APPLICATION OF MATRIX EQUATIONS IN THE AIDA METHOD

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Abstract: The article presents the problem of a decision-making process based on the method known as the analysis of interconnected decision areas (AIDA). Author described the basic assumptions of the AIDA method and the classic method of its implementation with the usage of a decision tree. Crucially, the new and innovative improvement in development of the AIDA method is connected with the replacement of the decision tree by the matrix equation to speed up the cost assessment of decision variants.

Keywords: decision-making, decisive problem, matrix, decision area, elementary decision, decision trees.

JEL: C10, H12.

1 Introduction

Decision-making process is an intrinsic element of enterprise management that constitutes its procedural and technological feature with multiple economic and psychosocial determinants. Making decisions can be considered in two connotations (Targalski, 1986, p.194):

- extensive – as a complex process that consists of registration and evaluation of information, identification of a decision problem and application of an adopted selection criteria, determination of a decision-making task, and registration of an information on its implementation,
- restricted – as a particular step of a decision-making process that represents a conscious act of will of a decision-maker, causing actually non-randomized choice of a specific variant from a collection of possible variants, when solving a decision-making problem.

A decision-maker has to make a decision using one of the decision-making models elaborated by management sciences: (a) rational, (b) procedural, (c) comprehensive, (d) intuitive, and (e) visionary, which allows for the identification of possible decision variants in three cross-sections (Kowalczyk and Roszyk-Kowalska 2016, p.59):

- maximization of the effect intended,
- minimization of the effect intended,

- the achievement of the effect within a specified range.

Irrespective of the chosen model, the decision-making process can be divided into three basic stages:

- first stage – recognition of a problem,
- second stage – development of solution variants,
- third stage – selection of decision.

The results of rudimentary steps of the decision-making process are obtained as a consequence of applying a specific method that supports the decision-making process. This includes (Trzaskalik, 2014, pp.241-249):

- Additive methods:
 - simple additive weighting method (SAW),
 - fuzzy simple additive weighing method (FSAW),
 - simple multi-attribute ranking technique (SMART),
 - simple multi-attribute ranking technique exploiting ranks (SMARTER),
- Analytical hierarchy method and similar methods:
 - analytical hierarchy process (AHP),
 - fuzzy analytic hierarchy process (F-AHP),
 - analytic network process (ANP),
 - fuzzy analytic network process (F-ANP),
 - Ratio Estimation in Magnitudes or deci-Bells to Rate Alternatives which are Non-Dominated (REMBRANDT),

- measuring attractiveness by a categorical based evaluation technique (MACBETH),
- verbal decision analysis (VDA),
- elimination et choix traduisant la realia (ELECTRE),
- preference ranking organization method for enrichment evaluations (PROMETHEE),
- methods of using reference points:
 - the technique for order preference by similarity to ideal solution (TOPSIS),
 - fuzzy technique for order preference by similarity to ideal solution (F-TOPSIS)
 - visekrterijumska optimizacija i kompromisno resenje (VIKTOR).

An actual use of the appropriate method supporting a decision-making process depends on the type of a decision (operational, tactical, strategical), type of data characterizing the decision areas and decision variants within such areas (data: quantitative, qualitative), decision complexity, number of selection criteria (decision areas), time for a decision making, and so on. One of the methods supporting a decision-making process based on a simple procedure of universal character is the analysis of interconnected decision areas (AIDA).

Author present a classic approach for implementing the method using a decision tree, in order to visualize decision variants. In the author opinion, the use of a decision tree lengthens the process of the method implementation. Therefore, aim of author to present an alternative way of evaluating decision variants, using a matrix equation that in consequence reduces significantly the time required to obtain costs assessment of the decision-making variant.

1 Method of interconnected decision areas

The AIDA method developed by J. Luckman is a technique supporting the decision-making process. The application of the method allows to formulate the model of a decision problem and generate cost assessment¹ for satisfactory decisions solving the

given decision problem for the adopted cross-section:

- maximization of costs' assessment,
- minimization of costs' assessment,
- maintenance of costs' assessment within a range.

The presented computation example and the description of the classic procedure for solving a decision problem using the AIDA method were cited after Krupa and Ostrowska (2012, pp.25-31).

