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Flow Rate Dependence of Ventilation*

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SUMMARY

A quantitative model describing the effects of puffing conditions on the level of filter ventilation was developed and evaluated. The development of the model was based on a quadratic flow-pressure drop relationship which was validated with experimental measurements for numerous plug wraps, tipping papers, and combinations of the two. This relationship was used to derive an equation describing the level of filter ventilation as a function of the flow rate of air exiting the filter. This equation was shown to accurately predict the measured ventilations of six brands of commercial cigarettes over a range of continuous flow rates. The instantaneous ventilation values predicted by the equation were utilized to model ventilation during a puff by integrating the equation with respect to flow rate over the duration of the puff. This method for predicting the effects of specific puffing conditions on ventilation was demonstrated for sinusoidally shaped puffs spanning a wide range of volume and duration. Finally, the effects on the flow dependence of ventilation of different combinations of plug wrap and tipping papers were described qualitatively based on experimental measurements of paper flow-pressure drop linearity.

ZUSAMMENFASSUNG

Zur Beschreibung der Auswirkungen der Zugbedingungen auf die Filterventilation wurde ein quantitatives Modell entwickelt und ausgewertet, das auf einem für eine Vielzahl von Filterumhüllungs- und Mundstücksbelagpapieren sowie deren Kombinationen experimentell nachgewiesenen quadratischen Verhältnis zwischen Strömung und Druckabfall basiert. Mit Hilfe dieser Beziehung wurde eine Gleichung abgeleitet, die die Filterventilation in Abhängigkeit von der Strömungsgeschwindigkeit der am Filterende austretenden Luft beschreibt. Bei sechs Marken handelsüblicher Zigaretten ließen sich mit dieser Gleichung durch experimentelle Messung ermittelte Ventilationsgrade bei einer Reihe verschiedener konstant gehaltener Strömungsgeschwindigkeiten genau voraussagen. Unter Einsatz der über die Gleichung bestimmten augenblicksbezogenen Ventilationsgrade konnte die Ventilation während eines Zuges modelliert werden, indem die Gleichung bezüglich der Strömungsgeschwindigkeit über die Zugdauer integriert wurde. Diese Methode der Vorherbestimmung der Ventilation unter bestimmten Zugbedingungen wird anhand sinusförmiger Zugprofile bei einer ganzen Reihe unterschiedlicher Werte für Zugvolumen und Zugdauer beispielhaft dargelegt. Auf der Grundlage der experimentell ermittelten Linearität zwischen der Luftdurchlässigkeit des Papiers und dem Druckabfall wird schließlich qualitativ beschrieben, inwieweit sich verschiedene Kombinationen von Filterumhüllungspapier und Mundstücksbelag auf die Abhängigkeit der Ventilation von der Strömungsgeschwindigkeit auswirken.

RESUME

Un modèle quantitatif décrivant les effets des conditions de tirage sur le taux de ventilation du filtre a été mis au point et évalué. Ce modèle a été établi sur la

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base d'une relation quadratique entre le débit gazeux et la chute de pression, relation vérifiée expérimentalement dans le cas d'un grand nombre de papiers d'enrobage de filtres et de papiers de manchette ainsi que de la combinaison des deux. A partir de cette relation, il a été possible de déduire une équation qui donne le taux de ventilation du filtre en fonction du débit de l'air sortant à l'extrémité du filtre. Pour six marques de cigarettes vendues dans le commerce, cette équation a permis de prévoir exactement les taux de ventilation déterminés expérimentalement, et ce pour différents débits maintenus constants. Les valeurs instantanées de la ventilation prévues grâce à l'équation ont été utilisées pour moduler la ventilation pendant une bouffée en intégrant l'équation considérant le débit pendant la durée de la bouffée. Cette méthode de prévision des effets de conditions spécifiques de tirage sur la ventilation a été démontrée pour des profils de bouffée de forme sinusoïdale, et ce pour un grand nombre de valeurs du volume et de la durée des bouffées. Enfin, les effets de différentes combinaisons de papiers d'enrobage et de papiers de manchette sur la variation de la ventilation en fonction du débit ont été décrits qualitativement sur la base de la linéarité déterminée expérimentalement entre la perméabilité à l'air du papier et la chute de pression.

