COMPARISON OF A MODIFIED AND CLASSIC FAMA-FRENCH MODEL FOR THE POLISH MARKET

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Abstract

This paper shows a comparison of the results of return, risk, and risk price simulation by a modified and classic Fama-French model. The modified model defines the new ICAPM state variable as a function of the structure of a company's past financial results. The model tests are run on the basis of stocks listed on the Warsaw Stock Exchange. In light of the classic model the risk price, on the tested market, turned out univariate due to *HML*, however, in light of the modified model, risk price turned out to be three-dimensional due to the proposed factors, and market portfolio. The factors of the modified model, compared with the *HML* and *SMB*, are widely perceived by portfolio managers, and the simulation results indicate a greater possibility to use this pricing application by large institutional investors.

Introduction

Most of the existing pricing procedures of securities are based on the capital pricing theory (CAPM) or arbitrage pricing theory (APT). The APT developed by Ross (1976) assumes that returns are generated by an unknown number of unknown factors. Similarly, the ICAPM generalized by Merton (1973) is based on a market portfolio and k factors dependent on unknown state variables. We can see considerable similarities between both theories; however, differences are due to methodological reasons. The differences between the theoretical assumptions have no impact on the pricing. The practical implementations of both theories come down to testing the models which assume a linear form of the stochastic discount factor.

Designing new applications of pricing is justified because neither ICAPM nor APT define the known pricing factors. On the other hand, even the most famous literature algorithms do not always generate the correct returns on the tested markets.

The Fama-French (1993) model (FF hereafter) propose *HML* and *SMB* factors, as the functions of state variables: capitalization and book to market value indicator. Extensive research conducted since the mid-twentieth century has shown a significant relationship between these variables and future returns. Examples are offered by the works of Stattman (1980), Rosenberg, Reid and Lanstein (1985), Reinganum (1981), Lakonishok and Shapiro (1986) or Fama and French (1992). However, only in 1995, did Fama and French indicate that direct factor generating the future returns is the structure of earnings in the last five-year period, while the capitalization and book to market values, dependent on earnings, indirectly affect pricing. Fama and French (1996) underlined those factors simultaneously generating returns and earnings are still unknown, which should be investigated. This logic conclusively confirms the need for the building of new pricing procedures.

At the turn of the 20th and 21st centuries, various tests of CAPM and ICAPM applications were proposed. Also, CCAPM applications were investigated. Examples include the work of Carhart (1995), Lettau and Ludvigson (2001), Yogo (2006) or Petkova (2006), which were tested on the US market. The most famous research study is probably that of Carhart (1995) in which the author modifies the classic FF three factor model, proposing a fourth factor as a one-year momentum. Lettau and Ludvigson (2001) propose conditional applications of the CAPM and CCAPM models. They argue that conditional applications better describe returns in comparison with unconditional applications. Yogo (2006) proposes the factorial application of CCAPM, using durable and nondurable consumption. The author concludes that durable consumption well describes changes in business cycles, and should be the main factor explaining

returns. Petkova (2006) shows that the replacement of *HML* and *SMB* by innovations in the aggregate dividend yield, term spread, default spread, and one-month T-bill increase the model's explanatory power as compared with the classic FF model.

Research on the Polish market focused on testing the classic CAPM. Examples testing the ICAPM applications are the works of Zarzecki et al. (2004–2005), Kowerski (2008), Czapkiewicz and Wójtowicz (2014) or Urbański (2011, 2012). Kowerski tested the classic FF model on the basis of monthly observations in the historical period of 1995–2005. Czapkiewicz and Wójtowicz (2014) tested the Carhart (1995) model in 2003–2012.

Urbański (2011, 2012) used the research results of FF (1995) and proposes the aggregated two and three factor models as the ICAPM applications. Urbański, in his previous works, tested the proposed model on 15 portfolios formed in one direction, using Cochrane's (2001, p. 435) advice. The tests performed on the Polish market in 1996–2005 show far lower pricing errors in the case of an aggregated model as compared with the FF three factor model. However, the procedure of testing both models (although complying with Cochrane's recommendations) differs from the FF (1993) proposition. The classic FF three factor model is based on 25 portfolios formed in the two directions, which is reflected by state variables of the ICAPM application.[1]

Therefore, to make an unambiguous assessment I have compared the results of pricing (in the light of theoretical assumptions and practical possibilities of the use by portfolio managers and investors) by the classic FF model and three factor aggregated model (modified FF model hereafter) under the proposed 25 testing portfolios, formed in the two directions, reflecting the state variables of the two models.

Section 2 presents the pricing methodology proposing new state variables, in the light of FF (1995) propositions, and my own considerations. Section 3 presents data used for calculations. Section 4 shows the results of calculations and discusses differences in the results of the performed tests from the perspective of practical applications by portfolio managers. The final section presents the conclusions.

1. Theoretical basis of the Fama and French model modifications

The basic pricing equation of any asset can be presented by dependency (1) if and only if the law of one price is reasonable (see Cochrane, 2001, pp. 63–65):

$$p_{it} = E_t(m_{t+1}x_{i,t+1}) \tag{1}$$

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[1] In the case of the classic FF model these are the capitalization and book to market indicator.

where: p_{it} is the current asset price i, m_{t+1} is the stochastic discount factor (SDF), $x_{i,t+1}$ is a future pay-out.