The decision problem is modeled by disjunctive decision areas and their characteristics. Every decision area is marked with the symbol D_i , and the symbol d_{ji} is the element of the area, accordingly for [j-th] element of such area.

The analysis area of a given decision problem D is represented in the form $D_1 \times D_2 \times \dots \times D_m$. or in the form of vectors collection $\{<d_{j1}, d_{j2}, \dots, d_{jm}>\}$, assuming that $d_{ji} \in D_i$ and that the supremacy of any decision area $|D_i|$ is determined as a finite measure. The solution for a decision problem is to define a set of D_i decision areas and establish and evaluate the correct vectors $<d_{j1}, d_{j2}, \dots, d_{jm}>$ in the decision tree (Fig. 1):

The essence of the AIDA method comes to:

- building the model of the decision problem that is related to understood as
 - separation of decision areas and quantification of elementary decisions,
 - marking up pairs of elementary decisions being in contradictory relations,
 - determination of significance weights² and decision areas D_i on a percentage scale and significance weight v_{ji} (costs to sum 1 in each decision area of D_i) for elementary d_{ji} decisions on the scale (0..1),
- creation of a collection of acceptable decisions that do not contain pairs of elementary decisions in a contradictory relation,
- making a choice and taking up a decision that is associated with
 - the execution of costs assessment for all appropriately created decisions (without contra-

¹ The cost assessment of the solution to the decision problem is determined based on the sum of the products of the significance of the decision area and the costs of the elementary decision for all decision areas in the decision problem.

² The significance of the decision area means the degree of its significance in relation to other recognized decision areas.

- dictory associations) and organize them in a descending order of costs;
- analysis of obtained solutions to the problem, selection of groups of the anticipated decision

variants, selection of one of such variant, and implementation of the established decision; and (c) analysis of the consequences of the taken decision.

Figure 1. Decision variant model (elementary decisions d_{21} , d_{32} , and d_{23} are linked with the dashed line) on the model of decision-making process (Source: Krupa and Ostrowska, 2012, p.27)

The structure of a graphical model of the decision area starts with the elimination of decision areas that contain only one elementary decision. Elementary decisions in a given decision area should be alternative to each other, only one elementary decision from each decision area collection on solution. This is because the elementary decisions of one decision area cannot be carried out in conjunction with elementary decisions that constitute a solution to another decision area.

In this case, a contradiction arises in elementary decisions and develops pairs in the decision-making model, which are drawn with an edge. It should be

noted at this point that contradictory relationships occur in elementary decisions that allow to subsistence of such a pair when resolving the decision-making problem, provided, however, that additional costs of implementing the considered solution shall be sustained. Nevertheless, these cases do not affect a possibility to apply the matrix equation for the determination of the costs of decision variants, although they are not considered in the article.

Percentage limits evaluating the significance of V_i decision areas and weight v_{ji} of all elementary decisions in all decision areas, the sum of which in each decision area is equal to 1, are given in Table 1.

Table 1. Values of the significance of decision areas as well as elementary decisions of the decision problem model presented in Fig. 1 (Source: Krupa and Ostrowska, 2012, p.28)

$V_1 = 20$	$V_2 = 30$	$V_3 = 50$	$\sum V_i = 100$
$v_{11} = 0,75$	$v_{12} = 0,50$	$v_{13} = 0,40$	
$v_{21} = 0,25$	$v_{22} = 0,10$	$v_{23} = 0,30$	
-	$v_{32} = 0,40$	$v_{33} = 0,30$	
$\sum v_{j1} = 1$	$\sum v_{j2} = 1$	$\sum v_{j3} = 1$	

Cost estimation Q of a single D_i decision is calculated as the sum of the products of the weights of importance (materiality) of decision areas and weights of the significance of elementary decisions from the corresponding decision areas according to the formula (1):

$$Q = \sum V_i * v_{ji} \quad (1)$$

where:

V_i – is the significance weight of decision area D_i on the open scale (0..100)%,

v_{ji} – is the significance weight of the elementary d_{ji} decision on the open scale (0..1).

