Practical Modeling of Filter Filtration Efficiency*

by J. Kao

Philip Morris Research Center, Richmond, Virginia, U.S.A.

SUMMARY

A simple and general mathematical model has been developed to calculate filter-filtration efficiency, which is based on practical design parameters: pressure drop, filter dimension, flow rate, and filament denier. This model has been verified by examining published experimental data, which include a broad range of design parameters. This model surpasses other models currently described in the literature in terms of accuracy. The drag force of the filter (which is accounted for by the pressure drop times the circumference squared, \( \Delta P \cdot C^2 \)) appears to be the most important contribution to filter-filtration efficiency, but its contribution can be drastically reduced by varying design parameters.

INTRODUCTION

Cigarette-filter efficiency (E) is conventionally defined as the ratio of the weight gained by the filter to the total smoke weight exposed to the filter; the total smoke weight exposed to the filter is experimentally determined as the sum of the material weights collected by the cigarette filter and a high-efficiency particulate filter pad, which is connected to the cigarette filter. Alternatively, it may be defined as equation 1, where \( \text{C}_{\text{in}} \) is the smoke/aerosol concentration impinging on the filter and \( \text{C}_{\text{out}} \) is the corresponding concentration leaving the filter. Problems associated with smoke/aerosol generation, concentration, etc. appear to have no major effect on filter-filtration efficiency and, thus, filtration efficiency.
Figure 1.

Calculated vs. experimental ln(1 - E) values for Dwyer's data (2).

For a single point, the symbol "o" is used to denote the location. For multiple points, "2", "3", ..., "9" are used to denote the number of points at a location. The character "M" is used, when the number of points is greater than 9.
efficiency is fairly constant for a given filter at a given condition. It is this property which makes filtration efficiency the most common way to compare filters.

\[ E = \frac{C_{in} - C_{out}}{C_{in}} \]  

[1]

There have been many reports on filtration efficiency in the past decades and filtration-efficiency data are abundant (1 - 10). These reports present different experimental results (observations) under various experimental conditions/constraints. Thus, some of the results may appear contradictory and many different models in various forms have been proposed. There is no universal model to account for all existing data. Thus, the objective of this study is to attempt to provide a simple and unique model to describe filter performance based on filter-design parameters. The proposed model is expected to fit the existing data which cover broad ranges of design parameters. Hopefully, the proposed model will enable us to correctly optimize the design parameters and to design novel filters. The proposed model may also furnish a framework for modeling a tobacco rod for delivery prediction.

There are two distinct basic approaches in deriving models, namely statistical (mathematical) regression and in-depth theoretical derivation by solving flow equations. The equations derived from the first approach may not be physically meaningful. On the other hand, every term of an equation derived from theory has a specific meaning. Ideally, a physical model from theory is preferred over a statistical model. Nevertheless, in practice, a real system may be too complex to have exact equations and exact solutions. A compromised approach is to derive a physically meaningful mathematical model. We have now obtained a physically meaningful mathematical model and the proposed model together with the units of measure used throughout this paper is shown in equation 2:

\[ \ln P = \ln (1 - E) = \frac{1}{Q} \left( a_0 \cdot C^2 + a_1 \cdot \Delta P \cdot C^2 + a_2 \cdot \frac{L}{d} \right) \]  

[2]

where

- \( P \): penetration efficiency,
- \( E \): filtration efficiency,
- \( Q \): flow rate (cm\(^3\)/s),
- \( \Delta P \): pressure drop (cm w.g.),
- \( L \): length (cm),
- \( d \): filament denier (g / 900,000 cm),
- \( C \): circumference (cm).

The right-hand side terms of equation 2 in parentheses characterize a filter, while the \( 1/Q \) term relates the flow rate to filtration efficiency. The coefficients of equation 2, \( a_0, a_1, \) and \( a_2 \), have negative values and represent composite contributions from various possible mechanisms (such as diffusion, interception, and impaction). Specifically, the \( \Delta P \cdot C^2 \) term corresponds to the drag force of the filter. The \( a_2 \) term corresponds to the mean/constant contributions per cross-section area of the intrinsic properties of a filter material (cellulose acetate here) and experimental conditions (such as constant butt length). Many different filters can be fabricated to have the same pressure drop, yet their filtration efficiencies can be drastically different. It is known that the longer the filter, the higher the efficiency. It is also known that the finer the filament (or the smaller the circumference of a filter), the higher the efficiency for cellulose-acetate filters. Thus, the \( L/d \) term is added to our model to account for this non-linearity. The \( L/d \) term has been used previously in Keith's model (1). The proposed model appears to satisfy the boundary conditions: \( P \to 0 \) as \( Q \to 0 \), and \( P \to 1 \) as \( Q \to \infty \).