Defining an asset return as: $r_{i,t+1} \equiv {x_{i,t+1}}/{p_{it}}$, equation (1) can be written as follows:

$$E_t(m_{t+1}r_{i,t+1}) = 1 \tag{2}$$

In the case of the three-factor ICAPM application SDF is a linear function (3) of return of the market portfolio $R\tilde{M}$, and two factors which take into account the influence of two hypothetical state variables $\tilde{S}_1$ i $\tilde{S}_2$. These variables should secure all future nature states.

$$m = a + bR\tilde{M}_t + c\tilde{S}_{1t} + d\tilde{S}_{2t} \tag{3}$$

If a risk free asset exists, and its return is (see Cochrane, 2001, p. 13):

$$RF_t = \frac{1}{E_t(m_{t+1})},$$

the pricing model (2) takes the following form:

$$E(r_i) - E(RF) = \gamma_M \beta_{i,M} + \gamma_{S_1} \beta_{i,S_1} + \gamma_{S_2} \beta_{i,S_2} \tag{4}$$

where:

$\gamma_M = -RF_t \, \text{var}(bRM_t),$

$\gamma_{S_1} = -RF_t \, \text{var}(S_{1t}),$

$\gamma_{S_2} = -RF_t \, \text{var}(S_{2t}),$

$\beta_{i,M} = \text{var}(bRM_t)^{-1} \text{cov}(bRM_t, r_{it}),$

$\beta_{i,S_1} = \text{var}(S_{1t})^{-1} \text{cov}(S_{1t}, r_{it}),$

$\beta_{i,S_2} = \text{var}(S_{2t})^{-1} \text{cov}(S_{2t}, r_{it}),$

$bR\tilde{M}_t$ is rate of return of a market portfolio approximated by excess rate of the return of the stock index – WIG over the risk-free rate, and $S_{1t} = c\tilde{S}_{1t}$, as well as $S_{2t} = d\tilde{S}_{2t}$ are factors of the assumed ICAPM application.

FF (1993) claim that their proposed three-factor model is the Merton ICAPM application. In this model the size and book to market value ratio (*BV/MV*) are risk factors, while $S_{1t} = c\tilde{S}_{1t} = HML_t$ and $S_{2t} = d\tilde{S}_{2t} = SMB_t$ are known functions of the size and *BV/MV*.

In the further work FF (1995, pp. 153–154) conclude: "If the average-return relations are due to rational pricing, then (i) there must be common risk factors in returns associated with size and *BE/ME*, and (ii) the size and book-to-market patterns in returns must be explained by the behaviour of earnings. … The evidence presented here shows that size and *BE/ME* are related

to profitability." At the end of the paper the authors pose two questions which have not been answered yet: "(i) What are the underlying economic state variables that produce variation in earnings and returns related to size and *BE/ME*? (ii) Do these unnamed state variables produce variation in consumption and wealth that is not captured by an overall market factor and so can explain the risk premiums in returns associated with size and *BE/ME*?" (see FF, 1995, p. 154).

The paper will attempt to answer to the first question. On the basis of FF (1995, pp. 134–140, Figures 1–2, and Table 1) I modify an FF (1993) three factor model, expecting that the following conjecture is true:

Conjecture

The economic state variable that produces variation in the future earnings and returns related to size and *BV/MV* is a vector of structure of the past long-term differences in profitability.

The adopted general state variable can be reflected by functional *FUN*, defined by equations (5), (6) and (7).

$$FUN = \frac{NUM}{DEN} = \frac{nor(ROE) \times nor(AS) \times nor(APO) \times nor(APN)}{nor(MV/E) \times nor(MV/BV)} \tag{5}$$

where:

$$ROE = F_1; \quad AS = F_2 = \frac{\sum\limits_{t=1}^{i} S(Q_t)}{\sum\limits_{t=1}^{i} S(nQ_t)}; \quad APO = F_3 = \frac{\sum\limits_{t=1}^{i} PO(Q_t)}{\sum\limits_{t=1}^{i} PO(nQ_t)};$$

$$APN = F_4 = \frac{\sum\limits_{t=1}^{i} PN(Q_t)}{\sum\limits_{t=1}^{i} PN(nQ_t)}; \quad MV/E = F_5; \quad MV/BV = F_6 \tag{6}$$

$F_j (j = 1, \ldots, 6)$ are transformed to normalized areas $<a_j; b_j>$, according to Eq. (7):

$$nor(F_j) = \left[a_j + (b_j - a_j) \times \frac{F_j - c_j \times F_j^{\min}}{d_j \times F_j^{\max} - c_j \times F_j^{\min} + e_j} \right] \tag{7}$$

In Equations (6) and (7), the corresponding indications are as follows: *ROE* is a return on book equity; $\sum\limits_{t=1}^{i} S(Q_t), \sum\limits_{t=1}^{i} PO(Q_t), \sum\limits_{t=1}^{i} PN(Q_t)$ are values that are accumulated from the beginning of the year as net sales revenue (S), operating profit (PO) and net profit (PN) at the

end of "i" quarter (Q_i); $\sum_{t=1}^{i} \overline{S(nQ_t)}$, $\sum_{t=1}^{i} \overline{PO(nQ_t)}$, $\sum_{t=1}^{i} \overline{PN(nQ_t)}$ are average values, accumulated from the beginning of the year as *S*, *PO* and *PN* at the end of Q_i over the last *n* years (the present research assumes that *n* = 3 years); *MV/E* is the market-to-earning value ratio; *E* is the average earning for the last four quarters; *MV/BV* is the market-to-book value ratio; a_j, b_j, c_j, d_j, e_j are variation parameters. In equilibrium modelling F_j (*j* = 1, ..., 6) can be transformed into the equal normalized area <1; 2> (see Urbański, 2011).