Q is the significance weight of a single decision (decision variant) $\langle \{d_{21}\}, \{d_{32}\}, \{d_{23}\} \rangle$:

$$Q = V_1 \times v_{21} + V_2 \times v_{32} + V_3 \times v_{23} = 20 \times 0.25 + 30 \times 0.40 + 50 \times 0.30 = 32$$

In the considered example, actually seven decisions are possible, for which costs' estimation (assessment) of the decision can be calculated (Table 2).

Table 2. Decision significance weights for an example of the decision problem of Figure 1
(Source: Krupa and Ostrowska, 2012, p.27)

Decision No	Decision	Decision's significance weight
1	$\langle \{d_{11}\}, \{d_{12}\}, \{d_{13}\} \rangle$	$Q_1 = 20 \times 0.75 + 30 \times 0.50 + 50 \times 0.40 = 50$
2	$\langle \{d_{11}\} \{d_{12}\} \{d_{33}\} \rangle$	$Q_2 = 20 \times 0.75 + 30 \times 0.50 + 50 \times 0.30 = 45$
3	$\langle \{d_{21}\} \{d_{22}\} \{d_{23}\} \rangle$	$Q_3 = 20 \times 0.25 + 30 \times 0.10 + 50 \times 0.30 = 23$
4	$\langle \{d_{21}\} \{d_{32}\} \{d_{13}\} \rangle$	$Q_4 = 20 \times 0.25 + 30 \times 0.40 + 50 \times 0.40 = 37$
5	$\langle \{d_{21}\} \{d_{32}\} \{d_{23}\} \rangle$	$Q_5 = 20 \times 0.25 + 30 \times 0.40 + 50 \times 0.30 = 32$
6	$\langle \{d_{11}\} \{d_{22}\} \{d_{23}\} \rangle$	$Q_6 = 20 \times 0.75 + 30 \times 0.10 + 50 \times 0.30 = 33$
7	$\langle \{d_{11}\} \{d_{22}\} \{d_{33}\} \rangle$	$Q_7 = 20 \times 0.75 + 30 \times 0.10 + 50 \times 0.30 = 33$

The procedure of creating decision variants is based on the decomposition of the graphical model in the given decision area. Decomposition consists of the systematic separation of variants that are stable internally. The term stable internally stands for a collection of elements aggregated for all decisions from all decision areas that meet the following two conditions:

- the collection contains as many elementary decisions as decision areas,
- the collection does not contain pairs of contradictory decisions.

For instance, for a decision problem defined in Fig. 1, internally stable decisions are from cluster 7 $\{d_{11}, d_{22}, d_{33}\}$ or decision 1 $\{d_{11}, d_{12}, d_{13}\}$.

Establishment of the decision variants (Fig. 2) can apply the following strategies:

- a) weight of each decision area is determined,
- b) decision areas are structured in accordance with the decreasing order of their weights,

c) decision areas are split into as many groups of collections as imposed by the weight of the largest decision area,

d) the next vertex of the decision tree is enacted and the outgoing edges are entered, to which appropriate groups of decision areas are allocated.

When establishing groups of decision areas, it should be assumed that they should not contain decisions alternative to decision against to which the split of collections takes place when the weight of any decision area equals 0. Therefore, the given decision area is eliminated from the splitting process and marked up as eliminated decision area (EOD). If the weight of all decision areas in a certain group is equal to 1, a given group is a variant of an internally stable collection of decisions and is marked up as SWOD (internally stable decision area [ISDA]).

Operations (a)–(d) are repeated as long and entire collection that consists exclusively of groups marked up as EOD and SWOD (ISDA). Groups marked up as SWOD (ISDA) constitute the collection of all

possible decision variants, meeting the criteria that no variant contains a pair of alternative decisions.

The implementation of operations (a)–(d) is illustrated in Fig. 2. In the decision tree, the ISDA decision variants are marked with gray.

Figure 2. Decision tree for the problem presented in Figure 1
(Source: Krupa and Ostrowska, 2012, p.29)

The decomposition procedure of a decision tree specifies distinctly the procedure that should be carried out in order to obtain decision variants (tuples³ of elementary decisions).