**VERIFICATION OF THE MODEL**

The accuracy, usefulness and characteristics of any model are best explored by examining the model against the experimental data in the literature.

**Verification**

Dwyer et al. (2) have recently reported the total particulate matter (TPM) efficiencies of unventilated cellulose-acetate (CA) filters for aged tobacco smoke by varying several design parameters at a constant circumference (2.47 cm). There are a total of 34 data points (\( E = 39\% - 93\% \)) and the range of parameters are as follows: \( \Delta P = 0.3 \text{ cm} - 66.5 \text{ cm w.g.}, \ Q = 1.7 \text{ cm}^2/\text{s} - 55.0 \text{ cm}^2/\text{s}, \ L = 1.9 \text{ cm} - 2.5 \text{ cm}, \) and \( d = 2.1 \text{ denier} - 3.4 \text{ denier} \). The derived equation (equation 3) is shown in the following as well as the associated statistics. Both the statistics and the scatter plot of calculated and experimental \( \ln (1 - E) \) values (Figure 1) clearly indicate the accuracy of the model. The average percentage deviation from experimental \( \ln (1 - E) \) values is only 5.4\%, which is presumably about the same size as the expected normal experimental errors (variations) for filtration-efficiency measurements (3, 4).

\[ \text{TPM: } \ln (1 - E) = \frac{1}{Q} \left( -0.120 \cdot C^2 - 0.188 \cdot \Delta P \cdot C^2 - 1.686 \cdot \frac{L}{d} \right) \]  

[3]

\( \text{S.E.E.*} = 0.059, \ r = 0.995, \ S.D. = 0.070. \)

*Standard estimated error.*
Figure 2.
Calculated vs. experimental ln(1 − E) values for Lee's data (5).
For a single point, the symbol "o" is used to denote the location. For multiple points, "2", "3", ..., "9" are used to denote the number of points at a location. The character "M" is used, when the number of points is greater than 9.
After development of our model, we became aware of the experimental data and the model reported by Lee (5). In his report, there are 42 data points containing information about tar and nicotine-removal efficiencies ($E = 25\% - 70\%$) at constant flow rate ($17.5 \text{ cm}^3 / \text{s}$), but at different circumferences. The variations of design parameters are as follows: $\Delta P = 3.5 \text{ cm} - 18.7 \text{ cm}$ w.g., $d = 1.8 \text{ denier} - 5.0 \text{ denier}$, $L = 2.1 \text{ cm} - 2.9 \text{ cm}$, and $C = 2.2 \text{ cm} - 2.6 \text{ cm}$. The derived equations from our model as well as the related statistics are shown in equation 4, while the corresponding results from Lee's model (5) are depicted in equation 5. For our model, the average percentage deviations from experimental $\ln(1 - E)$ values are $6.5\%$ and $6.4\%$, respectively, for tar and nicotine, while the corresponding values for Lee's model are slightly larger, i.e. $7.2\%$ and $6.4\%$, respectively, for tar and nicotine. By comparison, our model performs slightly better than Lee's model for this set of data. Both the statistics and the scatter plot (calculated vs. experimental $\ln(1 - E)$ values (Figure 2)) clearly demonstrate the accuracy of our model.

Our model:

\[ \text{tar: } \ln (1 - E) = \frac{1}{Q} \cdot \left( -0.385 \cdot C^2 - 0.118 \right) \cdot \Delta P \cdot C^2 - 3.590 \cdot \frac{L}{d}, \]  
\[ \text{S.E.E.} = 0.045, \quad r = 0.970, \]  
\[ \text{S.D.} = 0.034. \]

\[ \text{nicotine: } \ln (1 - E) = \frac{1}{Q} \cdot \left( -0.186 \cdot C^2 - 0.083 \right) \cdot \Delta P \cdot C^2 - 5.282 \cdot \frac{L}{d}, \]  
\[ \text{S.E.E.} = 0.032, \quad r = 0.981, \]  
\[ \text{S.D.} = 0.028. \]

