Mutants as Patches: Towards a formal approach to Mutation Testing

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Abstract. *Background:* Mutation testing is a widely explored technique used to evaluate the quality of software tests, but little attention has been given to its mathematical foundations.

Aim: We provide a formal description of the core concepts in mutation testing, relations between them and conclusions that can be drawn from the presented model.

Method: We introduce concepts of mutant space and patch space, and refer to patch merging procedure from the patch theory. We explicitly present constraints, such as location-dependence, that affect mutation operators. We also present a way to use introduced formalism with traditional operators proposed in other papers.

Results: The proposed formalism allows to describe interactions between separate mutations using well-known abstract algebra notation.

Conclusion: The presented formalism may substantially decrease the number of tested weak mutants and increase the number of valuable ones, while giving tools to partially address the problem of equivalent mutants, particularly for higher-order mutation testing. However, additional empirical evaluation is still needed.

Keywords: mutation testing, fault-based testing, patches, equivalent mutants

1. Introduction

Mutation testing is a fault-based technique used for assessing test quality. In the process artificial changes are introduced to the source code. Such modified programs—called mutants—are then used to evaluate test suite. If any test case fails on modified

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program, the mutant is said to be killed. A perfect test suite should kill all mutants that do not behave exactly like the original program. Despite its long history, starting at end of 70s [17, 12, 11], surprisingly little focus has been given to the theoretical foundation of mutation testing. While mutation testing is analyzed and developed, its basic assumptions still slip out of formal specification with rare exceptions [47, 5]. Hence, the aim of the article is to provide a minimal theoretical foundation for further work in mutation testing area.

This paper makes several contributions. The primary one is providing a distinct from existing ones and unequivocal definition of the basic concepts of mutation testing. Another contribution is an advanced but useful mathematical tool for further analysis, as well as preparation of domain-specific terms, definitions, constraints and goals that will be used in further research. It also sets the focus on mutation operators, which, are core elements of mutation testing. Consequences of imposing certain restrictions on operators are discussed in detail.

Some of the constraints introduced by the framework may seem too heavy for practical usage. While we believe that this is not the case, it is still too early to judge its usability. Even if some of the requirements are indeed too strict for general usage, they may turn out to be a reasonable choice for constrained, domain-specific languages (for example for logical expressions). The abstractions are introduced one by one, and in general each abstraction imposes additional constraints on mutation operators. Implementors and users are free to choose which level of abstraction is acceptable for them and utilize only the guarantees provided by these abstractions.

Our conjecture is that by utilizing the formal framework presented in this article it will be possible to substantially reduce both the testing effort related to useless Higher-Order Mutants (HOMs) and the number of equivalent mutants reported to the end user [33]. This can be done by introducing formal rules, structured as equations, and transforming the set of mutations into its minimal mathematical representation. This paper intentionally leaves some of the defined parameters, such as distance metric in Section 3 or location scheme in Section 6, without a specific instantiation. While we are aware that this might impact perceived expressiveness of presented model, at the same time we cannot present a single gold standard for this parameters—best values for given situations are still subject to further research. Therefore, not to lose generality, we abstract them and present them as subject to be instantiated during development of mutation testing tool (or possibly even at runtime). While we acknowledge that empirical evaluation of such framework would be beneficial, this is still subject for further research.

We structured the paper as follows: Section 2 introduces the closely related work (on which we will build upon) to the reader, Section 3 familiarizes the reader with the basic definitions used throughout the paper, Section 4 discusses the constraints on mutation testing we propose to impose, Section 5 presents the axioms, while Sections 6, 7 and 8 focus on our specific approach, discussion of the findings and conclusions.

2. Related work

A general theory for fault-based testing (which is more general than mutation testing) was described by [37, 38]. These papers introduce requirements which must be fulfilled in order for programs to be differentiable, discuss generation of reliable test sets and theoretically prove some theorems about them, like whether such sets can be generated for an arbitrary application.

There is a significant amount of research related to mutation testing in general. However, most of it focuses either on empirical analysis of certain features of mutation testing as a whole [44, 34, 4], analyzing features of operators [48, 49, 14, 19]), overcoming the equivalent mutant problem [33] or efficiency [16, 46, 53]). In the area of mutation operators, substantial amount of research was put into evaluating Statement Deletion (SDL) operator [9, 13]. Overcoming equivalent mutant problem and efficiency are related to this article, as formal tools that will be provided may allow to detect some of the equivalent mutants a priori (before compilation). We also argue that analyzing features of mutation operators is the core activity to use provided mathematical framework.

A different formal approach to mutation testing — based on directed graphs — was given by [5]. Their framework focuses on representing program structure for mutation purposes and mutating on the level of program graph. While the model differs from presented in this paper, it is in fact complementary to it. That is because, since directed graphs are a useful mathematical concept that may be used for constraining program structure to provide addressing model, introduced in Section 6.3.

A general, behavior-based framework for mutation-based methods, not only for mutation testing, was presented by [47]. We focus exclusively on mutation testing, mostly from structural point of view (as opposed to behavior-based approach presented by [47]). Our main focus are mutation operators and their construction, therefore our set of definitions and terms differ. For example, *test differentiator*, a core concept introduced in [47], is only implicitly present in this paper in the form of an assertion in a test set. On the other hand, *behavioral difference* presented by [47] is close to being a special case of a more generic *distance* relations, which we leave abstract.

Finally, there is a number of real-world applications of abstract algebra that make it possible to solve problems in various areas of science and engineering. These include, amongst others, chemistry [6], physics [30], computer science [22] and cryptography [29]. Since the same mathematical concepts are used in such broad spectrum, we suppose that it may also prove being useful when applied to mutation testing.