2 The proposal to modify the determination of costs assessment in the AIDA method

Resolving of the decision problem requires determination of all stable tuples for elementary decisions and calculation of relevant cost estimates. The classic procedure for the application of the AIDA method uses for such a purpose a decision tree that graphically illustrates all the possible decision variants, solving the problem defined by question. Im-

portantly, the use of the decision tree is advantageous when there are multiple contradictory pairs of elementary decisions. Thus, branches of decision tree should not be developed when such contradictions arise. In the case of a small number of contradictions or when such pairs do not exist, the decision tree is extremely complicated, which makes the visualization of the results difficult and requires a lot of time for its establishment.

The author proposal reduces significantly the time needed for generation of all stable decision-making variants and performance of costs' assessment for all acceptable solutions in the case of a given decision problem that is connected with the replacement of the decision tree with the matrix equation. In order to present the decision problem in the form of a matrix equation, decision areas should be written in the collection format.

³ Tuple – a solution to the decision problem containing the number of elementary decisions v_{ji} equal to the number of decision areas of D_i (one elementary decision from each decision area). Tuples are determined based on a Cartesian product for all elementary decisions from the entire decision areas.

Therefore, the decision problem takes the following form (2):

$$\begin{aligned} D_i &= \{d_{1,i}; d_{2,i}; \dots, d_{m,i}\} \\ D_{i+1} &= \{d_{1,i+1}; d_{2,i+1}; \dots, d_{m,i+1}\} \\ \dots &= \{\dots, \dots, \dots, \dots\} \\ D_n &= \{d_{1,n}; d_{2,n}; \dots, d_{m,n}\} \end{aligned} \quad (2)$$

where:

n – is the n -th decision area,

m – is the m -th elementary decision in the considered decision area.

Determination of record of all decision areas in the notation of collections leads to a necessity to indicate all tuples, which can be the solution to a given decision problem. Therefore, from the new collection of such tuples, their pairs with contradictory characteristics are eliminated. Consequently, the catalog of stable decisions, solving the considered decision problem, is created.

The evaluated collection of tuples can be saved in the matrix form, where the columns represent successive decision areas and the rows contain stable decisions (Table 3).

Table 3. The matrix of possible solutions to a decision problem presented in Figure 1
(Source: own elaboration)

	$d_{m,1}$	$d_{m,2}$	$d_{m,3}$
Decision 1	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$
Decision 2	$d_{1,1}$	$d_{1,2}$	$d_{3,3}$
Decision 3	$d_{2,1}$	$d_{2,2}$	$d_{2,3}$
Decision 4	$d_{2,1}$	$d_{3,2}$	$d_{1,3}$
Decision 5	$d_{2,1}$	$d_{3,2}$	$d_{2,3}$
Decision 6	$d_{1,1}$	$d_{2,2}$	$d_{2,3}$
Decision 7	$d_{1,1}$	$d_{2,2}$	$d_{3,3}$

By substituting the significance values of elementary decisions v_{ji} in place of the symbols of elementary decisions d_{ij} , the matrix of the significance of elementary decisions is created. Multiplying this matrix

by the significance matrix of particular decision areas V_i allows to establish costs' assessment of individual solutions (variants) to the decision (Table 4).

Table 4. An example of calculating the value of a cost estimate of solutions to a decision problem from Figure 1 (Source: Own elaboration)

	$v_{m,1}$	$v_{m,2}$	$v_{m,3}$		V_i		Cost assessment
Decision 1	$v_{1,1}$	$v_{1,2}$	$v_{1,3}$		V_1		$(v_{1,1} * V_1) + (v_{1,2} * V_2) + (v_{1,3} * V_3)$
Decision 2	$v_{1,1}$	$v_{1,2}$	$v_{3,3}$		V_2		$(v_{1,1} * V_1) + (v_{1,2} * V_2) + (v_{3,3} * V_3)$
Decision 3	$v_{2,1}$	$v_{2,2}$	$v_{2,3}$		V_3		$(v_{2,1} * V_1) + (v_{2,2} * V_2) + (v_{2,3} * V_3)$
Decision 4	$v_{2,1}$	$v_{3,2}$	$v_{1,3}$	*		=	$(v_{2,1} * V_1) + (v_{3,2} * V_2) + (v_{1,3} * V_3)$
Decision 5	$v_{2,1}$	$v_{3,2}$	$v_{2,3}$				$(v_{2,1} * V_1) + (v_{3,2} * V_2) + (v_{2,3} * V_3)$
Decision 6	$v_{1,1}$	$v_{2,2}$	$v_{2,3}$				$(v_{1,1} * V_1) + (v_{2,2} * V_2) + (v_{2,3} * V_3)$
Decision 7	$v_{1,1}$	$v_{2,2}$	$v_{3,3}$				$(v_{1,1} * V_1) + (v_{2,2} * V_2) + (v_{3,3} * V_3)$