Lee's model:

\[ \text{tar: } \ln (1 - E) = -0.135 - 0.016 \cdot \Delta P \cdot C \]  
\[ - 0.218 \cdot \frac{L}{d}, \]  
\[ \text{S.E.E.} = 0.048, \quad r = 0.966, \]  
\[ \text{S.D.} = 0.038. \]

\[ \text{nicotine: } \ln (1 - E) = -0.066 - 0.011 \cdot \Delta P \cdot C \]  
\[ - 0.302 \cdot \frac{L}{d}, \]  
\[ \text{S.E.E.} = 0.036, \quad r = 0.956, \]  
\[ \text{S.D.} = 0.028. \]

Recently, Eastman Kodak Company kindly provided us with two reports which include extensive lists of cellulose-acetate cigarette-filter properties (6, 7). In the first report (6), there are 144 data points containing information about dry-particulate-matter (DPM) removal efficiencies ($E = 29\% - 76\%$) at constant flow rate ($17.5 \text{ cm}^3 / \text{s}$) and constant circumference ($2.48 \text{ cm}$). The variations of design parameters are as follows: $\Delta P = 2.7 \text{ cm} - 22.2 \text{ cm}$ w.g., $d = 1.8 \text{ denier} - 5.0 \text{ denier}$ and $L = 1.7 \text{ cm} - 3.3 \text{ cm}$. The derived equations from our model as well as the associated statistics are shown in equation 6, while the corresponding results from Eastman's model shown in the report (6) are depicted in equation 7. The fittings are about the same for both models and they appear to be adequate for this data set. However, by comparison, our model is superior, since Eastman's model contains four regression coefficients. The average percentage deviation from experimental $\ln(1 - E)$ values is only $2.8\%$ for our model.

Our model:

\[ \text{DPM: } \ln (1 - E) = \frac{1}{Q} \cdot \left( -0.572 \cdot C^2 - 0.114 \right) \cdot \Delta P \cdot C^2 - 3.512 \cdot \frac{L}{d}, \]  
\[ \text{S.E.E.} = 0.024, \quad r = 0.994, \]  
\[ \text{S.D.} = 0.019. \]

Eastman's model:

\[ \text{DPM: } \ln (1 - E) = a_0 + a_1 \cdot \Delta P + a_2 \cdot L \]  
\[ + a_3 \cdot \frac{L}{d}, \]  
\[ \text{S.E.E.} = 0.027, \quad r = 0.993, \]  
\[ \text{S.D.} = 0.020. \]

For the second report (7), there is a collection of 672 data points containing information about dry-particulate-matter removal efficiencies ($E = 20\% - 80\%$) at constant flow rate ($17.5 \text{ cm}^3 / \text{s}$) and at different circumferences. The ranges of design parameters are as follows: $\Delta P = 0.8 \text{ cm} - 38.2 \text{ cm}$ w.g., $d = 1.8 \text{ denier} - 14.7 \text{ denier}$, $L = 1.7 \text{ cm} - 3.3 \text{ cm}$, and $C = 2.02 \text{ cm} - 2.67 \text{ cm}$. The derived equations from our model as well as the related statistics are shown in equation 8. As can be seen from equation 8, this set of data is also well correlated by our model. For comparison, the same
Figure 3.
Calculated vs. experimental ln (1 - E) values for EASTMAN data (6, 7).
For a single point, the symbol "o" is used to denote the location. For multiple points, "2", "3", ..., "9" are used to denote the number of points at a location. The character "M" is used, when the number of points is greater than 9.
data set is also employed to fit into both Lee's (5) and Keith's (1) models as shown in equations 9 and 10. For this collection of data, the average percentage deviations from experimental $\ln (1 - E)$ values are 4.0%, 6.8%, and 8.9% for our model, Lee's model, and Keith's model respectively.

Our model:

\[
\text{DPM: } \ln (1 - E) = \frac{1}{Q} \left( \frac{-0.628 \cdot C^2 - 0.105 \cdot Q \cdot C^3 - 3.622 \cdot \frac{L}{d}}{\Delta P \cdot C^2} \right), \tag{8}
\]

S.E.E. = 0.052, $r = 0.981$,
S.D. = 0.028.

Lee's model:

\[
\text{DPM: } \ln (1 - E) = -0.209 - 0.013 \cdot \Delta P \cdot C
- 0.253 \cdot \frac{L}{d}, \tag{9}
\]

S.E.E. = 0.068, $r = 0.968$,
S.D. = 0.049.