3. Definitions

Let us introduce some basic terms and definitions on which we will build upon:

S space of all possible text strings

P space of all programs syntactically valid in any programming language

P_{Lang} space of all programs syntactically valid in programming language $Lang$

$p \in P_{Lang}$ the application that is being analysed

$P_{Interface}$ space of all programs in P_{Lang} with the same public API as p

m a mutant; in general, $m \in S$, but only $m \in P_{Lang}$ is usually useful, so only $m \in P_{Lang}$ will be analyzed

O_p space of all mutation operators

O_{pc} a subspace of mutation operators which represent ones that are commutative¹

op mutation operator; in general $op : P_{Lang} \rightarrow P_{Lang}$, but most operators represent $op : P_{Interface} \rightarrow P_{Interface}$ ($P_{Lang} \rightarrow P_{Lang}$ may be useful for analyzing effects of breaking changes in public APIs)

t a test case, i.e. set of assertions that verify some property of a program and may either pass or fail

T set of all possible pairs of (input, output) of a given program p in form of test cases. This is a set that contains all possible scenarios of application execution, so it is not feasible to obtain it, but it is useful to set up the context for its relevant subsets.

T_0 set of all relevant test cases that can be written for a program, i.e., its formal specification (usually not present explicitly). This is mostly a concept that is useful for definitions and theorems. In most practical cases it will be approximated. Usually T_0 contains less constraints than T , simply because some application behaviors remain unspecified.

T_1 set of existing valid test cases, that may be executed at any mutant, i.e., ones that were already written; of course, $T_1 \subseteq T_0$

$test(t, m)$ result of running a test case on mutant m . $test(t, m) = true$ if all assertions from t are satisfied by m , $false$ otherwise

$fail(T_x, m_x)$ result of an operation that verifies whether a given set of tests T_x detects mutant m_x , i.e. $fail(T_x, m_x) = true \iff \exists t \in T_x : test(t, m_x) = false$. This is an operation used to introduce relation between a test set (such as T_0 , T_1 or any of their subsets) and mutants. We decided to use this notation instead of $T_x(m_x) = true \iff \forall t \in T_x : test(t, m_x) = true$ to clearly separate data (test case) from evaluation procedure ($fail$)

$dist(m_x, m_y)$ a number representing metric of similarity between m_x and m_y . We intentionally do not define the meaning of similarity here, as various distance metrics may lead to various results. This is also a subject for further research, thus no specific metric can be given as the gold standard yet. The metric should

¹operations f_1 and f_2 are commutative $\iff \forall x f_1(f_2(x)) = f_2(f_1(x))$

fulfil mathematical requirements of a *metric*, i.e., $dist(x, y) \geq 0 \wedge dist(x, y) = 0 \iff x = y \wedge dist(x, y) = dist(y, x) \wedge dist(x, z) \leq dist(x, y) + dist(x, z)$ [3]. Example metrics include edit distance [39] and number of affected program paths [28], but many others can be used as well, as long as they fulfill the aforementioned requirements.

3.1. Example

A simple example that we will expand during the whole paper is mutation testing of bubble sort algorithm.

Listing 1: Original program - BubbleSort

```
public static void bubbleSort(int[] arr) {
    for(int i = 1; i < arr.length - 1; ++i) {
        for(j = i; j < arr.length - 1; ++j) {
            if(arr[j] > arr[j+1]) {
                int tmp = arr[j];
                arr[j] = arr[j + 1];
                arr[j + 1] = tmp;
            }
        }
    }
}
```

This program contains a bug—iteration starts from $i = 1$ instead of $i = 0$. As a result, T and T_0 differ. Example test case that belongs to T but not to T_0 is

$$bubbleSort([2, 1]) = [2, 1]$$

Example test case that belongs to T_0 but not to T is

$$bubbleSort([2, 1]) = [1, 2]$$

Domains of both T and T_0 are identical and they are both infinite sets—possible inputs include all arrays, possible outputs—all arrays arr that contain same elements as input array and $\forall arr[i] \leq arr[i + 1]$.

The set of mutation operators that we will use will contain the following ($A \leftrightarrow B$ means an operator that applied to A transforms it to B and applied to B transforms it to A):

- comparison mutations: $LEQ \leftrightarrow GEQ$, $LEQ \leftrightarrow LE$, $GEQ \leftrightarrow GE$
- arithmetic mutations: $+$ $\leftrightarrow$ $-$
- delete statement
- change variable in statement into other one with matching type

- change assignment to comparison
- change integer to 0

As a metric of similarity we will use edit distance, understood as number of characters that need to be added, deleted or modified to make both source codes identical.

4. Constraints

Only mutants that are interesting for language with static types are ones that are syntactically valid in the analyzed programming language. Syntactic issues are detected in the compile time, so they can be mitigated easily by the development team and do not require additional assistance from testing staff. This is not strictly true in case of dynamically typed languages, where syntactic error may not be detected until run-time. However, this kind of problems is usually fairly simple to detect using a variety of available tools, such as linters [23, 35, 7] and unit tests [52, 45]. Additionally, this type of errors is usually permanently visible as soon as code is executed (or imported into scope, depending on severity of the problem and language details), thus greatly reducing usefulness of $m \notin P_{Lang}$ —even though they may still be runnable it is immediately obvious that any test case that covers invalid statements would kill the mutant. Since coverage analysis is a tool that is easier to apply than mutation testing, we will not focus on mutants that violate language rules—simply because any problems that they may pose are already addressed by different tools. Therefore, the only mutants that are worth analyzed further are $m \in P_{Lang}$. It is also reasonable to restrict analysis further to $m \in P_{Interface}$ —otherwise it is possible that one will not be able to execute same test suite against mutated application, which might be easily interpreted as killing a mutant (which renders these mutants useless, as will always be killed). Of course this is true only if mutated lines get executed (methods called, configurations used etc.), but if they do not, mutation testing is not the right tool—again, simple coverage analysis would suffice. Lines that are not executed may contain any code or no code at all and do not affect program output.

It is important to understand that—in sense of above definitions—program p is just a special case of a mutant. Transforming between programs by applying mutation operators is no different than implementing a feature or fixing a defect. In fact, one may understand fixing a defect as applying some operation to original source code (with defect) to obtain modified source code (without defect). In typical scenarios this „operation” would be a change applied by a software developer. This change is then communicated to all parties interested in modified source code, often in form of a patch—machine-readable instruction for transforming between two structures of source code.