The constructed functional *FUN* represents an investor constructing a portfolio, using the structure of the past long-term differences in profitability, which consists of the best fundamental and undervalued stocks. *FUN* is dependent on company evaluation indicators, occurring in the numerator and company market pricing indicators in the denominator. As far as the classification of companies to the portfolio is concerned, I base this on the criterion that I define as optimal the *FUN* value calculated for all companies listed in a given market. F_j variables are functions of company evaluation indicators (for *j* = 1, ..., 4) and functions of pricing indicators (for *j* = 5, 6). Given that F_j may change considerably, *FUN* value may be frequently impacted in a major or minor way. For this reason, it is necessary to transform all F_j variables to match the appropriately defined standardized areas, in accordance with Equation (7). It must be noted that parameters a_j and b_j define the border of the F_j variable standardized area, and have a varying impact on *FUN* value by given variables. The constructed portfolio contains *N* companies, for which *FUN* assumes *N* highest values (for long positions). Functional *FUN* is a gauge of securities that are assessed well by *NUM* and at the same time priced lowly by *DEN*. *NUM* represents an investor building a portfolio, comprising the best fundamental firms, while *DEN* represents an investor who purchases undervalued stocks. Investors construct the portfolios by maximizing *FUN* and *NUM* or minimizing *DEN* (if long investments are considered). *FUN* contains a clear economic interpretation and may constitute a criterion for selecting securities for the portfolio. The investment is more attractive if the *FUN* value is greater (Urbański, 2011).

A representative investor can successively achieve above-average returns on condition of the correct predictions of economic states that determine the future value of assets. If investments on the basis of *FUN* allow for achieving above-average returns, then the relation between *FUN* and the resultant of different, both known and unknown investment methods, can be concluded. These investment methods should predict future states of nature. In other words – functional *FUN* should determine state variables which will secure future investment payments.

Research conducted by Urbański (2011) shows the possibility of an investment decision based on *FUN*, and achievement of above-average returns on WSE in 1995–2005. Therefore,

it is assumed that the conjecture about the relation between *FUN* and the resultant of investment methods (that should predict future states of nature) is true. Thus, pricing procedures designed on the basis of such state variables are ICAPM applications.

In the case of the proposed multifactor model, as the modification of the FF three factor model, parameters of equation (4) are defined as follows:

$$b\tilde{RM}_t = RM_t - RF_t \tag{8a}$$

$$S_{1t} = c\tilde{S}_{1t} = HMLN_t \tag{8b}$$

$$S_{2t} = d\tilde{S}_{2t} = LMHD_t \tag{8c}$$

where $HMLN_t$ (high minus low) is the difference between the returns from the portfolio with the highest and lowest NUM_t values in period t; $LMHD_t$ (low minus high) is the difference between the returns from the portfolio with the lowest and highest DEN_t values in period t; $RM_t - RF_t$ is the market factor, defined as the excess return of stock index – WIG over the risk-free rate.

Considering (8), it can be shown that pricing model (2) and the following representation are equivalent (e.g. Balvers, 2001, pp. 136–137):

$$r_{it} - RF_t = \beta_{i,HMLN}HMLN_t + \beta_{i,LMHD}LMHD_t + \beta_{i,M}(RM_t - RF_t) \tag{9}$$

The proposed financial pricing model (9) can be tested in two passes (10a) and (10b).

$$r_{it} - RF_t = \alpha_i + \sum_{k=1}^{K}\beta_{ik}F_{kt} + e_{it}; \quad t = 1,...,T; \quad \forall i = 1, ..., m \tag{10a}$$

$$r_{it} - RF_t = \gamma_0 + \sum_{k=1}^{K}\gamma_k\hat{\beta}_{ik} + \varepsilon_{it}; \quad i = 1,...,m; \quad t = 1,...,T \tag{10b}$$

where: F_{kt} is the vector of factors ($k = HMLN$, $LMHD$, RM-RF), $\beta_{i,k}$ and γ_k ($k = HMLN$, $LMHD$, M) are vectors of systematic risk and risk price components due to the $HMLN$, $LMHD$ and market portfolio M.

2. Data and range of research

Pricing Model Simulations are carried out on the basis of all stock quoted on the Warsaw Stock Exchange (WSE) main market in 1995–2012, characterized by positive book value during the last reporting period. Data referring to the fundamental results of the tested stocks is taken

from the database created by the Notoria Service company,[2] while data for returns computing is provided from the Warsaw Stock Exchange database.

Returns and components of systematic risk and risk premium are modelled by the classic, and modified FF models described by equations (10). In the case of the classic FF model F_{kt} is the vector of FF factors *HML*, *SMB* and *RM-RF*, $\beta_{i,k}$ and γ_k are vectors of systematic risk and risk price components due to the *HML*, *SMB* and market portfolio *M*. In the case of the modified FF model F_{kt} is the vector of the FF factors *HMLN*, *LMHD* and *RM-RF*, $\beta_{i,k}$ and γ_k are vectors of systematic risk and risk price components due to the *HMLN*, *LMHD* and market portfolio *M*.