Keith's model:

\[
\text{DPM: } \ln (1 - E) = -0.084 \cdot L - 0.013 \cdot \Delta P
- 0.240 \cdot \frac{L}{d}, \tag{10}
\]

S.E.E. = 0.102, $r = 0.963$,
S.D. = 0.063.

Both of Eastman's data sets may be combined to generate a collection of 816 data points, and the resultant equation from our model is depicted in equation 11. The average percentage deviation from experimental $\ln (1 - E)$ values is only 3.8% for this combined data set.

\[
\text{DPM: } \ln (1 - E) = \frac{1}{Q} \left( \frac{-0.620 \cdot C^2 - 0.105 \cdot Q \cdot C^3 - 3.671 \cdot \frac{L}{d}}{\Delta P \cdot C^2} \right), \tag{11}
\]

S.E.E. = 0.049, $r = 0.982$,
S.D. = 0.027.

Verification 4

It has been known for many years that filtration efficiency is strongly influenced by flow rate (3). The advent of tip-ventilated filters has further demonstrated the practical importance of the dependence of filtration efficiency on flow rate. For such filters, smoke flow rate in the upstream filter segment can be markedly reduced at the standard smoking conditions. The reduced flow rate results in a concomitant increase of efficiency. Recently, Norman et al. (3) have measured DPM and nicotine-removal efficiencies ($E = 41\% - 63\%$) of a cellulose-acetate cigarette filter ($L = 2.50$ cm, $C = 2.48$ cm, $d/D = 3.3/35,000$, $D$ is the total denier) at a constant flow rate ($17.5$ cm$^3$/s) with tip dilutions from 0.0% to 70%. The corresponding flow rate in the upstream filter segment would thus be $5.3$ cm$^3$/s — $17.5$ cm$^3$/s. There are ten relevant data points in this report (3). However, our model has to be rearranged (simplified) to fit their data, because of the use of a single tow item at a constant dimension in the experiment. The necessary operations are straightforward and shown in equation 12. These data points can then be used to validate our modified model. The derived equations from our model as well as the associated statistics are shown in equation 13, while the corresponding scatter plots are shown in Figure 4. For this collection of data, the average percentage deviations from experimental $\ln (1 - E)$ values are 4.3% and 2.5%, respectively, for nicotine and TPM. Thus, the fits are excellent for both nicotine and DPM-removal efficiencies.

\[
\ln P = \ln (1 - E) = \frac{1}{Q} \left( a_0 \cdot C^2
+ a_1 \cdot \Delta P \cdot C^2 + a_2 \cdot \frac{L}{d} \right), \tag{12}
\]

Here, $\ln (1 - E) = \frac{1}{Q} \left( a'_0 + a_1 \cdot \Delta P \cdot C^2 \right)$

\[
= \frac{1}{Q} \left( a'_0 + a'_1 \right),
\]

($\Delta P \cdot C^2/Q$ is a constant).

nicotine: $\ln (1 - E) = -1.328 \cdot \frac{1}{Q} - 0.167, \tag{13a}$

S.E.E. = 0.018, $r = 0.957$,
S.D. = 0.013.

DPM: $\ln (1 - E) = -2.635 \cdot \frac{1}{Q} - 0.170, \tag{13b}$

S.E.E. = 0.016, $r = 0.990$,
S.D. = 0.011.
Figure 4.
Calculated vs. experimental \( \ln(1 - E) \) values for Norman's data (3).
For a single point, the symbol "o" is used to denote the location. For multiple points, "2", "3", ..., "9" are used to denote the number of points at a location. The character "M" is used, when the number of points is greater than 9.
Verification 5

In his studies of smoke-filtration mechanisms, Kertes (8) reported TPM-removal efficiencies \( E = 33\% - 61\% \) of a cellulose-acetate cigarette filter \( L = 2.00 \text{ cm}, C = 2.46 \text{ cm}, d/D = 3.1/41,000 \) at a constant flow rate \( 17.5 \text{ cm}^3/\text{s} \) with tip dilutions from 0.0% to 75%. The corresponding flow rate in the upstream filter segment would thus be \( 4.4 \text{ cm}^3/\text{s} - 17.5 \text{ cm}^3/\text{s} \). There are seven relevant data points in this report (8). Similarly, these seven data points can be used to validate our modified model shown in equation 12. The derived equations from our model as well as the related statistics are shown in equation 14, while the corresponding scatter plots are shown in Figure 5. The fittings are excellent, since the average percentage deviation from experimental \( \ln (1 - E) \) values is only 1.6%.