Changes applied by mutation operators are a subset of all possible changes. Therefore application of any mutation operator may be understood as application of a *patch* (modification) over a program source code. Thus the only two constraints are discussed:

1. its output must always be a syntactically valid program in a given programming language,
2. the public API specification (most notably function signatures) must remain unchanged. This should not be understood as preservation of implicit contracts (e.g. HTTP or SOAP ones), but only as minimal sanity (ability to run local instance of application).

A subtle, yet important detail is that T and T_0 are not necessarily related. T takes inputs and outputs from program p as-is, which means that for T and all its outputs of p are meaningful. T_0 on the other hand may discard some irrelevant outputs (e.g. random number requests, system time accesses etc.). An implicit effect is that T *may contain elements, which are not part of* T_0 , i.e., in general $T_0 \not\subseteq T$ (acceptable outputs—ones in T_0 —may include various versions of log messages, message ordering, number of retries etc., but T contains only single output for each input) and assumption that $T \subseteq T_0$ is equivalent to assuming that program does not contain defects (all input-output pairs are acceptable from the perspective of formal specification). However, for simplicity, further on it is assumed that test cases picked by the programmer are picked from the set of valid test cases, as, according to our current knowledge, an automatic tool has little chance of detecting non-trivial invalid test cases without understanding of program specification (program understanding is another wide area or research, but it is currently out of scope of mutation testing), which means that for further discussion we will assume $T_0 \approx T$ —i.e., we will treat p as an oracle and set of all its input-output pairs as the formal specification of p . This assumption is not entirely obvious, but reasonably intuitive—if we want to verify equivalence according to specification, but we have no specification, we verify for all possible input sequences.

5. Axioms

Mutation testing makes two assumptions about the original program:

1. $fail(T_1, p) = false$, i.e., all tests pass on the original program,
2. $fail(T_0, p) = false$, i.e., original program is the actual specification

The second assumption might seem strange, since it might be interpreted that the tested program is compliant to its specification. This is not a correct interpretation—the program does not have to be compliant to its specification (whatever its specification is—often it is not even known), but during mutation testing process we search for programs that exhibit same behavior as P , regardless if it is valid from business perspective. Therefore, we can assume that for generated mutants T_0 is the *important* subset of P behavior (whole behavior is defined as T and may contain also outputs and actions irrelevant to business, like random number acquisition, time clock checks, logging etc.).

Since a mutation tester aims to create mutants that help to discover missing test cases, one will aim to create mutants m that fulfill three requirements:

1. $fail(T_0, m) = true$ —the mutant is not equivalent (according to formal specification), i.e., there does exist a test case, t_f that fulfills specification ($t_f \in T_0$) and fails on mutated program. Mutants for which $fail(T_0, m) = false$ are called *equivalent mutants*,
2. $fail(T_1, m) = false$ —the mutant is not detectable with current test set, i.e., $t_f \notin T_1$ — t_f is not part of existing executable test suite,
3. $m \approx p$ —the mutant is similar to the original program p . This means that there exists some similarity metric, which indicates that mutant is close to the original program and its value is relatively small. Exact meaning of *small* may depend on multiple factors, such as planned effort for mutation testing.

These three requirements may lead to a formal definition of mutation testing:

Definition 1 *Mutation testing is a process of evaluating test set T_1 by finding a set M_ϵ of programs such that for given ϵ and p : $m \in P_{Interface}$ and $fail(T_0, m) = true$ and $fail(T_1, m) = false$ and $dist(p, m) < \epsilon \iff m \in M_\epsilon$*

For instance, for example described in Subsection 3.1 one such mutant appear when we apply „change integer to 0” operator to $i = 1$ resulting in $i = 0$. This will generate a mutant m that definitely cannot be detected by T_1 because of our assumption that $T_1 \subset T_0$ and m is actually the correct implementation. Other mutant in M_ϵ will be obtained by changing $arr[j] > arr[j + 1]$ into $arr[j] \geq arr[j + 1]$ —this one is equivalent. Depending on the exact contents of T_1 , M_ϵ may contain also other mutants.

When defined as such, every mutation testing algorithm shall have at least three parameters — *fail* operation, *dist* operation and ϵ parameter².

A key factor for finding programs satisfying Definition 1 is to define proper way of exploring the search space. A typical way of achieving it is to apply *mutation operators* to certain locations in program. When applying multiple mutations, an important factor is whether mutations are applied at once or as a sequence of changes. Applying a sequence of changes increases risk of conflicts, such as reversing own change, while using a set of changes allows the mutation tester to apply techniques developed for *patch theory* [2]—detect conflicts and resolve them (possibly by rejecting some of conflicting changes). Using *patch theory* is possible for both sets and sequences of changes, but may require more development effort and additional bookkeeping when implemented in the context of sequences of changes.

In the subject literature [21, 40] mutants with multiple changes are called *higher order mutants (HOMs)*³—contrary to mutants with only a single application of one mutation operator, called *first order mutants (FOMs)* [18]. This classification may give a false impression that FOMs are, generally speaking, more similar to original program than HOMs. This impression is caused by the underlying assumption (often not explicitly expressed) that every mutation (application of mutation operator)

²Theoretically it is sufficient to merge *dist* and ϵ into a single operation, but we will not apply this step, as it removes orthogonality from the solution.

