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FUZZIFICATION - DECISION MAKING IN TERMS OF UNCERTAINTY

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SUMMARY

The theory of fuzzy sets allows to analyze insufficiently precise, accurate, complete phenomena which can not be modeled by the theory of probability or interval mathematics. We define fuzzy sets as sets where the boundary of the set is unclear and depends on subjective estimation or individual preference. In addition to the standard interpretation scale, described above, a set of numbers to each qualitative attribute must be assigned. In addition to the standard interpretation scale a set of numbers to each qualitative attribute must be assigned. First of all, it is necessary to determine the procedure for determining fuzzy numbers describing the attributes. One of the imperfections of the fuzzy sets is subjectivism when defining the boundaries of fuzzy sets and functions of belonging, which can significantly influence the final decision. The decision maker's subjectivity is also present in the determination of weighted coefficients. However, in case of giving weight, fixed values are necessary. Some decisions require multidisciplinary knowledge, so the decision-making process includes more group decision-makers, who independently give their grades.

Keywords: fuzzification, uncertainty, qualitative attributes, weight coefficients

INTRODUCTION

Fuzzification is a suitable mathematical apparatus for modeling different processes dominated by uncertainty, subjectivity, indeterminacy. Uncertainty is also present when decision is made at all known decision parameters, but some criteria of a qualitative nature and the value of the attributes for assessing options depend on the subjective judgment of the decision maker and the relative weight of the selected criteria. The decision maker's subjectivity can not be avoided in solving real problems, but uncertainties must be taken into account in the decision-making process.

Every human being is exposed to the need to make decisions on a daily basis, whether personal or decisions related to the work he is doing. We may say that uncertainty is present in every decision we make, no matter how simple the problem is. The question is how much the decision maker is prepared to ignore the uncertainty and how much he is willing to accept the risk of the wrong decision. Uncertainty can be divided into uncertainties caused by wrong assumptions and uncertainties due to Force Majeure. Wrong assumptions are the result of insufficient information or ignorance of the nature of things that will happen in the future, whether it is an event that has an impact on the decision and is not foreseen, or an event is foreseen but not all the effects it carries with it, or the specific time of event is not foreseen. A Force Majeure is an event that has harmful consequences that could not be anticipated or prevented, such as weather, catastrophes or other events of a similar nature.

Apart from some statistical methods, where based on the knowledge of the past, we can conclude about future, there is no way to predict future events in all segments. The decision maker assumes certain conditions based on the collected data (state of resources, available capacities, employee motivation, etc.), which in most cases are accurate and slightly changeable in function of time.

Overall, uncertainty can be observed in the following situations:

- when given terms, which characterize the concept, do not determine the uniquely expected result - these phenomena are usually modeled by the theory of probability;
- when it is not possible, and it is not necessary, to know precisely the observed values - such uncertainties are usually treated with interval mathematics;
- when uncertainty arises from inaccuracy in communication among people (eg high people, low temperature, poor sales) - such uncertainties are modeled by the theory of fuzzy sets.

LITERATURE REVIEW

A large number of authors use fuzzy sets to analyze uncertainty in the decision-making process, but the most famous name and founder of the theory of fuzzy sets, Zadeh, set the basic postulates of this theory (Zadeh, 1965). The most common phasing refers to the use of fuzzy numbers with a predefined confidence interval (Retaei, Fahim, Tavasszy, 2014). Fuzzification can also be carried out by using the variable confidence interval of a fuzzy number. Thus, some authors (Pamučar, Ćirović, Sekulović, Ilić, 2011) believe that the confidence interval depends on the level of conviction of the decision-makers in their own claims. Based on the belonging level of individual values of the output variable, defazification is performed, i.e. the choice of one value of the output variable (Seiford, 1996). As a part of this step, the weight coefficients are aggregated using some of the known methods. In this paper, aggregation was carried out using arithmetic weighting (Srđević, Zoranović, 2003). One of the imperfections of the fuzzy sets is subjectivism when defining the boundaries of fuzzy sets and functions of belonging, which can significantly influence the final decision (Pamučar and Ćirović, 2015).