\[
\text{TPM: } \ln (1 - E) = -3.116 \cdot \frac{1}{Q} - 0.241, \quad [14] \\
\text{S.E.E.} = 0.012, \quad r = 0.998, \quad \text{S.D.} = 0.008.
\]
Other Verifications

Other verifications have also been completed against many published data such as those appearing in reference Nos. 7–9. Keith and Derrick (9) studied the dependence of filtration efficiency on flow rate by using homogeneous pyrene aerosols (at various particle sizes). Yamamoto et al. (4) reported the effect of circumference on filtration coefficients using two different blends of tobacco. Fordyce et al. (10) reported filtration data by varying several design parameters. All of these data are relevant and can be used to validate our model. Again, the proposed model correlates those data very well, and the correlation coefficients are typically greater than 0.950.

The work of Fordyce et al. (10) requires further comments here. In their paper, they derive the following two equations (equations 15 and 16):

\[ \Delta P = \varepsilon \cdot Q \cdot L / A, \]
\[ C_{\text{out}} = C_{\text{in}} \cdot e^{-\mu \cdot L} \quad \text{or} \]
\[ \ln(1 - E) = -\mu \cdot L, \]

where \( A \) is the filter cross-section area, \( \varepsilon \) is the impedance, which is analogous to the specific electrical resistance, and \( \mu \) is defined as filtration coefficient. They also deduced that, if filtration is governed solely by \( \Delta P \), \( \mu \) is constant. In reality, \( \mu \approx \text{constant} \) for a filter material (cellulose acetate, paper, tobacco, etc.). Their model can be related to ours by the following operations:

\[ -\mu \cdot L = \frac{1}{Q} \cdot (a_0 \cdot C^2 + a_1 \cdot \Delta P \cdot C^2 + a_2 \cdot \frac{L}{d}). \]

Thus,

\[ -\mu = \frac{1}{Q \cdot L} \cdot (a_0 \cdot C^2 + a_1 \cdot \Delta P \cdot C^2 + a_2 \cdot \frac{L}{d}). \]

From equations 15 and 17,

\[ \frac{\mu}{\varepsilon} = \frac{1}{\Delta P \cdot A} \cdot (a_0 \cdot C^2 + a_1 \cdot \Delta P \cdot C^2 + a_2 \cdot \frac{L}{d}) \]
\[ = a_1' + \frac{1}{\Delta P \cdot A} \cdot (a_0 \cdot C^2 + a_2 \cdot \frac{L}{d}). \]

Therefore, the last two terms of equation 18 are equivalent to contributions of non-linearity in their paper.

Variations of Coefficients

Summarized in Table 1 are the regression coefficients derived from our model using the various data sets. The \( a_0 \) values for Keith’s data (8) shown in Table 1 is deduced from the relation \( a_0' = a_0 \cdot \Delta P \cdot C^2 / Q \) (equation 12), where \( \Delta P \cdot C^2 / Q \) is about 1.94. A similar deduction cannot be made for Norman’s data (3), since no pressure-drop data were reported in the paper. The \( a_1 \) term appears to be fairly constant for all data sets and it is, thus, less sensitive to experimental conditions. On the other hand, both the \( a_2 \) and \( a_3 \) terms are more vulnerable to experimental conditions (Table 1). The \( a_2' \) values for Norman’s (3) and Keith’s data (8) can be compared with the estimates obtained from the relation \( a_2' = a_0 \cdot C^2 + a_2 \cdot L / d \) (equation 12) by applying the appropriate coefficients of equations 3, 4 and 11. The estimated values for Norman’s data are -5.145 for DPM and -6.594 for nicotine, while that for Keith’s TPM data is -1.814. These estimated values are several times larger than the values obtained from regression. It is interesting to note that the sum of \( a_2 \) terms for nicotine and tar, derived from Lee’s data, is about the same as the \( a_0 \) of equation 11 for DPM. Since these experiments were carried out at the same laboratory, it is reasonable to suggest that the \( a_0 \) contributions are additive. Nevertheless, the \( a_0 \) and \( a_3 \) terms of equation 11 appear to have intermediate values between the corresponding values for tar and nicotine derived from Lee’s data.