3 Implementation of the modified AIDA method in the IT tool

The decision problem considers concerns of three decision areas, D_i and $i \in \{1,2,3\}$. A collection of ele-

mentary decisions is indicated in every decision area. The relative significance values of decision areas and elementary decisions are presented in Table 5.

Table 5. List of relative significance values of decision areas and elementary decisions
(Source: Krupa and Ostrowska, 2012, s.26)

Decisions areas		
$D_1 (V_1 = 20)$	$D_2 (V_2 = 30)$	$D_3 (V_3 = 50)$
Elementary decisions		
$d_{1,1} (v_{1,1} = 0,75)$	$d_{1,2} (v_{1,2} = 0,50)$	$d_{1,3} (v_{1,3} = 0,40)$
$d_{2,1} (v_{2,1} = 0,25)$	$d_{2,2} (v_{2,2} = 0,10)$	$d_{2,3} (v_{2,3} = 0,30)$
	$d_{3,2} (v_{3,2} = 0,40)$	$d_{3,3} (v_{3,3} = 0,30)$

Elementary decisions remaining in the contradiction relationship are listed below:

$$\{d_{1,1} - d_{3,2}\}; \quad \{d_{2,1} - d_{1,2}\}; \quad \{d_{2,1} - d_{3,3}\}; \\ \{d_{2,2} - d_{1,3}\}; \quad \{d_{1,2} - d_{2,3}\}.$$

Table 6 presents all variants of solutions to the considered decision problem. The decisions containing pairs of contradictory elementary decisions are marked in gray.

Table 6. The matrix of solutions to a decision problem presented in Figure 1
(Source: Own elaboration)

	$d_{m,1}$	$d_{m,2}$	$d_{m,3}$
Decision 1	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$
Decision 2	$d_{1,1}$	$d_{1,2}$	$d_{2,3}$
Decision 3	$d_{1,1}$	$d_{1,2}$	$d_{3,3}$
Decision 4	$d_{1,1}$	$d_{2,2}$	$d_{1,3}$
Decision 5	$d_{1,1}$	$d_{2,2}$	$d_{2,3}$
Decision 6	$d_{1,1}$	$d_{2,2}$	$d_{3,3}$
Decision 7	$d_{1,1}$	$d_{3,2}$	$d_{1,3}$
Decision 8	$d_{1,1}$	$d_{3,2}$	$d_{2,3}$
Decision 9	$d_{1,1}$	$d_{3,2}$	$d_{3,3}$
Decision 10	$d_{2,1}$	$d_{1,2}$	$d_{1,3}$

Table 6. The matrix of solutions to a decision problem presented in Figure 1 (cont.)
(Source: Own elaboration)

Decision 11	$d_{2,1}$	$d_{1,2}$	$d_{2,3}$
Decision 12	$d_{2,1}$	$d_{1,2}$	$d_{3,3}$
Decision 13	$d_{2,1}$	$d_{2,2}$	$d_{1,3}$
Decision 14	$d_{2,1}$	$d_{2,2}$	$d_{2,3}$
Decision 15	$d_{2,1}$	$d_{2,2}$	$d_{3,3}$
Decision 16	$d_{2,1}$	$d_{3,2}$	$d_{1,3}$
Decision 17	$d_{2,1}$	$d_{3,2}$	$d_{2,3}$
Decision 18	$d_{2,1}$	$d_{3,2}$	$d_{3,3}$

Table 7 presents the considered decision problem in the form of a matrix equation.