FUZZY SETS TERM

The first work devoted to fuzzy sets was published in 1965 by American professor Lotfi Zadeh (Lotfi Zadeh). The theory of fuzzy sets allows to analyze insufficiently precise, accurate, complete phenomena which can not be modeled by the theory of probability or interval mathematics. When uncertainty arises from inaccuracy in communication among people, such uncertainties are modeled by the theory of fuzzy sets.

Unlike the classical theory of sets, which very precisely defines a boundary that separates elements belonging to a set from elements that do not belong to it, the theory of fuzzy sets does not define the boundary of separation sufficiently. We define fuzzy sets as sets where the boundary of the set is unclear and depends on subjective estimation or individual preference. Affiliation to a set is determined on the basis of the level of reliability and is denoted by So, has a characteristic of probability in the claim that an element belongs to or does not belong to the observed set. As an example, let's take a set of banks where it is crowded.

Depending on the relationship betwen these parameters and their weighted decision can be made about belonging to the set. For a specific unit of the bank, one can determine the number of users waiting in line as the limit over which it can be said it is a big crowd. However, in addition to the number of users, creating of the crowd in the bank is also determined by the time of retention of users in the bank, the average duration of the transaction, the state of the bank's resources (cash, personnel, etc.), the user's structure and a lot of other factors. For the above reasons it is not possible to determine the exact boundary of the set only based on the number of users in the waiting line. If we add a situation where there are multiple rows and the number of users in one row is disproportionately larger in than other rows, then the perception of the users in smaller row is that the bank is not very crowded, while the user in a bigger row has completely different opinion. Each perception can be described by the function of belonging which depends on the type of problem and the number of other elements. Most real business problems have described parameters that are unclear and which are consider linguistic variables and which can be described with some of the functions of belonging.

Depending on the relation between these parameters and their weight coefficients, a valuable decision can be made about belonging to the set.

A fuzzy set A is defined as a set of ordered pairs

A= $\{X, \quad \mu A(x)\}$,

where:

$X = x_1 \cup x_2 \cup ... \cup x_n$, final set of elements and

$\mu A(x)$ - the function of affiliation (the level of affiliation of element A).

In the theory of fuzzy sets, the belonging of group A is described by the function belonging to $\mu A(x)$ in following way:

$\mu A(x) = 1$, if and only if it belongs to A,

$\mu A(x) = 0$, if and only if it does not belong to A.

The function can take any value from the interval $[0, \quad 1]$. If $\mu A(x)$ is bigger, then there is more truth in the claim that x belongs to set A.

When set X is a final set, as in the starting assumption, then the fuzzy set A defined in it, is represented in the form:

$$\frac{\mu A(x_1)}{x_1} + \frac{\mu A(x_2)}{x_2} + ... + \frac{\mu A(x_n)}{x_n} = \sum_{i=1}^{n} \left[\frac{\mu A(x_i)}{x_i} \right]$$

In case that X is not a final set, the fuzzy set is defined as:

$$A = \int x \left[\frac{\mu(x)}{x} \right] dx$$

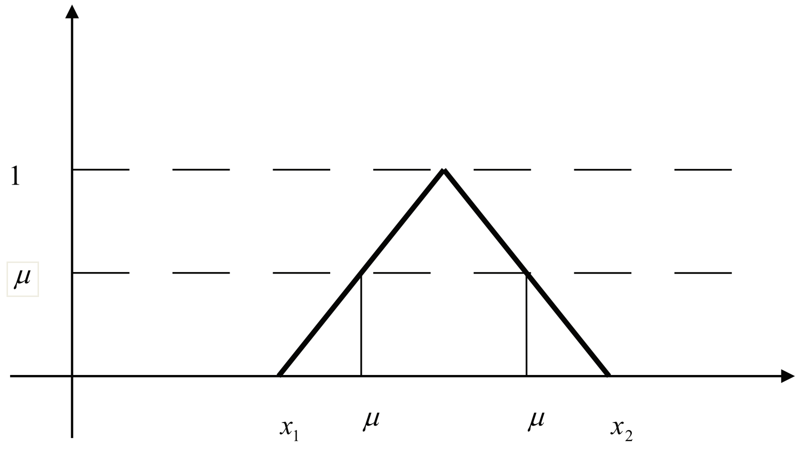
The requirement that some function of affiliation should be a fuzzy number is that it is normalized and convex over the confidence interval. The operations on fuzzy sets and fuzzy numbers are defined in a way that further on it is discussed about fuzzy programming and fuzzy logic that are suitable for modeling and solving real problems. More and more fuzzy theory is used in traffic regulation, intelligent machines, robotics and other areas where the management of some processes entails to machines.