INTRODUCTION

The smoker's perception of a cigarette is often explained on the basis of cigarette deliveries which have been measured by a machine operating under standard smoking conditions. Machine smoking may not accurately simulate the smoke experienced by the smoker, however, because individual smoking behavior can differ significantly from standard smoking conditions. Therefore, to be valid, studies of the subjective aspects of smoking must account for individual smoking patterns. This requires a thorough understanding of the effects of smoking conditions on cigarette performance.

The importance of smoking conditions to cigarette performance is readily demonstrated by altering the puff duration while holding the puff volume constant at 35 cm^3. For example, a filter cigarette ventilated at 50% will show a significant increase in dry total particulate matter delivery with decreasing puff duration, even though puff volume remains constant. The dry total particulate matter available from tobacco combustion and the measured efficiency of the mouth-end filter segment are only moderately affected by puff duration. A substantial decrease does occur, however, in ventilation and in the removal efficiency of the upstream filter segment (1). The dependence of removal efficiency on smoke flow rate has been studied extensively and is well understood (2, 3), but the effects of smoking conditions on ventilation are not well understood. This paper will, therefore, examine the flow rate dependence of ventilation and develop models which quantitatively describe this phenomenon.

EXPERIMENTAL

The level of filter ventilation is controlled by the balance of resistance to flow between the filter vent system and the portion of the filter and tobacco column upstream of the vent system (4). Flow through the filter and tobacco column is predominantly laminar (5), so the resistance to flow through these elements increases linearly with increasing flow velocity. Filter vents, however, follow a non-linear flow-pressure drop relationship which, ultimately, causes ventilation to vary with flow (6). In order to model the flow dependence of ventilation, flow through cigarette vent systems must first be described. Once a quantitative model has been developed for flow through the vents, an equation can readily be derived which describes ventilation as a function of flow through the cigarette.

The nature of flow through the vents was characterized by measuring the dependence of flow rate on pressure drop for a sample of plug wrap paper. These measurements were made by using pressure drops exceeding those which exist in normal cigarettes to guarantee broad applicability for the resulting model. Model generality was tested by measuring the dependence of flow on pressure drop for pre-perforated tipping papers, plug wrap papers, and combinations of the two over the range of pressure drops typical in ventilated cigarettes.

Mathematical models describing the effect of flow rate on steady state ventilation were evaluated by measuring the ventilation of ten commercial cigarettes with a continuous flow ventilation meter at five flow rates from 8 to 45 cm^3/s. The usefulness of these models for describing the effects of puff volume and duration on ventilation was evaluated by measuring cigarette ventilation with a spirometer. The measurements were made for sinusoidally shaped puffs with a range of puff volume and duration.

