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BEHAVIOURAL PRESENT VALUE DEFINED AS FUZZY NUMBER – A NEW APPROACH*

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Abstract

The behavioural present value is defined as a fuzzy number assessed under the impact of chosen behavioural factors. The first formal model turned out to be burdened with some formal defects which are finally corrected in the presented article. In this way a new modified formal model of a behavioural present value is obtained. New model of the behavioural present value is used to explain the phenomenon of market equilibrium on the efficient financial market remaining in the state of financial imbalance. These considerations are illustrated by means of extensive numerical case study.

Keywords: Behavioural finance, present value, fuzzy number, financial equilibrium, market equilibrium

JEL classification: C02, C44, G10

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Introduction

The considerations outlined in this article had their genesis in the acceptance of a view that – without the interest theory – the present value of future cash flow may be imprecise. The natural consequence of this approach is the assessment of the present value (for short PV) using fuzzy numbers.

Fuzzy PV is usually defined as the discounted value of imprecisely estimated future cash flow, which is shown e.g. in (Piasecki, 2013). A different approach is presented in (Piasecki, 2011a) where imprecise estimation of PV was based on the current market price of a financial asset. The lack of precision in the PV estimation was there justified by behavioural premises. Hence, the described PV is called a behavioural present value (for short BPV). The BPV model defined in (Piasecki, 2011a) will be called an old BPV model. Some uses of the old BPV model show its few defects.

The purpose of this paper is to present a new BPV model free of those. Preliminary corrections of the old BPV model were presented in (Piasecki, 2013). In this paper we present the final version of the new BPV model which will be used to explain the paradox of maintaining market equilibrium on a highly efficient financial market.

1. Behavioural present value

Let us consider any financial instrument which is the subject of trade on the highly efficient financial market. The market equilibrium condition is the state of the financial market in which the demand for this financial instrument is equal to its supply.

The market price $\check{C}$ may vary over time. This allows us to talk about the market price trend. The financial equilibrium condition is the state of the financial market in which the market price trend is constant. Then the value of the market price $\check{C}$ is equal to the equilibrium price C_0, determined by a technical or fundamental analysis.

In (Piasecki, 2012) it is shown that PV is the utility function defining the arrangement of the cash flows set. This utility may be of subjective character. Then any cash flow PV depends both on objective and subjective conditions. Let us consider any security, understood as the right to receive future income. The normative finance theory suggests that PV of the analyzed security should be equal to the market price $\check{C}$. On the other hand, the substantially justified equilibrium price C_0 can influence the PV deviation from the observed market price $\check{C}$. This deviation is highly dependent on the investor's susceptibility to internal and external behavioural factors. The natural question here is whether taking into account the behavioural factors impact is necessary in determining the PV.

The accrued market knowledge is the unique basis for determining the substantively justified equilibrium price C_0. In investors' considerations, the equilibrium price plays a role of a synthetic image of knowledge about the financial market. On the highly efficient financial market each investor defines the same value C_0, which is objective in this situation. At the same time, all market players observe the market price $\check{C}$ which is objective in its essence. The knowledge of both of these values is sufficient to rational justification of investment decision-making. In case of

$$\check{C} < C_0 \tag{1}$$

the rationale unambiguously suggests buying the considered financial instrument. This bargain is possible only when an offer for sale is proposed. The natural question here is what were the premises that the investor who sells such a security was driven by. Similarly, for the case

$$\check{C} > C_0 \tag{2}$$

the rationale unambiguously suggests selling the considered financial instrument. The bargain is again possible only when noffer for its purchase is proposed. This raises the question what premises make the investor buy this security. There is only one answer to the above two questions. The alternative of the conditions (1) and (2) describes the financial imbalance which is opposed to the market equilibrium. If the highly efficient financial market is under financial imbalance then the market equilibrium remains due to irrational premises. It shows that each investor's decision is taken under irrational premises. It is obvious that these premises may be of behavioural nature. Thus, taking the behavioural factors' impact into account helps to explain the paradox of maintaining the market equilibrium on the highly efficient market remaining under the influence of financial imbalances.

Let us consider the PV evaluation determined, inter alia, under the impact of behavioural premises. In their essence, behavioural environmental conditions are defined imprecisely. For this reason, the PV deviation from the market price is at risk of imprecision. Each behavioural evaluation is subjective. Subjective assessment of PV is ambiguous. Each of the considered valuation alternatives will be called a potential present value (for short PPV). The set of all PPV will be called a behavioural present value (for short BPV). The dependence of PV on subjective financial factors makes each investor appoint their own version of the BPV. Thus, all further considerations we will conducted for any fixed investor.