FUZZIFICATION OF QUALITATIVE ATTRIBUTES

In addition to the standard interpretation scale, described above, a set of numbers to each qualitative attribute must be assigned. First of all, it is necessary to determine the procedure for determining fuzzy numbers describing the attributes.

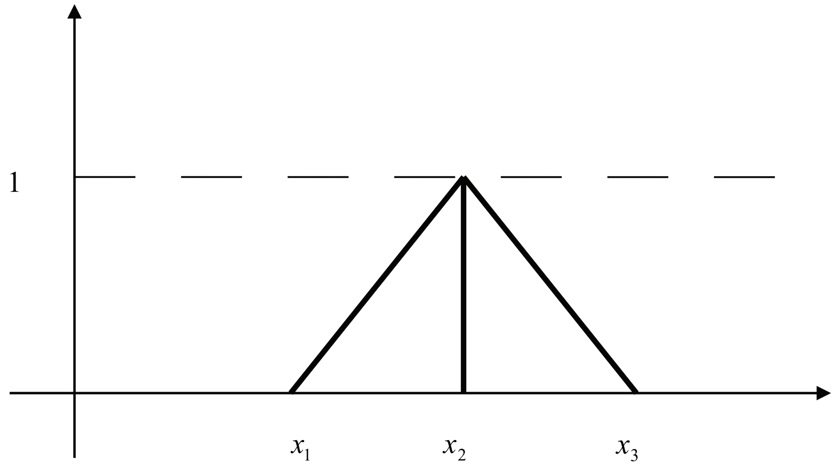
The Fuzzy number (Figure 1) represents a normalized and convex fuzzy set that characterizes the confidence interval $[x_1, \quad x_2]$ and degree of security (can be in intervals $[0, \quad x_2]$).

Picture 1 Fuzzy number



A triangular fuzzy number is shown in Figure 2.

Picture 2 Triangular fuzzy number



The triangular fuzzy number is conditioned by the form of belonging function and is defined by the form $A = (x_1, \quad x_2, \quad x_3)$, where:

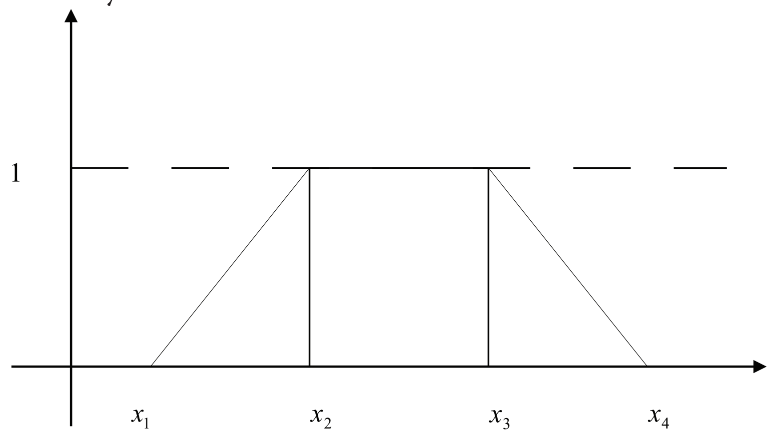
x_1 – bottom, left limit of fuzzy number,

x_2 – the value of the fuzzy number with the highest level of belonging and

x_3 – upper, right limit of fuzzy number.