³Strongly subsuming higher order mutants are of particular interest [41].

introduces a similar level of difference. This assumption can be formally expressed as:

$$\forall op_1 \in O_p, \forall op_2 \in O_p, \forall p \in P_{Lang}, \forall T_1 \subseteq T_0 : \\ dist(p, op_1(p)) = dist(p, op_2(p)) \quad (1)$$

Considering huge number of proposed mutation operators [10], it seems very unlikely for such assumption to hold for any *dist* function that is not defined in terms of *op* (and according to definition in Section 3 *dist* cannot be defined in terms of *op*, as arguments of *dist* are programs, not initial program and sequence of mutations). There are no widely accepted constraints regarding what can and what cannot be a mutation operator [25, 31, 48]—thus any general theory would need to be able to handle arbitrary operators, including ones that remove whole implementation or rewrite it. Second, even the same mutation operator applied in different places may result in various distances. Consider the two code snippets on Listings 2 and 3:

Listing 2: An obvious defect introduced by a mutation

```
int add2(int a) {
    return a + 2;
}

int add2Mutated(int a) {
    return a - 2;
}
```

Listing 3: A change introduced by mutation—it is not obvious whether this is a defect

```
int calculate(int a) {
    return a + 2;
}

int calculateMutated(int a) {
    return a - 2;
}
```

From the syntax perspective, both are identical. However, the first function (`add2Mutated`) does something contrary to the name, which—to a minimally competent developer—indicates a place where a defect is likely to be found. The other one `calculateMutated` has meaningless name, thus it is not clearly visible whether the implementation is correct or not. While usually relying on name to provide semantics is troublesome (especially in software that might operate on AST or bytecode level), some other areas of software engineering, such as Elemental Design Patterns [50] or issue-commit link mining [51] have successfully adopted such approach. Using this additional semantic information available in the source code is reasonable, since it

represents a significant part of developers' knowledge. It is also possible, as this information is present in source code (thus can be used directly in static analysis), and usually can be retrieved at runtime from built software (e.g. using symbol maps).

Similar examples are often used during software development lessons—that verbose names mean much to people. Taking into account *The Competent Programmer Hypothesis*, the second defect is much more likely to appear, so its distance from the original program should be smaller.

The division between FOMs and HOMs is largely based on the assumption that FOMs are *much more similar* to original program than HOMs. While this is intuitively true, this definitely does not mean that all defects represented by FOMs are equally likely to be introduced in code [42] by developers, and, more importantly, it does not mean that this difference somehow scales with the order of mutant. For example, consider the following original program in Listing 4:

Listing 4: Original program for different HOM distance

```
double hourDifference(
    double nowTimestampInHours,
    double daysOtherStamp,
    double hoursOtherStamp,
    double minutesOtherStamp) {
    double thenTimestamp = daysOtherStamp * 24 +
        hoursOtherStamp + minutesOtherStamp / 60;
    return nowTimestampInHours - thenTimestamp;
}
```

And possible two Second Order Mutants in Listing 5:

Listing 5: Two example Second Order Mutants

```
double hourDifferenceM1(
    double nowTimestampInHours,
    double daysOtherStamp,
    double hoursOtherStamp,
    double minutesOtherStamp) {
    double thenTimestamp = -(
        daysOtherStamp * 24 + hoursOtherStamp +
        minutesOtherStamp / 60);
    return nowTimestampInHours + thenTimestamp;
}

double hourDifferenceM2(
    double nowTimestampInHours,
    double daysOtherStamp,
    double hoursOtherStamp,
    double minutesOtherStamp) {
    double thenTimestamp =
        daysOtherStamp * 24 + (-hoursOtherStamp) +
```

```

    minutesOtherStamp / 60;
    return nowTimestampInHour + thenTimestamp;
}

```

While neither of them appeals to a software developer, they present an interesting situation — the first mutant is equivalent, while the second one is different. While both differ syntactically from the original program by the same number of changed AST nodes (2), it would be counter-intuitive to say that the distance is same in both cases. This example covers only two mutation operators, both arithmetic. In real-world applications, there are dozens of mutation operators of all kinds (arithmetic, logic, polymorphic, container-based, semantic etc.)—one cannot simply assume that they all have similar impact.

Another important point in mutation testing theory is the structure of P (space of all possible programs) which was not yet thoroughly studied, and this article aims to scratch the surface of this problem. The widest approach to defining the space itself is to define it as *set of all possible character strings*. This is the upper bound, but this already gives an important hint, namely that the mutant space is—at most—countable.

Next reasonable attempt to make the space smaller is to accept only those strings, which are syntactically valid programs in selected language. This creates the entire set of mutant spaces, P_{Lang} —one for each programming language—but of all of them, only a single one is valid at any given time (assuming only single language mutations are allowed, which is a reasonable constraint). Creating mutants that are not valid programs makes little sense, as they are detected before the program even starts, during compilation phase (as discussed in Section 4). Next change in the mutant space may be done basing on the public API—if the mutation changes public API (function names and parameters), then programs will either not compile, or crash at runtime—so these mutations also are useless. The remaining space, $P_{Interface}$, remains infinite, but countable.

We define a concept of *neighborhood*:

Definition 2 $\forall m_1 \in P_{Interface} \forall m_2 \in P_{Interface} : neighbor(m_1, m_2) = true \iff \exists op \in O_p : op(m_1) = m_2$, where O_p is a set of mutation operators allowed to be used in current algorithm

This means that m_1 is a *neighbor* of m_2 if and only if there is a direct transition between them, i.e., there is an operator, that—when applied exactly once—performs transformation from m_1 into m_2 . While this might not be intuitive, *neighborhood* relation is not always commutative. This is due to the fact that set of used mutation operators—in most current mutation testing tools—is configurable by user, so it is not guaranteed to be invertible (i.e., for given operator op_1 it is not guaranteed that there exists an operator op_2 such that $\forall p \in P_{Interface}, op_2(op_1(p)) = p$. If such an operator exists in O_p , then we will call op_2 an operator *inverse* to op_1 . If op_2 is inverse to op_1 and op_1 is inverse to op_2 , they are *reverse* operators, and they are both *reversible* in O_p .

Definition 3 If for given operator op_1 from set of mutation operators O_p , $\forall p \in P_{Interface}, \exists op_2 \in O_p : op_2(op_1(p)) = p$ then we will call op_1 invertible and op_2 is an operator inverse to op_1

Definition 4 If every operator in O_p is invertible, O_p is an invertible operator set

We define that mutant m_1 is *reachable* from mutant m_2 if there exists a sequence of mutation operators (from defined set) which transforms mutant m_1 into m_2 .

Definition 5 $\forall m_1 \in P_{Interface}, m_2 \in P_{Interface} :$
 $reachable(m_1, m_2) = true \iff \exists op_1, op_2, \dots, op_n : op_1 \dots op_n \in O_p :$
 $op_n(\dots(op_2(op_1(m_1))\dots)) = m_2$ where O_p is the set of mutation operators used in the algorithm.