2. Interval representation of behavioural present value

The starting point for all further deliberations is to present the BPV as an interval. We begin our reflections on BPV by considering the financial equilibrium case, when the market price $\check{C}$ coincides with the equilibrium price C_0

$$\check{C} = C_0 \tag{3}$$

This equilibrium is momentary. This fact requires that the PPV value is specified, as a number approximating the market price. The assumed scope of the PPV variability is characterized by a specific investor's susceptibility to the influence of behavioural factors. Therefore each investor determines the following values:

– C_{min} PPV lower range assumed under conditions of financial equilibrium,
– C_{max} PPV upper range assumed under conditions of financial equilibrium.

In the case of the financial equilibrium investor must take into account the possibility of declines and increases in quotations. In this situation, the scope of PPV variability satisfies the condition

$$C_{min} < C_0 < C_{max} \tag{4}$$

The numerical interval $[C_{min}, C_{max}]$ is the BPV image for the case of the financial equilibrium.

Further considerations on BPV we lead for the case where the quoted market price $\check{C}$ is arbitrary. It is obvious that BPV should be dependent on the deviation

$$\Delta C = \check{C} - C_0 \tag{5}$$

of market price from the equilibrium price. Then each investor determines the following values:

– $\check{C}_{min}$ PPV lower range assumed for the case of market price $\check{C}$,
– $\check{C}_{max}$ PPV upper range assumed for the case of market price $\check{C}$.

In (Piasecki, 2011) we can find that both of these values are dependent on the number $\alpha \in [0; 1]$, determining the degree of the investor's susceptibility to changes. This degree value informs us about the intensity of the impact in which the deviation ΔC influences the investor's beliefs. This means that the value

$$\zeta = 1 - \alpha \tag{6}$$

describes the degree of the cognitive conservatism (Edwards, 1968) of the phenomenon impact. This phenomenon is taken into account in many behavioural models of the financial market.

The discussion on this subject can be found, for example, in (Barberis at al., 1998), where the value $\zeta \in [0; 1]$ is called the sentiment index. This proposal has been widely accepted in the literature. Thus it is appropriate to apply this concept in a new formal model of BPV. In this situation, we assume that the values $\check{C}_{min}$ and $\check{C}_{max}$ depend on the value ζ of sentiment index which is an individual investor's characteristic founded on the behavioural basis. In this way the proposed model will be more compatible with the subject literature than the previous model.

The investor determines the PPV lower range $\check{C}_{min}$ as the weighted average of the lower range C_{min} and the corrected lower range $C_{min} + \Delta C$. The weight of the lower range C_{min} is equal to the value ζ of the investor's sentiment index. In determining the PPV lower range $\check{C}_{min}$ the investor must take into account the fact that this range is always less or equal than the current market price $\check{C}$. We have here

$$\check{C}_{min} = \min\left\{(1-\zeta)\left(C_{min} + \Delta C\right) + \zeta \times C_{min}, \check{C}\right\} = \min\left\{C_{min} + (1-\zeta)\left(\check{C} - C_0\right), \check{C}\right\} \tag{7}$$

The investor determines the PPV upper range $\check{C}_{max}$, as the weighted average of the upper range C_{max} and the corrected upper range $C_{max} + \Delta C$. The weight of the upper range C_{max} is equal of the degree to the value ζ of the investor's sentiment index. In determining the PPV upper range $\check{C}_{max}$ the investor must take into account the fact that this range is always greater or equal than the current market price $\check{C}$. We have here

$$\check{C}_{max} = \max\left\{(1-\zeta)\left(C_{max} + \Delta C\right) + \zeta \times C_{max}, \check{C}\right\} = \max\left\{C_{max} + (1-\zeta)\left(\check{C} - C_0\right), \check{C}\right\} \tag{8}$$

It is easy to note that in case of

$$\check{C} \le \frac{C_{min} - (1-\zeta)C_0}{\zeta} \tag{9}$$

the lower range PPV $\check{C}_{min}$ is equal to the market price $\check{C}$. This means that when facing a large surplus of the equilibrium price C_0 over the market price $\check{C}$, the analyzed BPV model uniquely identifies the considered financial instrument as undervalued. Then the possibility of the quotation downtrend is excluded. Also when

$$\check{C} \ge \frac{C_{max} - (1-\zeta)C_0}{\zeta} \tag{10}$$

that when facing a large surplus of the market price $\check{C}$ over the equilibrium price C_0, the BPV model excludes the possibility of a rise in the quotation. Then the upper range PPV $\check{C}_{max}$ is equal to the market price $\check{C}$.