RESULTS AND DISCUSSION

Part 1: Flow through Plug Wrap and Tipping Paper

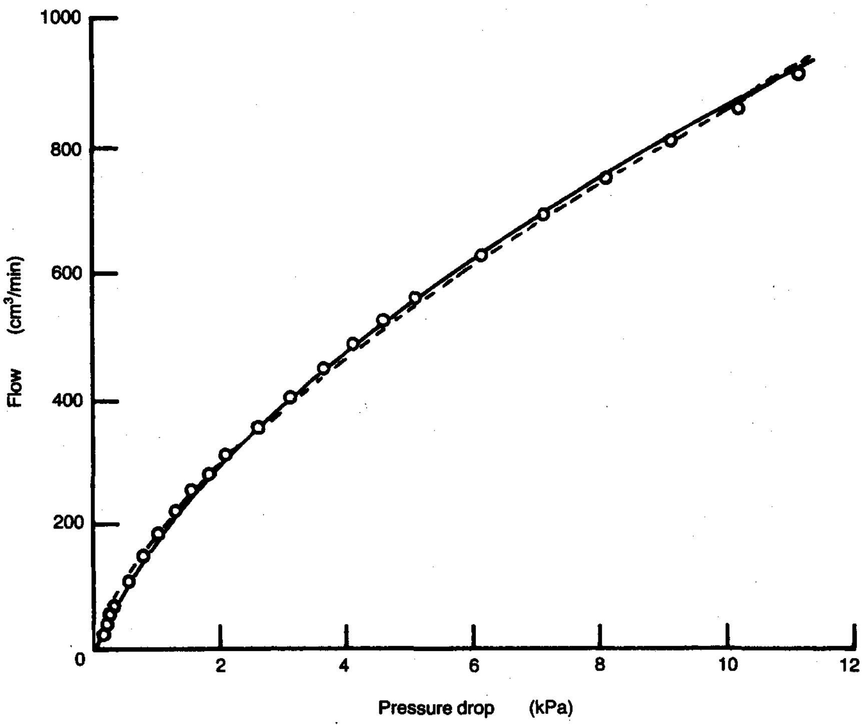
Two equations are commonly used to describe the dependence of vent flow on pressure drop. Equation 1, shown here, describes an exponential relationship between flow, F [cm^3/s], pressure drop, P [kPa], and area, A [cm^2] (6):

$$F = k_e A P^n \qquad (0.5 < n < 1.0) \quad . \qquad [1]$$

The limits of the unitless exponent's value range from 0.5 to 1.0 corresponding to the extreme cases in which flow is controlled entirely by either inertial or laminar effects. Equation 2 is the Ergun equation, which describes the pressure drop across the vent system as the sum of laminar and inertial contributions:

$$P = k_l F/A + (k_i F/A)^2 \quad . \qquad [2]$$

Figure 1.
Comparison of flow-pressure drop models.
Sample: Ecusta 06530 porous plug wrap.



symbol	*flow-pressure drop equation*	*root-mean-square deviation (%)*
----	$F = 189.0 \times P^{0.67}$	2.42
———	$P = 8.41 \times 10^{-6} \times F^2 + 4.07 \times 10^{-3} \times F$	1.25

The laminar flow contribution is represented by the linear term in the equation, k_l [kPa · s/cm], and the inertial contribution is described by the quadratic flow dependence term, k_i [$\sqrt{\text{kPa}}$ · s/cm]. The exponential and quadratic models are equivalent in the limiting cases of total laminar or total inertial flow. The models differ in their description of the intermediate flow cases typical of filter ventilation systems.

The accuracies of these two equations were compared by fitting them to flow-pressure drop data for Ecusta 06530 plug wrap measured over an extended range of pressure drops. Figure 1 shows that both models fit the data well, but the quadratic model had about half the root-mean-square deviation of the exponential model. An extensive comparison of the two models was then undertaken in which plug wraps, tipping papers, and combinations of the two were examined at pressure drops typical of those in ventilated cigarettes.

Flows through paper samples were measured at pressure drops of 0.25, 0.50, 0.75 and 1.0 kPa for 14 pre-perforated tipping papers, 40 plug wraps, and 96 combinations of tipping and plug wrap papers. These measurements were made with an unmodified commercial Phobos permeability tester. The data for each paper were fit with both the quadratic and the exponential models, and the relative root-mean-square deviations were calculated. The results of these calculations are shown in Table 1. For each type of paper studied, measured values of residual error were lower for the quadratic than for the exponential model although these differences were not statistically significant at

Table 1.
Comparison of model performance.