The analysis is carried out on the quarterly returns of formed portfolios. The quintile portfolios are formed in two directions on the values of state variables that are used by each model. In the case of the classic FF model the portfolios are formed on *BV/MV* and capitalization. Firstly, all tested companies are divided into five portfolios due to *BV/MV*. Then, each so formed quintile portfolio is divided into five portfolios due to capitalization.

In the case of the modified FF model the portfolios are formed on *NUM* and *DEN*. Similarly, the tested companies are divided into five portfolios due to *NUM*. Then each so formed quintile portfolio is divided into five portfolios due to *DEN*. In total, 25 portfolios are formed for each version of the FF model.

3. Results and analysis

The dependent variable of regressions (10) constitutes the excess of returns of 25 test portfolios. Tables 1 and 2 present their mean values for the portfolios testing the classic and modified FF model. Tables 3 and 4 show the mean values of capitalization and *BV/MV* for the portfolios testing the classic FF model. Tables 5 and 6 show the mean values of *NUM* and *DEN* for the portfolios testing the modified FF model.

The distributions of mean value of excess returns on portfolios formed on *BV/MV* and *NUM* are similar. The highest values are observed for the 1st *BV/MV* quintiles, and the lowest for the 5th quintiles. However, in the classic FF model the all mean values are statistically equal to zero.

The lowest values of capitalization are found for the highest *BV/MV* quintiles, while the values of *BV/MV* are similar for all capitalization quintiles and the first four quintiles of *BV/MV*. In the case of the highest *BV/MV* quintile the highest *BV/MV* value is for the 1st quintile of capitalization and monotonically decreases to the 5th quintile.

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[2] Notoria Service sells stock analysis tools and the provision of financial data and quotations, see http://ir.notoria.pl.

Table 1. Dependent variable of the classic FF model;
excess returns on 25 stock portfolios formed on capitalization and *BV/MV*

Size quintiles	Book-to-market value (*BV/MV*) quintiles									
	low	2	3	4	high	low	2	3	4	high
	means					standard deviations				
Small	−0.02	−0.00	0.03	0.05	0.04	0.20	0.19	0.23	0.28	0.26
2	0.00	0.04	0.02	0.02	0.05	0.19	0.24	0.19	0.20	0.29
3	0.00	−0.01	0.01	0.02	0.03	0.17	0.17	0.21	0.18	0.24
4	0.00	−0.00	0.01	0.04	0.03	0.17	0.16	0.17	0.20	0.21
Big	−0.01	0.01	0.03	0.02	0.07	0.15	0.15	0.18	0.17	0.38
	t-statistics for means									
Small	1.16	1.28	0.91	−0.11	−0.62					
2	1.36	0.87	0.71	1.43	0.08					
3	1.01	0.83	0.32	−0.59	0.21					
4	1.15	1.44	0.26	−0.16	0.19					
Big	1.50	1.05	1.45	0.60	−0.37					

Notes: The sample period is from 1995 to 2012, 64 Quarters. "Size" is the capitalization of the portfolio.

Source: own calculations.

Table 2. Dependent variable of the modified FF model;
excess returns on 25 stock portfolios formed on *DEN* and *NUM*

DEN quintiles	*NUM* value quintiles									
	low bad	2	3	4	high good	low bad	2	3	4	high good
	means					standard deviations				
Small cheap	−0.00	0.03	0.07	0.06	0.10	0.22	0.21	0.29	0.20	0.26
2	0.02	−0.01	0.04	0.04	0.05	0.26	0.21	0.23	0.19	0.20
3	−0.04	−0.03	−0.00	0.01	0.04	0.21	0.18	0.16	0.17	0.18
4	0.00	−0.01	−0.02	−0.00	0.06	0.21	0.20	0.14	0.16	0.22
Big priced	−0.04	0.01	−0.02	−0.01	0.04	0.25	0.33	0.18	0.15	0.21
	t-statistics for means									
Small cheap	−0.03	1.18	1.89	2.54	1.47					
2	0.62	−0.46	1.40	1.64	2.06					
3	−1.53	−1.30	−0.16	0.39	1.73					
4	0.13	−0.58	−1.04	−0.12	2.18					
Big priced	−1.13	0.26	−0.98	−0.32	3.18					

Notes: The sample period is from 1995 to 2012, 64 Quarters.

Source: own calculations.

Table 3. Capitalization values of 25 stock portfolios formed on capitalization and *BV/MV*

| Size quintile | Book-to-market value (*BV/MV*) quintiles | | | | | | | |
| | low | 2 | 4 | high | low | 2 | 4 | high |
	means				standard deviations			
Small	85,442	64,957	29,974	17,137	56,483	41,905	13,440	9,931
2	223,993	189,298	69,143	34,910	112,830	92,113	32,393	21,979
3	487,968	434,506	142,468	63,703	149,135	205,921	87,768	46,961
4	1,180,525	1,221,215	334,337	155,160	440,250	745,656	306,427	156,297
Big	6,938,272	8,186,226	2,999,759	2,871,182	4,346,260	5,688,788	3,066,362	4,221,733

Notes: The sample period is from 1995 to 2012, 64 Quarters. "Size" is the capitalization of the portfolio.