Hence we conclude that only in the case of significant deviations of the market price $\check{C}$ from the equilibrium price C_0, rationale is the only reason for investment decision-making. The scope of behavioural reasons' impact is determined by the following condition

$$\frac{C_{min}-(1-\zeta)C_0}{\zeta}<\check{C}<\frac{C_{max}-(1-\zeta)C_0}{\zeta} \tag{11}$$

Finally, for each investor we can determine a specific scope of the PPV variability

$$(\check{C})=\left[\check{C}_{min},\check{C}_{max}\right]=\begin{cases}\left[\check{C},C_{max}+(1-\zeta)(\check{C}-C_0)\right] & \text{for } (9)\\ \left[C_{min}+(1-\zeta)(\check{C}-C_0),C_{max}+(1-\zeta)(\check{C}-C_0)\right] & \text{for } (11)\\ \left[C_{min}+(1-\zeta)(\check{C}-C_0),\check{C}\right] & \text{for } (10)\end{cases} \tag{12}$$

forming the interval representation of BPV. In this way, we describe the impact of market conditions on the investor's beliefs. If the fixed PPV belongs to an interval representation of BPV then it is called a forecasted PPV. The position of the forecasted scope of the PPV variation depends on the following variables:

- $\check{C}$ observed market price,
- C_0 substantially justified equilibrium price,
- $\check{C}_{min}$ PPV lower range assumed for the case of market price $\check{C}$;
- $\check{C}_{max}$ PPV upper range assumed for the case of market price $\check{C}$.
- ζ sentiment index.

In Section 2 we point out that the observed market price $\check{C}$ and the substantially justified equilibrium price C_0 are objective in nature. Assumed under the financial equilibrium, the PPV scope and the sentiment index are dependent on the investor's susceptibility to behavioural impulses. Thus, individual investors will be characterized by different values of these variables. For BPV models built in this paper, the vector ($\check{C}$, C_0, C_{min}, C_{max}, ζ) will play a part of the parameters vector. For the fixed investor only the value of an observable market price $\check{C}$ is variable. Then the values of other parameters are constant. Therefore, to simplify, the description of the BPV model will be explicitly parameterized only by the market price $\check{C}$.

3. Fuzzy representation of behavioural present value

The interval image of BPV treats all acceptable PPV values equally. On the other hand, we can suppose that the investor accepts PPV more, when it is nearer to the market price. It implies that individual PPVs differ in their degrees of acceptance. We see that the interval model of BPV describes the complexity of the behavioural effects in an insufficient way. This makes it necessary to build a BPV model taking into account the variability of individual PPV importance. This leads directly to creating a fuzzy image of BPV. Fuzzy representation of BPV boils down to determining its membership function assigning acceptance degree to each PPV.

We keep our further discussion for a given value ($\check{C}$, C_0, C_{min}, C_{max}, ζ) of the parameters vector. Then the fuzzy BPV model is defined by means of its membership function $\mu\left(\cdot\middle|\check{C}\right): \mathbb{R} \to [0;1]$, which determines the acceptance degree of each PPV. We can assume that:

- market price of $\check{C}$ is fully acceptable PPV,
- PPV approaching to the market price $\check{C}$ does not cause decrease in the acceptance degree of PPV,
- all unforecasted PPVs are not acceptable.

For this reason, any BPV should be a fuzzy number (Dubois, Prade, 1979) determined by its membership function $\mu\left(\cdot\middle|\check{C}\right): \mathbb{R} \to [0;1]$ fulfilling the conditions

$$\mu\left(\check{C}\middle|\check{C}\right) = 1 \tag{13}$$

$$\forall x, y, z \in \mathbb{R}: x \le y \le z \Rightarrow \mu\left(y\middle|\check{C}\right) \ge min\left\{\mu\left(x\middle|\check{C}\right), \mu\left(z\middle|\check{C}\right)\right\} \tag{14}$$

$$\forall x \notin \mathbb{Z}\left(\check{C}\right): \quad \mu\left(x\middle|\check{C}\right) = 0 \tag{15}$$