Type of paper	No. of samples	Relative root-mean-square deviation (%) Quadratic model	Exponential model
Pre-perforated tipping	14	1.76	2.16
Porous plug wrap	40	8.07	8.34
Tipping-plug wrap combination	96	2.27	2.58
Average relative root-mean-square deviation (%)	150	3.77	4.08

95% confidence. Both models described the performance of pre-perforated tipping paper more accurately than the performance of porous plug wrap. Residual errors for plug wrap-tipping paper combinations were slightly greater than two percent, which is adequate accuracy for modelling flow through the cigarette vent system.

Although the two equations yield similar results, the quadratic model has several significant advantages. First, this work has shown that the quadratic model fits empirical flow-pressure drop data as accurately as the exponential model. Second, it is mathematically much easier to determine quadratic model coefficients which minimize flow residuals than it is to determine coefficients for the exponential model. Finally, use of the quadratic model makes it possible to derive an exact equation for ventilation, whereas the equation derived from the exponential model must be solved numerically. For these reasons, the quadratic model was chosen to describe flow through the filter vents.

Part 2: Dependence of Steady State Ventilation on Flow

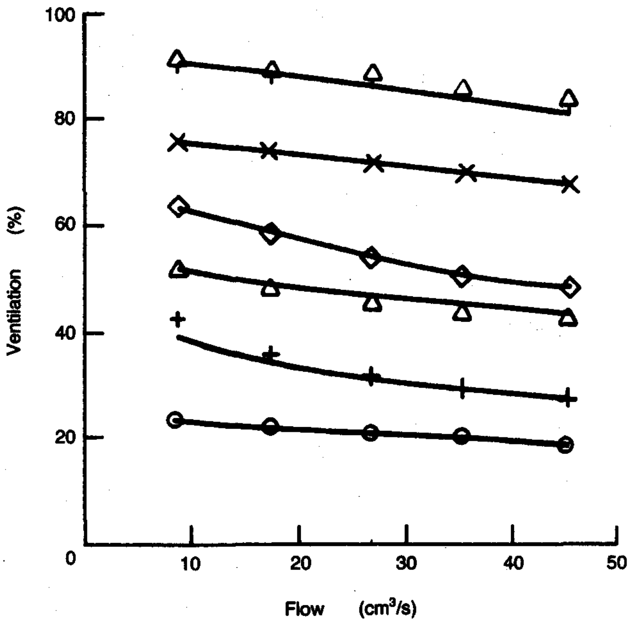
The use of the quadratic flow-pressure drop relationship to derive an equation for the dependence of filter ventilation on the flow velocity through a cigarette is shown in Appendix 1. The fundamental assumption of the derivation is that the pressure drop across the filter vents equals the cigarette pressure drop upstream of the vents. This assumption is reasonable because both pressure drops start at atmospheric pressure and end at the same location in the cigarette.

Several substitutions and mathematical manipulations are required to obtain the final result. The flow upstream of the vents is described with a linear dependence on the upstream pressure drop, and the total flow through the cigarette equals the sum of the upstream and the vent flows. The pressure drop across the vents is described with the quadratic model and filter ventilation is obtained from the ratio of vent flow to the total flow. The final result, shown in equation 3, describes the dependence of the filter ventilation level in percent (V) on the total flow rate through the cigarette in cm^3 per second (F_t) and on two constants (k', k") which are determined by cigarette design and the choice of plug wrap and tipping papers:

$$V = 100k'(\sqrt{1 + k''F_t} - 1)/F_t \quad . \qquad [3]$$

The equation for the flow dependence of ventilation was evaluated experimentally by measuring the flow-pressure drop characteristics of commercial cigarettes and comparing them to the results predicted by the equation. The linear coefficients for the upstream resistance to flow were determined by subtracting the pressure drops of the downstream filter segments from the total vents-closed cigarette pressure drops and by dividing the results by 17.5 cm^3/s, the flow rate used in the pressure drop measurement. The coefficients for

Figure 2. Flow dependence of ventilation.