The mathematical formula of our model (equation 2) indicates that, although the flow rate will inversely and significantly influence filter-filtration efficiency, the contributions of each term can remain a constant percentage to the overall efficiency by keeping constant pressure drop, circumference, filter length, and filament denier. The contributions of the second term, \( a_2 \cdot \Delta P \cdot C^2 \), vary from 10% to 95% for the data examined here. However, for the most common filters, the contributions from the term are probably about 50%. The larger contributions of the second term can be accomplished by increasing pressure drop and circumference, while simultaneously decreasing the \( L/d \) ratio. Conversely, the smaller contributions of the second term can be accomplished by decreasing pressure drop and circumference, while simultaneously increasing the \( L/d \) ratio.

In summary, in view of the broad ranges of experimental variables, the derived coefficients appear to be very consistent, although there are certain variations. The way of measurement (gravimetric or spectroscopic methods) and the material measured (tar, DPM, TPM, or nicotine) should cause differences in regression coefficients. These variations may also be attributed to the different experimental conditions. For instance, experiments were possibly performed by smoking different cigarettes to different butt lengths. It has been shown that filtration efficiency depends on butt length, presumably due to condensation (2, 11). Filtration efficiency may vary to a certain degree, if different cigarettes are...
Table 1.
Variations of regression coefficients.

<table>
<thead>
<tr>
<th>Author</th>
<th>Equation No.</th>
<th>Material</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a'_0$</th>
<th>$a'_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dwyer (2)</td>
<td>3</td>
<td>total particulate matter</td>
<td>-0.120</td>
<td>-0.188</td>
<td>-1.686</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lee (5)</td>
<td>4</td>
<td>tar</td>
<td>-0.385</td>
<td>-0.116</td>
<td>-3.590</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>nicotine</td>
<td>-0.186</td>
<td>-0.083</td>
<td>-5.262</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eastman (6, 7)</td>
<td>11</td>
<td>dry particulate matter</td>
<td>-0.620</td>
<td>-0.105</td>
<td>-3.871</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norman (3)</td>
<td>13</td>
<td>nicotine</td>
<td></td>
<td></td>
<td></td>
<td>-1.328</td>
<td>-0.187</td>
</tr>
<tr>
<td></td>
<td></td>
<td>dry particulate matter</td>
<td></td>
<td></td>
<td></td>
<td>-2.635</td>
<td>-0.170</td>
</tr>
<tr>
<td>Keith (8)</td>
<td>14</td>
<td>total particulate matter</td>
<td>(-0.123)</td>
<td></td>
<td></td>
<td>-3.116</td>
<td>-0.241</td>
</tr>
</tbody>
</table>

employed for testing (10). Unfortunately, not all experimental conditions are specified in the reports to allow us to narrow down the exact cause. Furthermore, the distribution/domain of design parameters may also mathematically exercise undue influence on the regression coefficients for a small set of data.

CONCLUSIONS

A simple, general and practical model has been proposed to accurately calculate filtration efficiency. The proposed model has been verified by using a variety of experimental data ($E = 20\% - 93\%$) from the literature, which cover broad ranges of design parameters: $\Delta P = 0.3 \text{ cm} - 66.5 \text{ cm w.g.}$, $Q = 1.7 \text{ cm}^3/s$ - $55.0 \text{ cm}^3/s$, $C = 2.2 \text{ cm} - 2.7 \text{ cm}$, $L = 1.7 \text{ cm} - 3.3 \text{ cm}$, and $d = 1.8 \text{ denier} - 14.7 \text{ denier}$. The drag force by the filter (which is accounted for by the pressure drop times the circumference squared, $\Delta P \cdot C^2$) can be the most important contribution to filter-filtration efficiency, but can be drastically reduced by varying design parameters: the contribution is $10\% - 95\%$ for the data examined here. The average percentage deviation from experimental $\ln(1 - E)$ values is no more than $5\%$, which is about the same size as the expected normal experimental errors (variations) for filtration-efficiency measurements. The proposed model contains both flow-rate and circumference terms and can be easily used for both ventilated and unventilated cellulose-acetate filters. This is important for designing cigarettes with unconventional circumferences and ventilation levels. Extensions of this model to other materials are under way.

REFERENCES

7. Eastman Chemicals Division (Filter Products Research Laboratory): Eastman's empirical data base on the effect of filter design parameters on the smoke UV removal efficiency of CA filters; A special technical service report, 1988, Eastman Kodak Company, Kingsport, Tenn.

Acknowledgement

Thanks to Dr. R. W. Dwyer and Dr. J. F. Whidby for their guidance, support and comments.

Author’s address:

Philip Morris Research Center,
P.O. Box 26383,
Richmond, Virginia, 23261, U.S.A.