In the considered case the interval $\mathbb{Z}\left(\check{C}\right) = \left[\check{C}_{min}; \check{C}_{max}\right]$ of PPV variability is determined explicitly. To simplify further consideration, the PPV variability will be standardized. We will use the standardization function $\beta: \mathbb{R} \to [-1;1]$ given by the identity

$$\beta(x) = \begin{cases} -1 & x < \check{C}_{min} \\ \dfrac{x - \check{C}}{\check{C} - \check{C}_{min}} & \check{C} \ne \check{C}_{min} \le x \le \check{C} \\ \dfrac{x - \check{C}}{\check{C}_{max} - \check{C}} & \check{C} \le x \le \check{C}_{max} \ne \check{C} \\ 1 & \check{C}_{max} < x \end{cases} \tag{16}$$

Let us consider the forecasted PPV equal to $x \in \mathbb{Z}\left(\check{C}\right)$. Then the value $|\beta(x)|$ determines. For this reason we define degree of PPV similarity to market price $\check{C}$ by the identity

$$\gamma(x) = 1 - \left|\beta(x)\right| \tag{17}$$

The similarity degree γ defined above determines simultaneously the relative distance between PPV and the limit of variability scope.

In addition, the BPV membership function can be represented as

$$\mu\left(x|\check{C}\right) = \kappa\left(\beta\left(x\right)|\check{C}\right) \tag{18}$$

where $\kappa\left(\cdot|\check{C}\right): \mathbb{R} \to [0;1]$ is the membership function of the standardized BPV model which is given as a fuzzy number. Moreover, from the conditions (13), (16) and (18) for any market price $\check{C}$ we obtain

$$\kappa\left(0|\check{C}\right) = \kappa\left(\beta\left(\check{C}\right)|\check{C}\right) = \mu\left(\check{C}|\check{C}\right) = 1 \tag{19}$$

Let us note also that if the PPV forecast is represented in a standardized BPV by the number β, then – in accordance with (17) – the degree of its similarity to the market price $\check{C}$ is equal to

$$\gamma = 1 - |\beta| \tag{20}$$

According to the results of the discussion held in (Buckley, 1987), (Gutierrez, 1989), (Kuchta, 2000) and (Lesage, 2001) further we will assume that for the case of the financial equilibrium (3) the standardized fuzzy BPV model will be formulated as a triangular number. The membership function of this number takes the form

$$\kappa_0\left(\beta\right) = \kappa\left(\beta|C_0\right) = 1 - |\beta| = \gamma \tag{21}$$

This function describes the equilibrated distribution of acceptance. This distribution will be regarded as a reference point for determining the acceptance distribution for the case of financial imbalance (1) or (2).

The second reference point for determining any acceptance distribution will be rational forecast changes in the quotation. It is known that:

– if the imbalance condition (1) is fulfilled, then rational premises exclude the decrease in quotation,
– if the imbalance condition (2) is fulfilled, then rational premises exclude the increase in quotation,
– if the equilibrium condition (3) is fulfilled, then any future quotation cannot be excluded.

Thus, the rational forecast may be described by its characteristic function $\Theta\left(\cdot|\check{C}\right): \mathbb{R} \to \{0;1\}$ given by the identity

$$\Theta\left(\beta|\check{C}\right) = \begin{cases} 0 & \beta \times \Delta C > 0 \\ 1 & \beta \times \Delta C \leq 0 \end{cases} \tag{22}$$

The characteristic function of rational forecast will be briefly called a rationale characteristic. For any market price $\check{C}$ the investor assesses the acceptance degree as a weighted average of the rationale characteristic and the equilibrated distribution of acceptance (21). In (Piasecki, 2011a) the old BPV model was built under the assumption that the rationale characteristic weight is directly proportional to the product $\gamma \times |\Delta C|$. Such weight acceptance implies that the importance of the rationale characteristic was dependent on the currency used for the security assessment (Piasecki, 2013). With high nominal market prices, the importance of the acceptance distribution had depreciated. All this caused that the established accounting convention could have a significant impact on the final form of the standardized fuzzy BPV model. This is contrary to the practice of economic modelling.

Therefore, in our new model, we propose to replace the absolute deviation ΔC by the relative distance

$$\delta C = \frac{|\Delta C|}{\check{C}} \tag{23}$$

of the market price $\check{C}$ from the equilibrium price C_0. In this way the importance of the rationale characteristic will be independent on the currency used for the security assessment.