Source: own calculations.

Table 4. *BV/MV* indicators of 25 stock portfolios formed on capitalization and *BV/MV*

| Size quintile | Book-to-market value (*BV/MV*) quintiles | | | | | | | | | |
| | low | 2 | 3 | 4 | high | low | 2 | 3 | 4 | high |
	means					standard deviations				
Small	0.26	0.58	0.82	1.17	2.23	0.15	0.21	0.30	0.47	1.12
2	0.28	0.56	0.81	1.16	2.08	0.11	0.20	0.30	0.47	1.05
3	0.27	0.56	0.81	1.11	1.98	0.10	0.20	0.30	0.43	0.91
4	0.26	0.56	0.80	1.11	1.80	0.08	0.20	0.29	0.41	0.75
Big	0.25	0.56	0.79	1.10	1.75	0.10	0.20	0.28	0.40	0.72

Notes: The sample period is from 1995 to 2012, 64 Quarters. "Size" is the capitalization of the portfolio.

Source: own calculations.

Table 5. *NUM* values of 25 stock portfolios formed on *DEN* and *NUM*

| DEN quintile | NUM value quintiles | | | | | | | | | |
| | low bad | 2 | 3 | 4 | high good | low bad | 2 | 3 | 4 | high good |
	means					standard deviations				
Small cheap	0.62	1.56	2.27	2.91	4.32	0.51	0.57	0.60	0.83	1.40
2	0.60	1.53	2.26	2.90	4.13	0.51	0.55	0.60	0.79	1.25
3	0.51	1.55	2.25	2.91	4.29	0.40	0.58	0.62	0.82	1.42
4	0.39	1.46	2.24	2.94	4.24	0.34	0.61	0.62	0.85	1.27
Big priced	0.33	1.41	2.22	2.94	4.49	0.26	0.58	0.67	0.83	1.52

Notes: The sample period is from 1995 to 2012, 64 Quarters. The small *DEN* value portfolios are characterized by the small values of *MV/BV* and *MV/E*, therefore they are called "cheap". The big *DEN* value portfolios are characterized by the big values of *MV/BV* and *MV/E*, therefore they are called "priced".

Source: own calculations.

Table 6. *DEN* values of 25 stock portfolios formed on *DEN* and *NUM*

DEN quintile	*NUM* value quintiles									
	low bad	2	3	4	high good	low bad	2	3	4	high good
	means					standard deviations				
Small cheap	1.51	1.14	1.12	1.12	1.11	0.50	0.07	0.05	0.05	0.05
2	2.15	1.36	1.22	1.21	1.22	0.75	0.31	0.10	0.08	0.10
3	2.79	1.76	1.34	1.21	1.30	0.49	0.64	0.23	0.13	0.14
4	3.13	2.27	1.60	1.39	1.44	0.26	0.73	0.49	0.19	0.19
Big priced	3.79	2.94	2.26	1.89	1.95	0.41	0.56	0.70	0.41	0.41

Notes: The sample period is from 1995 to 2012, 64 Quarters. The small *DEN* value portfolios are characterized by the small values of *MV/BV* and *MV/E*, therefore they are called "cheap". The big *DEN* value portfolios are characterized by the big values of *MV/BV* and *MV/E*, therefore they are called "priced".

Source: own calculations.

The values of *NUM* are similar for all *DEN* quintiles. However, *DEN* values decrease from the 1st to the 5th quintile of *NUM*. This means that companies presenting good financial results are characterized by smaller values of *MV/BV* and *MV/E*.

The loadings of explanatory variables of regressions (10a) and (10b) are systematic risk components, estimated in the first pass, and risk premium components, estimated in the second pass. The parameter values of regressions (10) are determined by means of the GLS method with the application of the Prais-Winsten procedure with first-order autocorrelation. The impact of model factor loadings on returns is shown in Figures 1 to 4.

In light of the modified FF model, investments in companies showing good financial results (high *NUM*), both cheap and priced companies, demonstrate growing returns for growing *HMLN* (see Figures 1a, 1c, and 2a), while investments in companies showing bad financial results (low *NUM*), both cheap and priced companies, demonstrate decreasing returns for growing *HMLN* (see Figures 1a, 1c, and 2b). On the other hand, investments in cheap companies, and showing both good and bad financial results (small *DEN*), demonstrate growing returns for growing *LMHD* (see Figures 3a, and 4a, 4c). Simultaneously, investments in priced companies, and showing both good and bad financial results (big *DEN*), demonstrate decreasing returns for growing *LMHD* (see Figures 3c, and 4a, 4c).