Table 7. A matrix equation of decision problem from Figure 1 (Source: Own elaboration)

	$V_{m,1}$	$V_{m,2}$	$V_{m,3}$		V_i		Cost assessment
Decision 1	0,75	0,5	0,4		20		$(0,75 * 20) + (0,5 * 30) + (0,4 * 50) = 50$
Decision 3	0,75	0,5	0,3		30		$(0,75 * 20) + (0,5 * 30) + (0,3 * 50) = 45$
Decision 14	0,25	0,1	0,3		50		$(0,25 * 20) + (0,1 * 30) + (0,3 * 50) = 23$
Decision 16	0,25	0,4	0,4	*		=	$(0,25 * 20) + (0,4 * 30) + (0,4 * 50) = 37$
Decision 17	0,25	0,4	0,3				$(0,25 * 20) + (0,4 * 30) + (0,3 * 50) = 32$
Decision 5	0,75	0,1	0,3				$(0,75 * 20) + (0,1 * 30) + (0,3 * 50) = 33$
Decision 6	0,75	0,1	0,3				$(0,75 * 20) + (0,1 * 30) + (0,3 * 50) = 33$

The results obtained (Table 5-Table 7) and the number of possible decisions resolving the given decision problem are consistent with the values from Table 2.

4 Practical applications

The article presents an innovative approach to modify the AIDA method that is based on the replacement of a decision tree with a matrix equation. Such improvement undoubtedly accelerates the process of calculating costs for stable decisions, solving

the decision problem. Crucially, the acceleration of the process of cost estimation (assessment) is particularly desirable in the era of the observed increase in the rate of economic changes.

Supplementary effect of the replacement of the decision tree with the matrix establishes a possibility of the analysis of an advanced number of decision areas relevant to the considered decision problem. When using a decision tree, visualization of a decision problem consisting of more than three decision areas containing actually three elementary decisions

has resulted in the necessity to generate an extensive decision tree.

In consequence, decisions meeting the accepted criterion were difficult to find (maximization of cost assessment, minimization of cost assessment, or maintenance of cost assessment in a given range). Utilization of the matrix equation introduces the possibility of considering unlimited number of decision areas (limited only by the computing power).

Thanks to the possibility of analyzing unlimited number of decision areas within the considered decision problem, the modified AIDA method can be used to solve decision problems related to the protection of facilities (included in the list of critical infrastructure⁴) and facilities (objects) supplementation of inherent components for the so-called key services⁵.

Security management of critical infrastructure facilities and objects providing key services components requires simultaneous consideration of the entire set of threats to identify a collection of safety measures allowing to maintain the adopted level of security.

This management area is particularly important because it is associated with the efficiency of facilities classified as critical infrastructure and availability of key services that determines the perception of the safety of citizens and the rate of economic growth, social satisfaction, state sovereignty, and the efficiency of public administration entities as well.

Limiting the efficiency of the critical infrastructure facilities and the availability of key services results in economic losses, contamination of the natural environment, and a real threat to the health and life of the population. Therefore, the ability to assess decision variants for the complex decision-making problems is of particular interest in this area.

Examples of the application of the modified AIDA method for solving flat decision problems⁶ and hierarchical⁷ decision problems in the area of critical infrastructure security management are presented in the monograph: *Situational Management of the Critical Infrastructure of the State* (Wiśniewski, 2019, pp.100-126).

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⁴ Critical infrastructure – systems and their functionally related objects, including construction objects, devices, installations, key services for the security of the state, and its citizens, and to ensure the efficient functioning of public administration bodies, as well as institutions and entrepreneurs (Journal of Laws as of 2019, item 730, art. 3, point 2)

⁵ Key services – a service that is key to maintaining critical social or economic activity, listed in the list of key services (Journal of Laws as of 2018, item 1560, art. 1, point 16)

⁶ Flat decision problem – a set of decision areas designated by threats to which a critical infrastructure object is susceptible, whose risk does not allow to reach the assumed safety threshold and in relation to which decisions are made at one decision level (critical infrastructure operator or a commune or poviát or country or country) (Wiśniewski, 2019, p.148)

⁷ Hierarchical decision problem – a set of decision areas designated by threats to which a critical infrastructure object is susceptible, the risk of which does not allow to reach the assumed safety threshold, for which resolution about security requires agreement on at least two decision levels (critical infrastructure operator, commune, poviát, voivodeship, country) (Wiśniewski, 2019, p.144)

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