The trapezoidal fuzzy number is defined by the shape $A = (x_1, \quad x_2, \quad x_3, \quad x_4)$, as shown on picture 3.

Picture 3 Trapezodial fuzzy number



FUZZY LOGIC

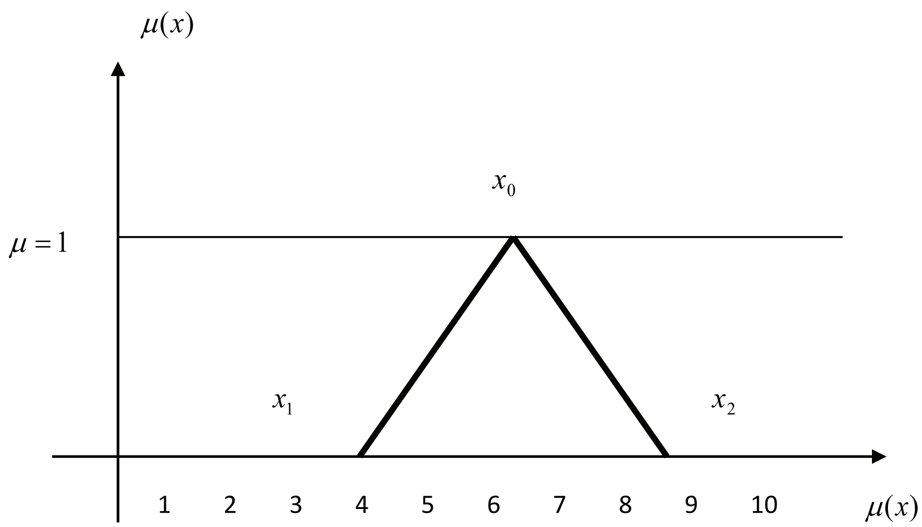
Fuzzy logic, as the basis of the fuzzy system, allows decisions making based on incomplete information and models based on fuzzy logic consist of the so-called IF-THEN rule. The input variables in the fuzzy system are so-called linguistic variables ("a small number of people in waiting row", "long waiting times", "high prices", etc.). The output is given in continuous form. All possible values of the output variable are associated to the corresponding level of belonging. Based on the level of belonging of individual values of the output variable, "defuzzification" is performed, i.e. the choice of one value of output variable is made. Fuzzy logic is most often used to model complex systems in which by applying other methods should be very difficult to determine the interdependencies that exist between some variables in the model.

Example: IF-THEN rule
IF value is variable x BIG,
THEN value of variable y SMALL.

Based on the theory of fuzzy sets, a whole range of its applications has been developed in practice, theory and the usage of fuzzy numbers and fuzzy logic have been developed. Practice shows that decision-makers quantify qualitative attributes by comparing them with extremes and predominantly statements of the type: "around *x* ", "not less than *x* and not more than y", "between *x* and y" (for example, flight from Banja Luka to Belgrade lasts "approximately 40 minutes") and similar, which in essence are linguistic expressions such that they can be represented by the fuzzy number of the "triangular" form. Sometimes, expressions as "between *x* and y, but not less than and not more than *q* " appear, which is a fuzzy number of trapezoidal shape, which can be approximated by the fuzzy number of the "triangular" form where $\mu(x)=1$ the mean value of the interval *x-y* is taken. Therefore, if the decision maker evaluates the qualitative attribute indefinitely, then this grade can be expressed as a fuzzy number. If we take a triangular fuzzy number as a form of a fuzzy number with which we describe the linguistic expressions of the decision maker, then the interval can be described with three discrete values, as in Picture 4.

$$p = x_0, \quad \forall\mu(x_0) = 1$$
$$p^- = x_1, \quad \forall\mu(x_1) = 0 \quad \wedge \quad x_1 \le x_0$$
$$p^+ = x_2, \quad \forall\mu(x_2) = 0 \quad \wedge \quad x_2 \ge x_0$$