Then, in agreement with assumptions given in (Piasecki, 2011a), the weights are appointed that the influence of the rationale characteristic increases with the increase in the relative distance δC and with the increase in the degree γ of PPV similarity to the market price. Therefore, without generality loss we can assume that the weight of the rationale characteristic is directly proportional to the product $\gamma \times \delta C$. This weight will be free of defects described above. Then the acceptance distribution is described by the identity

$$\kappa\left(\beta|\check{C}\right) = \frac{1}{1+\gamma\times\delta C}\times\kappa_0(\beta) + \frac{\gamma\times\delta C}{1+\gamma\times\delta C}\times\Theta\left(\beta|\check{C}\right) = \frac{\left(1-|\beta|\right)\times\left(1+\delta C\times\Theta\left(\beta|\check{C}\right)\right)}{1+\left(1-|\beta|\right)\times\delta C} \tag{24}$$

The membership function described above specifies the standardized BPV model. Next, using the identities (16) and (19) we assess BPV as a fuzzy number given by its membership function $\mu\left(\cdot|\check{C}\right):\mathbb{R}\to[0;1]$ as follows

$$\mu\left(x|\check{C}\right) = \begin{cases} \kappa\left(\dfrac{x-\check{C}}{\check{C}-\check{C}_{min}}\middle|\check{C}\right) & \check{C}\neq\check{C}_{min}\leq x\leq\check{C} \\ \kappa\left(\dfrac{x-\check{C}}{\check{C}_{max}-\check{C}}\middle|\check{C}\right) & \check{C}\leq x\leq\check{C}_{max}\neq\check{C} \\ 0 & x\notin\left[\check{C}_{min};\check{C}_{max}\right] \end{cases} \tag{25}$$

In the end, we obtain

– for $\Delta C \le 0$:

$$\mu\left(x|\check{C}\right)=\begin{cases}\dfrac{x-\check{C}_{min}}{\check{C}-\check{C}_{min}+\left(x-\check{C}_{min}\right)\times\delta C} & \check{C}\ne\check{C}_{min}\le x\le\check{C}\\ \dfrac{\left(\check{C}_{max}-x\right)\times\left(1+\delta C\right)}{\check{C}_{max}-\check{C}+\left(\check{C}_{max}-x\right)\times\delta C} & \check{C}\le x\le\check{C}_{max}\ne\check{C}\\ 0 & x\notin\left[\check{C}_{min};\check{C}_{max}\right]\end{cases} \tag{26}$$

– for $\Delta C > 0$:

$$\mu\left(x|\check{C}\right)=\begin{cases}\dfrac{\left(x-\check{C}_{min}\right)\times\left(1+\delta C\right)}{\check{C}-\check{C}_{min}+\left(x-\check{C}_{min}\right)\times\delta C} & \check{C}\ne\check{C}_{min}\le x\le\check{C}\\ \dfrac{\check{C}_{max}-x}{\check{C}_{max}-\check{C}+\left(\check{C}_{max}-x\right)\times\delta C} & \check{C}\le x\le\check{C}_{max}\ne\check{C}\\ 0 & x\notin\left[\check{C}_{min};\check{C}_{max}\right]\end{cases} \tag{27}$$

Due to (5), (7), (8) and (23), for any vector ($\check{C}$, C_0, C_{min}, C_{max}, ζ) the vector ($\check{C}$, $\check{C}_{min}$, $\check{C}_{max}$, δC) is assessed unambiguously. Thus each membership function $\mu\left(\cdot|\check{C}\right):\mathbb{R}\to[0;1]$ is determined uniquely.

4. Paradox explanation

In this section we will apply the BPV model for the explanation of the mechanism of maintaining the equilibrium between demand and supply on a highly efficient financial market. To each market price $\check{C}\in\mathbb{R}^+$, we can assign the value $\xi(\check{C})$ of PPV average which is defined by the identity

$$\xi\left(\check{C}\right)=\left(\int_{-\infty}^{+\infty}\mu\left(x\,|\,\check{C}\right)dx\right)^{-1}\times\int_{-\infty}^{+\infty}x\times\mu\left(x\,|\,\check{C}\right)dx \tag{28}$$

The average PPV $\xi(\check{C})$ determined for a given investor can be interpreted as their subjective PV evaluation. The objective assessment of the present value is identified with the equilibrium price C_0 which is only one of the reasons determining the subjective PV evaluation. In this situation, from the investor's point-view the average PPV $\xi(\check{C})$ is more reliable information than the equilibrium price C_0. This causes that the investor's decisions are dependent on the relationship between the market price $\check{C}$ and the average PPV $\xi(\check{C})$.