symbol	vent system (tipping / plug wrap)	k' (cm^3/s)	k" (s/cm^3)
○	mech./porous	35.640	0.0125
+	laser/none	8.822	0.1072
△	laser/porous	41.431	0.0263
◇	on-line laser	22.497	0.0616
×	laser/porous	108.843	0.0145
⍋	mech./none	136.235	0.0139

the quadratic model description of the vent flow were determined by measuring the flows through the vents at four pressure drops after removing the tobacco columns and sealing the tobacco end of the filters. The pressure drops across the vents were determined by correcting the experimental pressure drops for the pressure drops across the mouth end of the filters. The flow rate dependences of ventilation were determined with a continuous flow ventilation meter at five flow rates from 8 to 45 cm^3/s. These experimentally measured values were then compared to ventilations calculated from the model by using the measured coefficients.

The results of these evaluations for six commercial cigarette brands are shown in Figure 2. Experimentally determined ventilation values are represented by symbols, and the theoretically predicted dependences of ventilation on flow rate are shown with solid lines. In all cases, good agreement was observed between experimental and theoretical values, and the rate of change of cigarette ventilation with flow rate has been accurately predicted.

For all the cigarettes tested, ventilation decreased with increasing flow rate, but three of the cigarette brands

Figure 3.
Instantaneous ventilation for a standard puff.

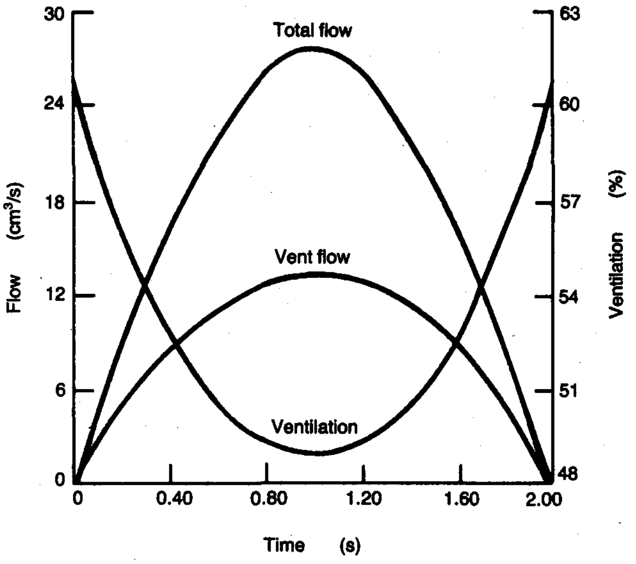
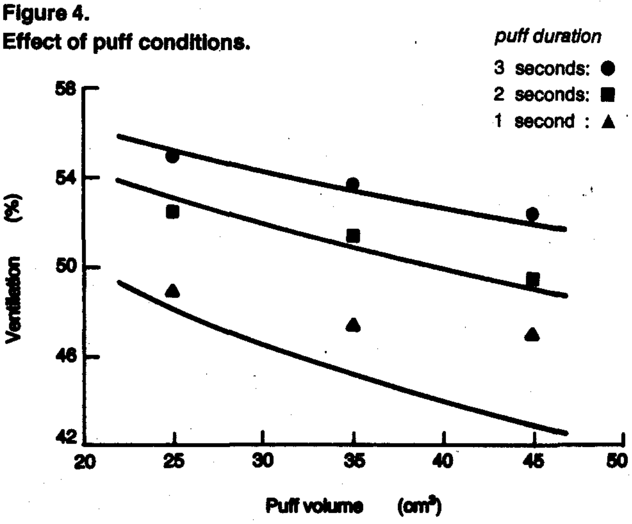


Figure 4.
Effect of puff conditions.



vent system (tipping / plug wrap)	k' (cm^3/s)	k" (s/cm^3)
laser/porous	25.782	0.0450

exhibited especially steep dependences of filter ventilation on flow rate. It is interesting to note how these brands differed from the others. One of the brands utilized laser perforation of the finished filter cigarette; another brand used a filter ventilation system composed of tipping paper without plug wrap; and the third brand utilized tipping paper with extremely large mechanical perforations. In these examples, large, orifice-like vents resulted in more flow-dependent ventilation than for the other brands studied, all of which consisted of fairly typical combinations of plug wrap and pre-perforated tipping paper.