5.1. Invertibility imposed on mutation operators

If every mutation operator in the used set is invertible, $P_{Interface}$ can be divided into clusters, P_C — every element of a cluster is reachable from other element in the cluster. Cluster might be infinite (depending on mutation operators), but for practical reasons the search should be constrained, e.g., by some heuristic on *dist*, usually just maximum number of mutations or maximum mutation time. Only a single cluster can be analyzed when using mutation testing, because the starting point for mutation (program p) is fixed, and single point can only belong to one cluster.

Definition 6 $\forall m_1 \in P_C, m_2 \in P_C : reachable(m_1, m_2) \iff reachable(m_2, m_1)$

Definition 7 $\forall m_1 \in P_C, m_2 \in P : reachable(m_1, m_2) \iff m_2 \in P_C$

5.1.1. Example

If we decide to use only invertible operators in our example, we need to inspect the original list. It contains comparison mutations, arithmetic ones, statement deletion, chariabile change and assignment to comparison change. Now, if we insist on leaving only invertible ones we definitely need to reject statement deletion (to invert it we would need to create an opeartor that inserts arbitrary code). Both comparison operators and arithmetic ones fulfill our requirements. Changing variable into other one may (but does not have to) match our requirements, depending on detailed implementation—for simplicity, we will reject it now. Changing assignment to comparison is not invertible according to our set of operators, but it is relatively simple to make it invertible—add a reverse operator, change of comparison to assignment. Note that this operator cannot be applied anywhere in the original program, but nevertheless needs to be part of our operator suite.

5.2. Mutation testing as a problem of reachability

In Section 5 we have defined the operational goal of mutation tester as finding other programs which are different than the original (but similar to it), and the difference is not detectable by existing test set (more widely, these are used to evaluate the test set and, possibly, improve it). Here we provide an additional constraint—the other program must be *reachable* from the original one by using a series (or set) of applications of chosen *mutation operators*. This requirement is somewhat similar to the one regarding similarity, but is much better defined (for given set of mutation operators), and easier to apply. It also does not cover some parts of the original requirement, since it is possible to create a program substantially different from original which will pass all tests. Up to now we did not impose any constraints on the nature of mutation operators—those have a critical impact on whole process of mutation testing and types of mutants that may be evaluated.

Theorem 1 $\forall m_1 \in P_{Interface} \forall m_2 \in P_{Interface}, \forall m_3 \in P_{Interface} :$
 $reachable(m_1, m_2) \wedge reachable(m_2, m_3) \Rightarrow reachable(m_1, m_3)$

Proof: Let $m_1, m_2, m_3 \in P_{Interface}$, op_1 be a mutation operator or a chain of n_1 mutation operators such that $op_1(m_1) = m_2$ and op_2 be a chain of n_2 mutation operators such that $op_2(m_2) = m_3$. Then, by definition, $reachable(m_1, m_2) = true$ and $reachable(m_2, m_3) = true$. Since $op_2(m_2) = op_2(op_1(m_1)) = m_3$, $reachable(m_1, m_3) = true$, and the mutation order (number of applications of mutation operators) is at most $n_1 + n_2$.

Reachability is always associative. Reachability is commutative if, and only if, mutation operator set is reversible.

6. Relations between mutation operators

While the general approach to the mutant space gives some insight into the structure of mutation testing, it may not be sufficient for analyzing *relations* between mutants and properties that might be preserved. For this reason we now introduce a new structure, that might be useful for higher order mutation testing.

6.1. Patch space

In the context of mutation testing, program p is a special one—it is a starting point for mutation testing process. We are going to transform $P_{Interface}$ —space of all reasonable mutants—into space of patches X_p . A *patch* is a function that can be applied to the original program giving a mutant as a result. We are going to call invocation of patch on a program a *patch application*, and use syntax: $x(p)$. *Patch application* is defined only for the analyzed application p_0 and is related to transformation $P_{Interface} \Rightarrow X_p : \forall m \in P_{Interface}, x \in X_p : m \Rightarrow x \iff x(p_0) = m$. Informally

speaking, patch application may be understood in the way UNIX tools do it (although structural requirements imposed on the patch itself differ). *merge* is defined as composing patch functions, i.e., $merge(x_1, x_2)$ is a higher-order function [15]—a function which returns a function—and $merge(x_1, x_2)(p) = x_1(x_2(p))$.

An important outcome from this analysis is that—since *merge* is associative— X_p creates a semigroup. In fact, it forms a monoid. *Proof.* Transformation of the original program, p_0 to X_p yields a null patch ($x_{null}(p) = p$). Since this null patch applied to any program returns it, $merge(x, x_{null}) = x$ and $merge(x_{null}, x) = x$. This means that x_{null} is an identity element in X_p , and X_p with *merge* operation form a monoid [27], $(X_p, x_{null}, merge)$ — from now on we will refer to them as $(X_p, 1, \cdot)$. #.

This reasoning holds under an assumption that any two patches may be merged and will produce a valid patch. Note that this requirement is not easy to guarantee, and has received attention in the industry mainstream. For example, well-known version control systems like Git or Mercurial do not allow arbitrary patches to be merged, and there is a sophisticated *patch theory* describing required relationships between patches [20], implemented in a less known tool, Darcs. While patches in Unified Diff Format [1] (typical UNIX format) may be used in the implementation of the above system, it would be required to specify a conflict resolution algorithm or accept the fact that *merge* would be only a partial function (of course, one can also try to mitigate it by not allowing conflicting mutations to appear in X_p , but that might require manual inspection). This is due to fact that typical patches were invented to provide a transition between two arbitrary states of the code. However, in the mutation testing analysis we do not require transition to an arbitrary program, since there is a requirement regarding interesting mutants—they have to be *similar* to the original program. Nevertheless, patches can be used as the implementation of mutation operators.

6.2. Mutation operators—requirements

It is worth noting that—in above discussion—mutation operators were treated as unary functions. While inference basing on this assumption was valid, it is not very intuitive during discussion about the nature of the *merge* operation and requirements it imposes on the mutation operators.