If the condition

$$\check{C} < \xi(\check{C}) \tag{29}$$

is fulfilled then the investor acknowledges that the financial market has undervalued the considered security. Therefore they expect a fast increase in the market price of this financial instrument. This expectation justifies the notification of an offer to buy the considered security. The demand value depends on the investor's strategy and their financial resources. If the condition

$$\check{C} > \xi(\check{C}) \tag{30}$$

is fulfilled then the investor recognizes that the financial market has devalued the considered security. Therefore, the investor expects a fast decrease in the market price of this financial instrument. This expectation justifies the notification of offer to sell the considered security. The supply value is limited from above by the value of the security held by the investor.

Let us note here, that:

– subjective condition (30) replaces the objective condition (1),

– subjective condition (31) replaces the objective condition (2).

It is obvious that on the effective financial market the conditions (1) and (2) could not be fulfilled at the same time.

On the other hand, in the highly efficient financial market each investor defines its BPV in a specific way. It causes the investors satisfying the condition (29), and investors satisfying the condition (30) to be found both on the effective financial market. In this situation the supply offered by the investors satisfying the condition (30) meets the demand caused by the investors satisfying the condition (29). If sales reduction or purchase reduction does not take place, then the observed market price of $\check{C}$ is the price of market equilibrium in the sense given by microeconomics. This price depends much on the investors' susceptibility to the behavioural environment impact.

However, the financial equilibrium price C_0 is determined by a technical or fundamental analysis. It means that on the efficient financial market we can observe the financial equilibrium price C_0 and the market equilibrium price $\check{C}$ whose values may differ. The conclusions fully explain the paradox described at the beginning that is the paradox of maintaining the market equilibrium on the highly efficient financial market.

Due to the detailed variability analysis of the acceptance distribution we can say that:

– the condition (9) is sufficient for (29) one,

– the condition (10) is sufficient for (30) one.

In this situation only the case (11) requires a detailed analysis.

5. Case study

Numerical complexity of the BPV model leads us to the study of its properties through computational experiments. In the first step, let us reconsider the numerical case study considered in (Piasecki, 2011) and (Siwek, 2015). The main goal of it will be a demonstration event in which offers to buy and offers for sale appear at the same time.

We consider a financial instrument characterized by the equilibrium price C_0 = 100. Investors Albert and Benjamin are interested in participating in trading this security.

Albert's susceptibility to the impact of internal and external behavioural factors is described by the values:

– $C^A{}_{min}$ = 95 lower bound of PPV assumed under conditions of financial equilibrium;
– $C^A{}_{max}$ = 110 upper bound of PPV assumed under conditions of financial equilibrium.

Benjamin's susceptibility to the impact of internal and external behavioural factors is described by:

– $C^B{}_{min}$ = 95 lower bound of PPV assumed under conditions of financial equilibrium;
– $C^B{}_{max}$ = 105 upper bound of PPV assumed under conditions of financial equilibrium.

The comparison of both of these scopes shows that, for the case of the financial equilibrium (3), Albert's market expectations are more optimistic than Benjamin's market expectations.

Albert's sentiment index is equal to ζ_A = 0.8. Analogous Benjamin's sentiment index is equal to ζ_B = 0.2. It is evident here that Benjamin's market reaction is stronger than Albert's market reaction.

It is easy to see that each investor has one advantage and one disadvantage. The advantages are: more optimistic Albert's market expectation and Benjamin's stronger market reaction. Disadvantages are: more pessimistic Benjamin's market expectations and weaker Albert's market reaction.

In agreement with (11), the scope of behavioural reasons' impact on Albert is determined by the following inequalities

$$93.75 < \check{C} < 112.50 \tag{31}$$

In analogous way, we obtain the scope of behavioural reasons' impact on Benjamin

$$50 < \check{C} < 125 \tag{32}$$

As it is easy to note Albert and Benjamin differ in their scopes of behavioural premise impact. Nevertheless, we can say that the behavioural factors' impact on the investment relationships between Albert and Benjamin is limited to the price range (32). Within this range we will be looking for an area where both conditions (29) and (30) are fulfilled simultaneously.