In light of the classic FF model, investments in value companies (high *BV/MV*), both small and big capitalization stocks, demonstrate growing returns for growing *HML* (see Figures 1b, 1d, and 2b), while investment in growth companies (low *BV/MV*), both small and big capitalization stocks, demonstrates decreasing returns for growing *HML* (see Figures 1b, 1d, and 2d). On the

a) Modified FF model   b) Classic FF model
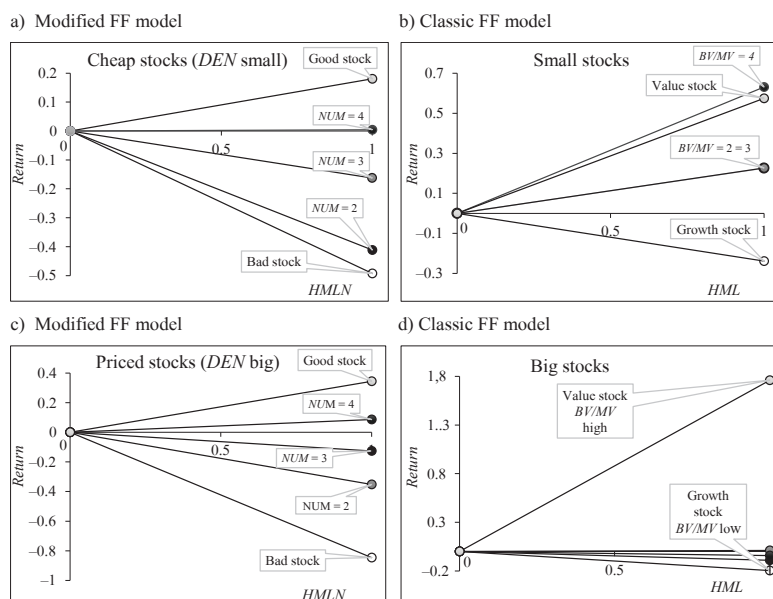
c) Modified FF model   d) Classic FF model

Figure 1. The impact of *HMLN* and *HML* loadings on returns for portfolios formed on: a) and
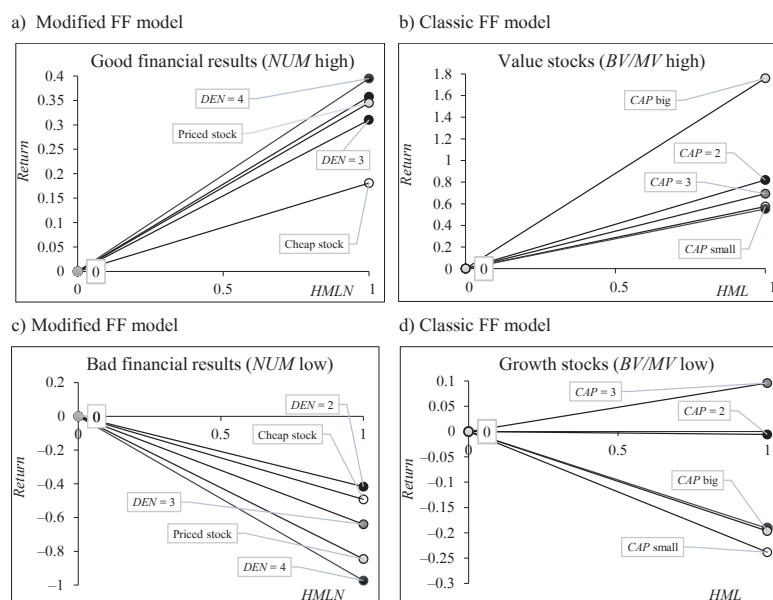c) *NUM*, and b) and d) *BV/MV*

Source: own elaboration.

a) Modified FF model   b) Classic FF model

c) Modified FF model   d) Classic FF model

Figure 2. The impact of *HMLN* and *HML* loadings on returns for portfolios formed on: a) and
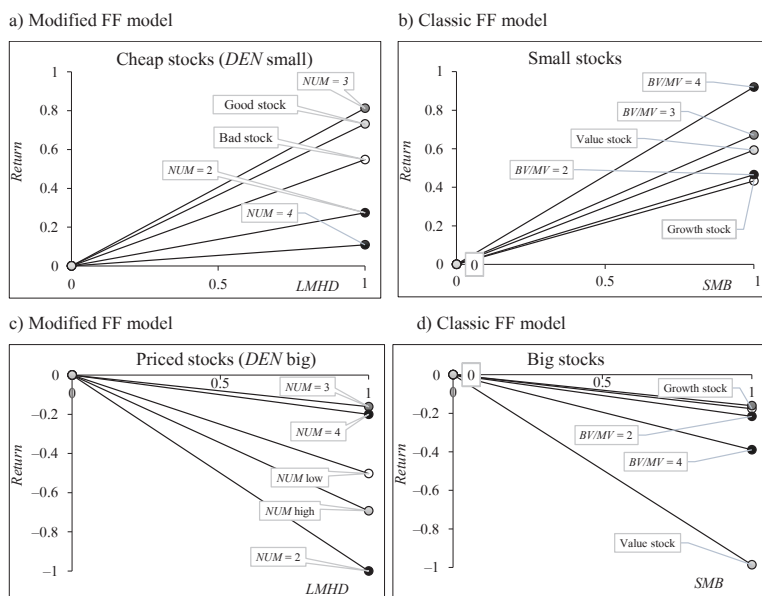c) *DEN*, and b) and d) Capitalization

Source: own elaboration.