Definition 8 *An addressing L is a way of uniquely identifying each statement of a program, which is preserved during the process of mutation testing.*

We do not propose any gold standard for *addressing*—various techniques can be used, starting from simple statement counter, up to complex relationship dependencies. As of now, we cannot reliably predict which of those will be most useful for mutation testing systems—for future discussion, we simply assume that such an *addressing* exists.

From now on we will use a concept of *mutator op*—a pair (f, l) , where f corresponds to a mutation function (the „proper” mutation operator — one that is usually referred to in literature) and l corresponds to a location, where f is applied. For

clarity for each program p we will define $op(p) = x = f(l, p)$, and O_{all} is a set of all mutators that can be represented in this way for program p . Let also O_u be a set of used mutators (of course $O_u \subset O_{all}$). *Mutator* is conceptually equivalent to a *patch*, but we use a separate term to denote its different origin.

6.3. Example

Continuing the bubble sort example, we provide a simple example addressing: statement/entity number. This means that *inti* would be statement 1, *inti = 0* - statement 2, *j* - statement 3, *arr.length* - statement 4, *arr.length - 1* - statement 5, *j < arr.length - 1* - statement 6 etc. This means that we made the number of statements/entities rigid, and our "delete statement" operator will actually become "replace with empty statement".

Requirement 1 *In order for $(X_p, \mathbb{1}, \cdot)$ to form a monoid, every code location l must be resolvable for every sequence of preceding mutators, i.e., let D_f be domain of $f_x(l_x, p_x)$. Then $\forall (f_x, l_x) \forall op_1, op_2, \dots, op_n \in O_{all} : (op_1 \cdot op_2 \cdot \dots \cdot op_n)(p_0) \in D_f$*

This means that no matter what mutations were performed earlier, it must be possible to unambiguously resolve l_x . This is equivalent to a statement that there is some structure in p_0 which must be preserved during every mutation—the structure is represented by l_x addressing. The nature of this structure does not need to be defined globally for every operator set—it can be defined per O_u . One can decide about the structure which has to be preserved—for example if the structure is the line or statement number— l_x represents a range of affected statements. Note that such structure means that operators which add or remove lines/statements are not allowed in this specific model (empty lines are allowed by addressing and/or empty addresses are left between statements), but can be enabled by using a different addressing scheme.

The requirement above leads also to a more subtle constraint imposed on the mutation operators. As we have already stated, they need to preserve some predefined structure. However, when designing mutation operator one cannot assume that during execution it will encounter same structure as in the original program — the nature of mutation operators becomes a more general function, that may need to handle several types of inputs. To illustrate this fact with example, let us consider two mutation operators defined in a more typical way— M_1 transforming each *and* operator it encounters into *or* and M_2 , transforming *and* operator into *xor*. Now let us consider the code in Listing 6.

Listing 6: Original function

```
int foo(int a, int b) {
// conditional statement 1
    return (a and b) ? a : b;
}
```

After applying only M_1 in conditional statement 1 we would get the code presented in Listing 7.

Listing 7: Function after M_1 mutation

```
int fooM1(int a, int b) {
// M1-mutated conditional statement 1
return (a or b) ? a : b;
}
```

However, M_2 was also applicable to original conditional statement 1, and would give in result code in Listing 8.

Listing 8: Function after M_2 mutation

```
int fooM2(int a, int b) {
// M2-mutated conditional statement 1
return (a xor b) ? a : b;
}
```

Both M_1 and M_2 were applicable to function `foo`, but it is clear that neither M_1 is applicable to `fooM2` nor M_2 to `fooM1`. This breaks Requirement 1, and as such, is not allowed. What is even more troublesome, even M_1 is not applicable to `fooM1` and M_2 is not applicable to `fooM2` because they were explicitly defined as „transform `<<and>>` to ...”, but there is no `<<and>>` any more.

Yet operators in this form are very common and reusing them and knowledge related to them is an important issue. It arises from the fact of moving the logic of *merge* operation to the operators. There are three ways to solve the problem when there is given mutator $op = (f, l_x)$ and operation of f is defined only for a specific subset of structures that might be present in location l_x :

1. If f is not defined for structure in l_x , leave the structure as-was (i.e., for conflicting $(f_1, l_x) : f(l_x, f_1(l_x, p_0)) = f_1(l_x, p_0)$).
2. Regardless of the structure that is in l_x , replace conflicting structure with the result of the original application (i.e., for conflicting $(f_1, l_x) : f(l_x, f_1(l_x, p_0)) = f(l_x, p_0)$).
3. Modify f to extend its domain. This might be done for example by merging several typical mutation operators into one. For example, we could take operator M_1 defined above and add two other rules to it — for instance "replace *or* to *xor* and *xor* to *and*" — which would make the resultant operator — let us call it M_3 — applicable to all the three: `foo`, `fooM1` and `fooM2`. It is also worth noting that $M_3(M_3(foo)) = M_2(foo)$, which means that they are not both simultaneously in the base of a monoid.

These solutions are used in virtually any version control system around the world in case of conflicting patches (sometimes they are called merge strategies). In mutation testing there is a need to perform every merge automatically, so during application

operator (or some controlling entity) must be "aware" of what do to during conflict. Of course, this *does not* discard the possibility of automated full-program analysis for best resolution.

Note that while the first two options can be applied in presence of any patches preserving l_x addressing, the third one cannot and generally might yield a partial function. This is not an acceptable constraint in the general patch theory, as applied in version control systems (that is when manual resolution is needed), but it is acceptable for the purposes of mutation testing. All three strategies boil down to transforming a partial function into a total one.

6.4. Patch theory

During studies related to patch theory [20, 2, 36] it was shown that if the system allows arbitrary patches, it can be represented as an *inverse semigroup*. However, during mutation testing only specific set of patches can be applied, and they do not need to keep this property (i.e., that for $\forall x \in X_p \exists x^{-1} : x \cdot x^{-1} \cdot x = x \wedge x^{-1} \cdot x \cdot x^{-1} = x^{-1}$). In the further discussion we will limit our concern to these subsets of mutation operators that form inverse semigroups, O_{is} – namely ones that are *invertible* in the meaning of Definition 3. Since it was proven earlier that every valid set of mutation operators forms a *monoid*, O_{is} forms an *inverse monoid* (but not necessarily a group).