Searching for the behavioural factors' impact on the market equilibrium in the financial market, we define by the identity (29) the functions:

– Albert's average PPV $\xi^A : \mathbb{R} \to \mathbb{R}$,
– Benjamin's average PPV $\xi^B : \mathbb{R} \to \mathbb{R}$.

If the condition

$$\xi^A\left(\check{C}\right) > \check{C} > \xi^B\left(\check{C}\right) \tag{33}$$

is fulfilled, then for the market price $\check{C}$ Albert declares the demand which is offset by the supply offered by Benjamin. If the condition

$$\xi^A\left(\check{C}\right) < \check{C} < \xi^B\left(\check{C}\right) \tag{34}$$

is fulfilled, then for the market price $\check{C}$ Albert offers the supply which meets with the demand declared by Benjamin.

All the considered average PPV values are calculated in the MATLAB software environment. The graphs of both averages PPV are presented on Figure 1

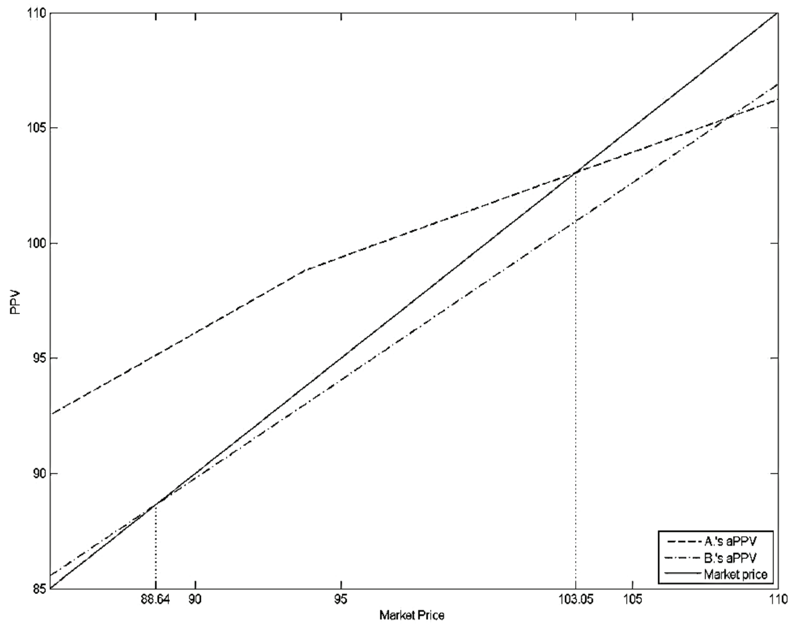

Figure 1. Albert's average PPV (A.'s aPPV) $\xi^A(\check{C})$ and Benjamin's average PPV (B.'s aPPV) $\xi^B(\check{C})$ for the increase of the market price $\check{C}$ from 85 to 110.

Source: own elaboration.

We see on the graph that if the market price $\check{C} \in [88,64; 103,05]$ then offered by Benjamin, the supply meets the demand declared by Albert.

In this way we have shown the possibility of applying the BPV model explaining the phenomenon of maintaining market equilibrium under the conditions of financial imbalance. The BPV model may be used to explain the market paradox of conducting contradictory transactions under the same set of rational premises.

However, let us note that this observation applies only to the investors present at the moment on the financial market. In the proposed model, premises which pushed the investor to enter the considered financial market are not taken into account.

Conclusions

In this paper we formulated a formal BPV model only. The next step should be empirical research dedicated to the problem of estimating the parameters of the presented model.

BPV may be used not only to explain the considered above financial market paradox. This model may be applied wherever the fuzzy evaluation of PV is used (e.g. Boussabaine, Elhag, 1999; Chiu, Park, 1994; Fang Yong et al., 2008; Huang, 2007; Piasecki 2011b, 2013, 2014 and Haifeng et al., 2012).

Using BPV we can define the return rate as a fuzzy probabilistic set which is at the composition of non-Knightian uncertainty risk and imprecision risk (Piasecki, 2011b). In this way we can simultaneously take into account the behavioral and empirical circumstances to make investment decisions.

Summing up, the use of fuzzy evaluation BPV significantly increases the possibility of the analysis of financial markets. This is a highly advantageous feature of the proposed model since it brings the possibility of real applications.

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