Picture 4 An example of a fuzzy number that joins an attribute



It is understood that there is no x where: $\mu(x) \succ 0, \quad \wedge \quad (0 \le x \prec x_1 \vee x_2 \prec x \le 10), \quad 0 \le x \le 10$
The presented fuzzy number, which must be normalized and convex, represents a subjective assessment of decision-maker in evaluating something that is not exactly defined, but is expressed in linguistic terms or quantized in a given value scale. Linguistic expressions are quantified in the interval of the value scale (1-10) so that the end values of the scales correspond to the terms "impermissibly bad" = 1 or "perfect" = 10.
In our example, the decision maker claims that the attribute has the value of "slightly over 6". To the question: which is the smallest and the highest value you would give the attribute, the decision maker has opted for "something more than 8" and "no less than 4" which when translated into the numeric form is x_1=4 and x_2=8,2. Hence, the decision-maker's belief is that the rating for the observed attribute can be from 4 to 8.2 and with the highest level of confidence the value . The term "about 6" implies that x_0=6 and x_1 and x_2 are also determined in a way that the decision maker must give the lower and upper limits of possible values for the attribute. Evaluation can take a rational value, which practically means that there is an unlimited number of values that an attribute can have in a defined value scale.
On the other hand, when subjective assessments are made, decision makers tend to give indefinite score, in terms of almost 8 or more than 6, approximately 7, or some other terms that can hardly establish the boundary intervals. It is necessary to insist on more precise definitions in order to define the values $x_i, \quad i \in \{0,1,2\}$ for each attribute that is expressed linguistically in order to establish a triangular fuzzy number in the same way. Thus, the fuzzy number can have different forms, but we can still say that in most cases the decision maker gives their linguistic expressions so they can approximate them with a triangular fuzzy number, where values for $\mu(x) = 0$ as well as for $\mu(x) = 1$ and allocation within the interval is linearized.
The decision maker's subjectivity is also present in the determination of weighted coefficients. However, in case of giving weight, fixed values are necessary, because it has to fulfill the requirement:

$$\sum_{j=1}^{k} w_j = 1 \quad ,$$

so a change in the value of a coefficient affects all other weight coefficients. It is possible to set multiple tasks with different weight coefficients and to analyze the ranking options in relation to the inscribed changes.
Some decisions require multidisciplinary knowledge, so the decision-making process includes more group decision-makers, who independently give their grades. Big differences in subjective ratings are possible, especially in terms of aesthetic nature, so the decision maker must determine the values for $x_i, \quad i \in \{0,1,2\}$ and weighting coefficients by statistical methods, which differes from case to case. In this way, group decision making gains its goal, and decisions are more precise.
Forming of fuzzy sets for each qualitative attribute would be based on the corresponding fuzzy number and selected level of attachment. Namely, if we determine for level of reliance $\mu(x) = 0{,}8$ then a fuzzy set is defined in a way that values for the attributes may take all the values for which it is $\mu(x) \ge 0{,}8$. Then calculations are made for:

$$\mu(x^-) = 0{,}8 \quad \wedge \quad x^- \le x_0$$

$$\mu(x^+) = 0{,}8 \quad \wedge \quad x^+ \ge x_0$$

CONCLUSIONS

The fuzzy set theory allows the analysis of insufficiently precise, accurate, complete phenomena that can not be modeled solely by theory of probability or interval mathematics. Instead of "solid" and precise values, "soft" - blurred values are accepted and processed and attempted to encompass the imprecisions and indeterminacy immanent to a man, such as opinion and resonance - based on the words. In addition, at the end of the 20th century, new computational discipline was introduced as an alternative to classical artificial intelligence, called computional inteligence, based on soft computing, which is the basis of computer intelligence, or a set of methodologies that enable conceptualization, design and application of intelligent systems. The main factors of soft computing are: fuzzy logic, neuro-computing, genetic computing and probabilistic computing.

For global optimization problems, so-called meta-heuristics are used, which, accordingly get fine solution; even in cases of very serious problems (nonlinear, nonconvenctive and/or combinatorial). These techniques are complementary rather than opposed to each other, and are often used in combination, thus leading to the formation of hybrid intelligent systems and their application in supporting decision making process.

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