6.5. Patch independence

We are going to define two patches, $x_1 = (f_1, l_1), x_2 = (f_2, l_2) \in X_p$ as *independent* $\iff l_1 \cap l_2 = \emptyset$.

Independence is an important feature, because independent patches are *trivially commutative* (see [20] for definitions of commutativity in the context of patch theory), i.e., $x_1(x_2(p)) = x_2(x_1(p))$. Note that there is no restriction as to what is represented by l_x —one example was unique number of affected statement, which is relatively generic. Another example—arguably more sophisticated—would be a set of all nodes in the program flow graph which might be affected by the result obtained in the node. This representation might be more interesting, as may allow to significantly (depending on the application) decrease number of necessary higher-order mutants. This is because if two independent patches, x_1 and x_2 cause respectively T_{f_1} and T_{f_2} test sets to fail, mutant obtained by applying a merged patch, $x_1 \cdot x_2$ will simply cause to fail a set of test cases $T_{f_1} \cup T_{f_2}$. Such conclusions are possible regardless of the actual details of structure of both patches, original program and content of test cases. This can be formalized in the following way:

Theorem 2 $\forall x_1, x_2 \ T_{f_1} : \{t \in T_1 : test(t, x_1(p)) = fail\}, T_{f_2} : \{t \in T_1 : test(t, x_2(p)) = false\} : x_2(x_1(p)) = x_1(x_2(p)) \implies T_{f_{12}} = T_{f_1} \cup T_{f_2}$

Proof. Let N_1 be the set of nodes traversed during evaluation of x_1 and N_2 be the set of nodes traversed during evaluation of x_2 . Since the assumption is that

$N_1 \cap N_2 = \emptyset$, $T_{f1} \cap T_{f2} = \emptyset$. If it was not the case, it would mean that there is a test case that fails and runs through both N_1 and N_2 . However, this is not possible, because in such case there would be a node $n_1 \in N_1$ that would affect $n_2 \in N_2$, which is not possible according to initial assumption. Therefore, since node sets are independent and failed test sets are independent, no interaction is possible between these two mutations, thus failed test case set must be a sum of failed test case sets for each of them. #. It is important to note that this feature is specific to a given *addressing*—one that includes whole dependent subgraph. It will not hold if the *addressing* simply uses line or statement number.

The nature of *patch independence*, as understood in this paper, is neither syntactic nor semantic itself—it depends solely on L addressing model. Change of addressing model may require a change of approach towards merging strategy—if the approach with dependency graph is used, there is little sense in using the first or second described general strategy. Instead, the patch should be usually applied to the code, partially overriding results of previous patches.

Definition 9 A *Mutation Testing System* is a tuple (O_p, L) where O_p is a set of mutation operators that can be applied in the system and L represents used addressing model.

Definition 10 n -th order mutant is a mutant that can be represented at minimum by n locations of application of commutative patches from the base.

Let us consider a simple example of when formal approach may yield useful results. Let our operator set contains two operators: *negate* (which negates a boolean) N_l and *logical* (which transforms each *and* to *or* and *or* to *and*) L_l . In both cases lowercase l represents location where operator shall be applied. Original function is simple:

Listing 9: Original function

```
boolean foo(boolean a, boolean b) {
    return (a or b) ? a : b;
}
```

after first application of *logical* operator we get:

Listing 10: Function after applying *logical* operator

```
boolean fooM(boolean a, boolean b) {
    return (a and b) ? a : b;
}
```

after first application of *negate* operator to result we get:

Listing 11: Function after applying *logical* operator and *negate* once

```
boolean fooMN(boolean a, boolean b) {
    return (!a and b) ? a : b;
}
```

and after second application of *negate* we get:

Listing 12: Function after applying *logical* operator and *negate* twice

```
boolean fooMNN(boolean a, boolean b) {
  return (!a and !b) ? a : b;
}
```

after third application of *negate* we get:

Listing 13: Function after applying *logical* operator and *negate* three times

```
boolean fooMNNN(boolean a, boolean b) {
  return !(!a and !b) ? a : b;
}
```

which is an equivalent mutant—one of the possible equivalent mutants from this set of operators. Let us describe it in notation used in the paper: $M_{2-3} \cdot N_2 \cdot N_3 \cdot N_1 = \mathbb{1}$. Since $\cdot$ operation is associative, we can write it as $M_{2-3} \cdot (N_2 \cdot N_3 \cdot N_1) = \mathbb{1}$. Now there are a few steps to be taken:

1. N_1, N_2 and N_3 are commutative
2. $N_1 \cdot N_2 \cdot N_3 = M_{2-3}$
3. $M_x \cdot M_x = \mathbb{1}$, i.e., $M_x = M_x^{-1}$

In the end it was provable that an equivalent mutant will be obtained. This was of course a simple example, but it demonstrates the power of the notation working in concert with specific features of patches and relations between mutation operators (and, consequently, resultant patches). When mutation operators are expressed as functions, such relations may be expressed as facts⁴ and stored in some knowledge database. This example also demonstrates risks of using operators which are not part of the monoid base—when operators are not part of base, the risk of achieving the same program by different series of mutations arises.

7. Discussion

In this paper a formal system that can be used for inference about mutant features before their creation was presented. This can be perceived as an extension of papers by [37, 38]—all theorems from these articles remain valid, but we have provided a more detailed description of a model tailored specifically for mutation testing. Structure constraint required in the presented model may be provided by research published by [5], extending its primary effect scope.