Figure 3. The impact of *LMHD* and *SMB* loadings on returns for portfolios formed on: a) and
           c) *NUM*, and b) and d) *BV/MV*
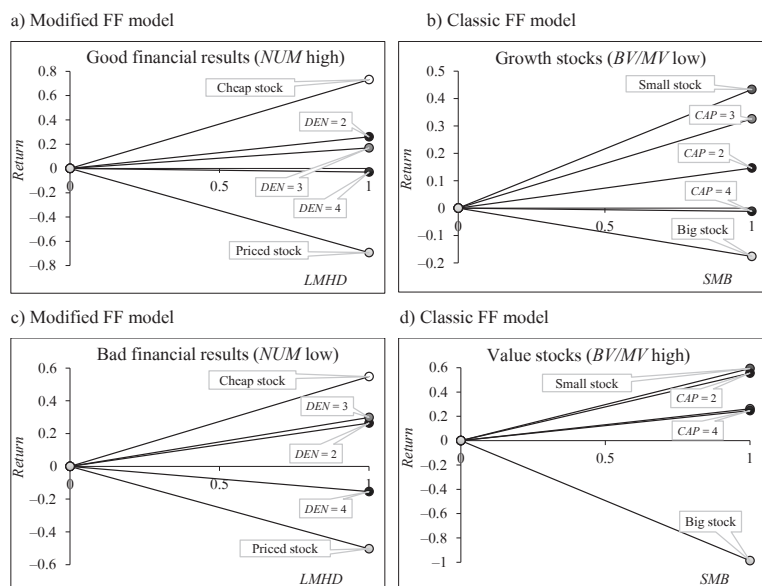
Source: own elaboration.

Figure 4. The impact of *LMHD* and *SMB* loadings on returns for portfolios formed on: a) and
           c) *DEN*, and b) and d) Capitalization

Source: own elaboration.

other hand, investments in small capitalization companies, both value and growth companies (high and low *BV/MV*), demonstrate growing returns for growing *SMB* (see Figures 3b, 4b, and 4d). However, investments in big capitalization companies, both value and growth companies (high and low *BV/MV*), demonstrate decreasing returns for growing *SMB* (see Figures 3d, 4b, and 4d).

Table 7 presents the values of parameters of regressions (10b), and statistics testing the multifactor efficiency of generated portfolios by the classic and modified FF models.

Table 7. Regressions of excess returns on the loadings of the classic
and modified FF model factors

$$r_{it} - RF_t = \gamma_i + \gamma_h \hat{h}_i + \gamma_s \hat{s}_i + \gamma_b \hat{b}_i + \varepsilon_{it}; \quad i = 1, \ldots, 25; t = 1, \ldots, 64$$

Parameter	Modified FF model	Classic FF model
γ_0	−0.09	−0.03
t	−2.89	−0.85
$t(S)$	−2.14	−0.78
$\gamma_{h(HMLN/HML)}$	**0.04**	**0.05**
t	2.82	2.77
$t(S)$	2.82	2.87
$\gamma_{s(LMHD/SMB)}$	**0.03**	−0.01
t	2.09	−0.70
$t(S)$	2.66	−0.76
γ_b	**0.10**	0.04
t	3.42	1.15
$t(S)$	2.74	1.08
GRS-F	3.20	**1.65**
p-value, %	0.07	8.25
$Q^4(F)$	**1.35**	**0.67**
p-value, %	20.42	83.15
R^2_{LL}, %	59.79	70.30

Notes: This table presents estimation results assessing the classic and modified FF models. RF_t is the 91-day Treasury bill rate of return. $\hat{h}_i$, $\hat{s}_i$, $\hat{b}_i$ are the loadings on the model factors. The response variable is excess return on 25 stock portfolios formed in period t. $t(S)$ corresponds to the statistic of Shanken (1992) adjusting for errors-in-variables. GRS-F is the F-statistic of Gibbons et al. (1989) that the generated portfolios are multifactor efficient. $Q^4(F)$ reports F-statistic and its corresponding p-value indicated below for the Shanken test (1985) that the pricing errors in the model are jointly zero. R^2_{LL} is a measure, follows Lettau and Ludvigson (2001), showing the fraction of the cross-sectional variation in average returns that is explained by each model and is calculated as follows: $R^2_{LL} = [\sigma^2_c(\overline{r_i}) - \sigma^2_c(\overline{e_i})] / \sigma^2_c(\overline{r_i})$, where σ^2_c denotes a cross-sectional variance, and variables with bars over them denote time-series averages. The sample period is from 1995 to 2012, 64 Quarters.

Source: own calculations.

In the case of the modified FF model the perceived risk price is multidimensional due to *HMLN*, *LMHD* and market portfolio factors, and amounts to 4, 3 and 10% per quarter,

respectively. However, in the case of the classical FF model the risk is priced only due to the *HML* factor, and amounts to 5% per quarter.

There is no basis to reject the zero hypothesis which presumes that both models generate multifactor efficient portfolios, which is confirmed by the values of the GRS-*F* and $Q^4(F)$ statistics.

Conclusions

This paper presents two ICAPM applications of stocks pricing on the Warsaw Stock Exchange – modified vs. the classic three factor Fama-French model. Both pricing procedures are tested on the basis of 25 portfolios, formed in two directions, according to the algorithm proposed by FF (1993). The adoption of exactly the same testing boundary conditions allows for comparing the information effectiveness of both models.

The results of research can be summarized as follows:

1. The state variable, generating future returns, of the modified FF model is the structure of past financial results in relation to the company value – modelled by *HMLN* and *LMHD* factors.

2. The state variable, generating future returns, of the classic FF model is a change of *BV/MV* in relation to the company value – modelled by *HML* and SMB factors.

3. The structure of past financial results is more widely perceived by investors and (according to the FF (1995)) has a direct impact on returns.

