfolios were obtained. However, comparing the P1 portfolio with P3 and P2 with P4, the more favourable results noted P1 compared to P3 and P4 compared to P2. At this threshold, there is no unambiguity in assessing the standard approach compared to the fuzzy one. On the background of the discussed portfolios the P0 has more favourable results than the portfolios selected using criteria 1–3 and 5 (the exception is P3 = P4 for threshold 0.45), and comparable or slightly worse results than the "standard" portfolios with criteria 1–4; it has also worse results than P3 at threshold 0.5.

Table 7. Profits/losses of effective portfolios (%) – annual average rates of return

Rate of profit of portfolio compared to 3.01.2018	S_i = 0.45			S_i = 0.5				S_i = 0.55			P0
	P1	P2	P3 = P4	P1	P2	P3	P4	P1	P2	P3 = P4	
31.01.2018	3.56	2.67	3.63	3.62	–0.96	4.30	1.45	3.62	–0.96	5.56	3.51
28.02.2018	–2.40	–8.73	–2.27	–2.40	–13.28	–1.12	–7.35	–2.40	–13.28	–1.38	–2.70
29.03.2018	–6.51	–20.24	–6.43	–6.52	–19.75	–4.97	–10.68	–6.52	–19.75	–3.65	–6.93
30.04.2018	–5.56	–14.79	–4.70	–4.76	–10.87	–3.44	–4.56	–4.76	–10.87	–0.95	–6.26
30.05.2018	–8.12	–21.98	–7.25	–7.30	–6.13	–5.78	–7.30	–7.30	–6.13	–16.20	–8.73
29.06.2018	–14.50	–20.72	–13.54	–13.60	–28.77	–12.06	–16.74	–13.60	–28.77	–10.04	–14.76
31.07.2018	–8.92	–12.67	–8.17	–8.17	–22.57	–6.60	–12.31	–8.17	–22.57	–3.07	–9..31

Source: own calculations.

In the case of annual average rates of return for threshold 0.45, the better results of portfolios obtained according to the fuzzy approach and not worse results of portfolios obtained using criteria 1–4 are visible. Taking into account the threshold of 0.5, portfolios generated using criteria 1–4 have better results than using criteria 1–3 and 5. In addition, the portfolios generated using the fuzzy approach have definitely recorded better results. At the threshold of 0.55, the portfolios using the fuzzy approach are identical (regardless of whether criteria 1–4 or 1–3 and 5 are taken into account) and with the exception of the end of May, they clearly record better results compared to their standard counterparts. It can be concluded that considering criteria 1–4 we get no worse results than using the second of the considered combination of criteria. The P0 portfolio has similar features to the P1 portfolios obtained using the standard approach with criteria 1–4.

Table 8. Profits/losses of effective portfolios (%) – semi-annual average rates of return

Rate of profit of portfolio compared to 3.01.2018	S_i = 0.45			S_i = 0.5				S_i = 0.55			P0
	P1	P2	P3 = P4	P1	P2	P3	P4	P1	P2	P3 = P4	
1	2	3	4	5	6	7	8	9	10	11	12
31.01.2018	4.37	2.68	4.90	4.89	–0.96	5.29	2.26	4.89	–0.96	4.34	4.37
28.02.2018	–0.28	–7.99	0.65	0.25	–13.28	1.28	–6.29	0.25	–13.28	–4.14	–0.49
29.03.2018	–5.43	–16.01	–4.08	–4.25	–19.75	–3.22	–9.51	–4.25	–19.75	–7.77	–5.74
30.04.2018	–5.36	–9.74	–3.39	–3.49	–10.87	–2.50	–3.96	–3.49	–10.87	–2.80	–5.85

1	2	3	4	5	6	7	8	9	10	11	12
30.05.2018	–7.54	–11.13	–6.44	–6.62	–6.13	–5.79	–7.96	–6.62	–6.13	–7.09	–7.99
29.06.2018	–12.94	–19.23	–11.11	–11.23	–28.77	–10.16	–15.47	–11.23	–28.77	–13.32	–13.14
31.07.2018	–6.22	–12.39	–3.94	–4.10	–22.57	–2.79	–11.38	–4.10	–22.57	–7.10	–6.58

Source: own calculations.

With semi-annual average return rates and a threshold of 0.45, the fuzzy approach allows for the generation of more advantageous portfolios (variant with criteria 1–4 is not worse than with criteria 1–3, 5). Applying the threshold 0.5, the regularity regarding the superiority of applying criteria 1–4 over the combination of 1–3 and 5 is preserved. In addition, the superiority of the fuzzy approach over the standard one is also visible (the exception is the end of May). At the threshold of 0.55, the use of criteria 1–4 gives no worse results than the application of the second considered combination of criteria. The P3 portfolio (fuzzy) compared to P1 (standard) is slightly worse, while comparing P4 (fuzzy) and P2 (standard), the first one shows better results. The P0 portfolio is comparable to the P1 portfolios and slightly worse than P3, the exception is P3 obtained at the 0.55 threshold.

Conclusions

The analyses carried out were aimed at comparing the profitability of effective portfolios generated on the basis of the initial selection carried out using the multi-criteria method of TOPSIS. The method was applied in two versions – standard and fuzzy. In addition, two sets of criteria were used to assess the companies and it was examined to find out which combination of criteria gives the opportunity to obtain more profitable portfolios.

In the proposed variants, in most cases the fuzzy approach enabled the determination of effective portfolios with a higher (not lower) profits or lower (not higher) losses compared to the standard approach. In addition, it was observed that the inclusion of criterion 4 (P/E) instead of criterion 5 (earnings per share) gives the possibility of obtaining portfolios with higher profits or lower losses. It can also be seen that the most consistent results, showing the superiority of the fuzzy approach over the standard one and the superiority of the application of the set of criteria 1–4 instead of 1–3 and 5, were obtained at the threshold of 0.5 for the indicator S_i (for average rates of return from different periods). In turn, for all three accepted thresholds, the most consistent results were obtained taking into account the annual average rates of return.

At this stage of research, it can be concluded that the use of fuzzy numbers can positively influence the results of effective portfolios determined using the proposed approach, so the hypothesis can be positively verified; nevertheless, the problem requires further analysis.

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