Presented theory—when implemented—may have significant impact on how to deal with the problem of equivalent mutants. Current approaches concentrate rather

⁴Logical sentences that are always true.

on post-creation detection whether mutant is equivalent to the existing program [43, 24, 26]. This is an approach different than presented in this article—we have presented a notation that might be used to avoid creating equivalent mutants in the first place, as well as avoiding to create mutants for which it is already certain that they would be killed. Such approach may minimize both testing and mutation effort (which might be significant, especially if compilation is required for analysis) and act as a useful filter, leaving dynamic analysis only for mutants, which cannot be inferred. Additionally, notation presented in this article may reduce test effort required for mutants obtained by different operators (thus with different source, AST and bytecode), but behaving the same way—this is a unique feature coming from defining proper relations between mutation operators, such as in Listings 9-13. Three similar concepts related to introducing changes were used, so it may be useful to precisely recapitulate their scope:

- Mutation operator is a function that can be applied to a specific location in code. Mutation operator can be applied multiple times in the same location and may yield different results for each application. Operators are defined by user or mutation testing application.
- Patch is a single result of mutation operator application. Patches can be merged to form other patches. Patches form at least a monoid, but in special cases may also form a group.

It is also critical to understand that, even though the theory goes into details of operators forming a group, it is not strictly required to use only those. Instead, the developer (or even user) of the system should make an informed decision whether to use a restricted set of operators with additional benefits (e.g., decreased probability of equivalence) or an extended set with additional risks. In practice we do not expect users to opt-in for all possible features, but to understand the trade-off and pick a set of operators that best suits their needs.

Additionally, in this paper a minimal set of entities required for a mutation testing algorithm were defined. These include:

fail a function determining whether a mutant is killed by a given test suite

dist distance metric between mutants

ε maximum distance in the search space which will be analyzed for interesting mutants

O_p a set of mutation operators

L addressing mechanism for mutation operators

A lot of focus was given to mutation operators, as we believe they are core elements of useful mutation testing tools, albeit they still do not get enough attention in the literature.

Most of the presented research may be also applied to the field of automated refactoring, as the community of automated refactorings seeks applications syntactically

different than the original one, but having same semantics, i.e., the same what community of mutation testing rejects. Since results from this paper can be used for analysing both equivalence and non-equivalence, one may use them to assess equivalence on refactored application (for a strictly defined set of refactoring operators).

7.1. Threats to validity

Since this paper is a theoretical one and presented concepts were not yet implemented, there is a chance that some or all of them will not have expected benefits to actual analysis.

It is possible that in real-world applications independent paths are rare, and, as a result, there will be a little gain due to their analysis. It is also possible that analysis of the mutation path, $x_1 \cdot x_2 \cdot \dots \cdot x_n$ will not be efficient enough to detect mutants that were already analyzed and a simple check sum algorithm after mutation will be better in practical approach.

It may also be that reversible mutation operators are virtually useless in real-world programs for a number of reasons—they may either not produce interesting outputs or be hard to create and validate.

Some of commonly used mutation operators cannot be used to form a group—in such cases, only theories applicable to monoids (or even semigroups) may be used. Further, relations between operators may be hard to formulate and their analysis may be inefficient.

While we put effort into making sure that formulas and proofs presented in the paper are correct, there is still a possibility for an error.

This paper also introduces a new threat for mutation testing—it increases difficulty of empirical evaluation of mutation testing strategies. While typically metrics like Mutation Score Indicator (*MST*) [32, 34, 33] can be applied, it is not clear how to apply them to an algorithm that removes mutants whose properties can be inferred statically.

7.2. Further work

Critical aspect of further work on this subject is to empirically validate theoretical results provided by this paper. Our conjecture is that this formalization may be useful to the reader during a study independent from our own. One of the possibilities we consider particularly attractive is to apply search-based software engineering as a way to filter Higher-Order Mutants before evaluating them against test cases.

Most important theoretical area that needs to be explored in relation to this study is the nature of mutation operators. Describing and analyzing dependencies between various mutation operators, relationship between applying same mutation operator in multiple locations in the source code and researching usefulness of sets of operators forming groups are at the core of further research.

It is also very important to analyze how do classic mutation operators fit into

presented theory—while in Section 6.3 we did present three ways of transforming them to make them fit, we do not claim that these are the only ones or the best ones. In particular, operator-specific strategies may bring in much benefit. Existing mutation operators—such as statement deletion—are already empirically evaluated and have a proven track of usefulness, therefore it would be beneficial to be able to use them.

To further expand possibility of analysis based on patches and simplify proofs, it may be useful to transform it into some well-known mathematical object, for example a boolean function. How to do this is still a subject of further research.

Performance comparison of different metrics for comparing programs (in order to decide whether further mutations should be applied, or is the application too far from the original one) will bring in great benefit, providing guidance for selection of one of the core parameters of mutation testing system.

Finally, a new metric for evaluating mutation algorithm quality is needed. Using classical metrics like *MSI* [32, 34, 33] may be biased when the algorithm aims to remove both mutants which will surely be alive and ones which will surely be killed (in unknown proportions). As a result, the numbers will be totally incomparable, and it will not be actually possible to decide whether an algorithm performs better or worse without higher-level evaluation metrics. Perhaps such metrics can be provided by co-evolution of test set and mutation set [8] (one simple example of such metric may be number of additional test cases needed to kill all killable mutants), but this is still subject to be researched.

8. Conclusions

Mutation testing is widely known and used for several decades already. Still, a fundamental theory that would make it more efficient and understandable, as well as usable in real-world applications is rather limited. Lack of basic definitions and requirements (such as the nature of mutation operators) makes the area much harder to develop and use in production.

This paper introduces a formal mathematical theory, based on abstract algebra, which can be applied to the mutation testing. It uses a notion of *patches*, but imposes several new constraints on mutation operators. While no effort in this paper was dedicated to define any set of operators, three approaches were presented to the problem of adjusting existing operators to new constraints. We have also formally defined other items required for a mutation testing tool to be able to apply this theory: a measure of similarity between applications and addressing scheme, which has to be invariant in respect to used operator set.

Then we have presented related theoretical results from *patch theory* and, as a result, proven that any set of mutation operators for which imposed requirements hold form an *inverse monoid*. We have also showed an example of how to extract a *group* from this monoid (imposing additional constraints on mutation operators).

In the end we demonstrated several use cases for the group theory in mutation testing and outlined main directions of further work on the principles of mutation

testing.

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