4. The results of pricing simulations by the modified FF model (compared with the classic FF model) are more widely perceived by investors and financial managers, and they are as follows:

   a) If the diversity of financial results increases (*HMLN* increases):
      – buy good financial companies (high *NUM*), the higher returns are for priced companies (big *DEN*),
      – sell bad financial companies (low *NUM*), the higher returns are for priced companies (big *DEN*),

   b) If the diversity of value increases (*LMHD* increases):
      – buy cheap companies (small *DEN*), the higher returns are for good financial companies (high *NUM*),
      – sell priced companies (big *DEN*), the higher returns are for bad financial companies (low *NUM*).

5. In light of the modified FF model the risk price is three-dimensional due to *HMLN*, *LMHD* and market portfolio factors, and amounts to 4, 3 and 10% per quarter, respectively.

6. In light of the classic FF model the risk price is univariate due to *HML*, amounting to 5% per quarter.

7. The results of Gibbons et al. (1989), and Shanken's (1985) tests are similar for both models.

Acknowledgements

References

Balvers, R.J. (2001). *Foundations of Asset Pricing*. West Virginia University.

Carhart, M.M. (1995). *Survivor bias and persistence in mutual fund performance*. Thesis (Ph.D.), Graduate School of Business, University of Chicago.

Cochrane, J. (2001). *Asset Pricing*. New Jersey: Princeton University Press.

Czapkiewicz, A., Wójtowicz, T. (2014). The four-factor asset pricing model on the Polish stock market. *Economic Research-Ekonomska Istraživanja*, *27* (1), 771–783. DOI: 10.1080/1331677X.2014.975518.

Fama, E.F., French, K.R. (1992). The Cross-Section of Expected Stock Returns. *Journal of Finance*, *47* (2), 427–465. DOI: 10.1111/j.1540-6261.1992.tb04398.x.

Fama, E.F., French, K.R. (1993). Common Risk Factors in the Returns on Stock and Bonds. *Journal of Financial Economics*, *33* (1), 3–56. DOI: 10.1016/0304-405X(93)90023-5.

Fama, E.F., French, K.R. (1995). Size and Book-to-Market Factors in Earnings and Returns. *Journal of Finance*, *50* (1), 131–155. DOI: 10.1111/j.1540-6261.1995.tb05169.x.

Fama, E.F., French, K.R. (1996). Multifactor Explanations of Asset Pricing Anomalies. *Journal of Finance*, *51* (1), 55–84. DOI: 10.1111/j.1540-6261.1996.tb05202.x.

Gibbons, M.R., Ross, S.A., Shanken, J. (1989). A Test of the Efficiency of a Given Portfolio. *Econometrica*, *57* (5), 1121–1152.

Kowerski, M. (2008). Trójczynnikowy model Famy i Frencha dla Giełdy Papierów Wartościowych w Warszawie. *Przegląd Statystyczny*, *55* (4), 131–148.

Lettau, M., Ludvigson, S. (2001). Resurrecting the (C) CAPM: A Cross -Sectional Test when Risk Premia Are Time-Varying. *Journal of Political Economy*, 109 (6), 1238–1287.

Lakonishok, J., Shapiro, A.C. (1986). Systematic Risk, Total Risk, and Size as Determinants of Stock Market Returns. *Journal of Banking and Finance*, *10* (1), 115–132.

Merton, R.C. (1973). An Intertemporal Capital Asset Pricing Model. *Econometrica*, *41* (5), 867–888.

Petkova, R. (2006). Do the Fama-French Factors Proxy for Innovations in Predictive Variables? *Journal of Finance*, *61* (2), 581–612. DOI: 10.1111/j.1540-6261.2006.00849.x.

Reinganum, M.R. (1981). A New Empirical Perspective on the CAPM. *Journal of Financial and Quantitative Analysis*, *16* (4), 439–462.

Rosenberg, B., Reid, K., Lanstein R. (1985). Persuasive Evidence of Market Inefficiency. *Journal of Portfolio Management*, *11* (3), 9–16.

Ross, S.A. (1976). The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory*. *13* (3), 341–360.

Stattman, D. (1980). Book Values and Stock Returns, The Chicago MBA. *A Journal of Selected Papers*, *4*, 25–45.

Shanken, J. (1985). Multivariate Tests of the Zero-Beta CAPM. *Journal of Financial Economics*, *14*, 327–348.

Shanken, J. (1992). On the Estimation of Beta-Pricing Models. *The Review of Financial Studies*, *5* (1), 1–33.

Urbański, S. (2011). *Modelowanie równowagi na rynku kapitałowym – weryfikacja empiryczna na przykładzie akcji notowanych na Giełdzie Papierów Wartościowych w Warszawie*. Katowice: Wydawnictwo Uniwersytetu Ekonomicznego w Katowicach.

Urbański, S. (2012). Multifactor Explanations of Returns on the Warsaw Stock Exchange in Light of the ICAPM. *Economic Systems*, *36* (4), 552–570.

Yogo, M. (2006). A Consumption-Based Explanation of Expected Stock Returns. *Journal of Finance*, *61* (2), 539–580.

Zarzecki, D., Byrka-Kita, K., Wiśniewski, T., Kisielewska, M. (2004–2005). Test of the Capital Asset Pricing Model: Polish and Developed Markets Experiences. *Folia Oeconomica Statitiensa*, *11–12* (3–4), 63–84.