Part 3: Effect of Puff Parameters on Ventilation

The effectiveness of the model for predicting continuous flow ventilation as a function of flow rate has been demonstrated, but the model can also be used to predict the effects of different puffing conditions. In order to study different puffing conditions, the puff must first be defined mathematically. Appropriate descriptions include values of either puff volume or flow rate as a function of time from the beginning to the end of the puff. This description can be an exact mathematical equation, such as a sine function, or just a list of individual values versus time. In the latter case, however, the accuracy and resolution of the analysis may be limited by the amount of data available.

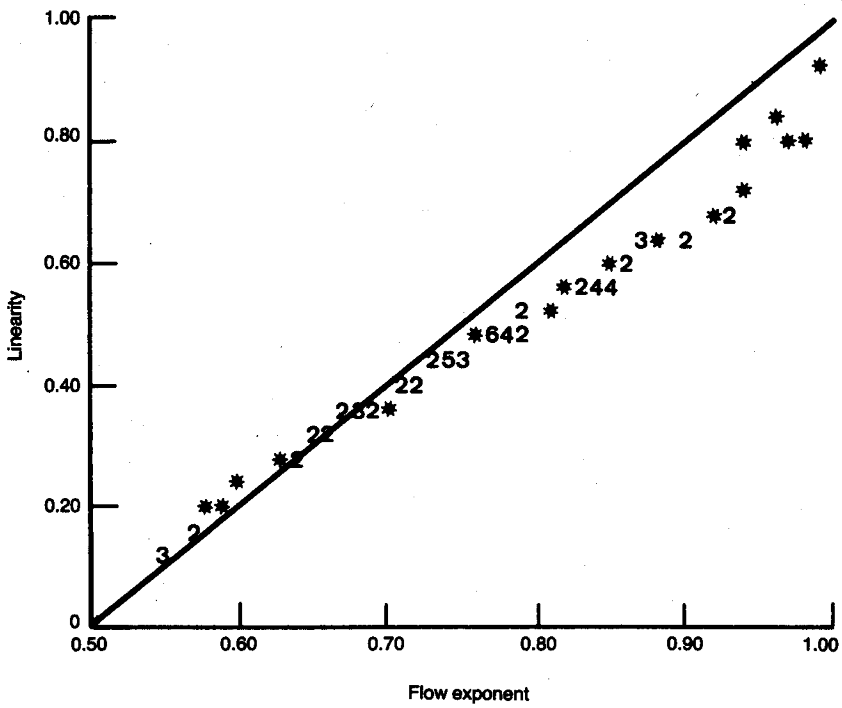
Once the puff has been defined, the flow rate through the vents and the instantaneous ventilation at any time during the puff can be calculated from the model. Appendix 2 demonstrates the general procedure for predicting the effects of puff variables on ventilation by using the standard 2 s, 35 cm^3, sinusoidally shaped puff as an example. Figure 3 shows the total flow, vent flow, and instantaneous ventilation calculated by this procedure. This figure shows that, even for the standard sinusoidally shaped puff, the instantaneous ventilation varies significantly with time such that it is highest at the beginning and the end of the puff and lowest during the middle of the puff. These theoretical model predictions for the time dependence of ventilaton during a puff have been verified by independent experimental measurements (7).

The validity of this procedure described in Appendix 2 was evaluated by measuring the ventilation of a single cigarette for sinusoidally shaped puffs with a range of puff volume and duration. The results of this evaluation are shown in Figure 4. Ventilation values determined with a spirometer are shown as symbols, and values predicted by integrating the results of theoretical model calculations are shown as solid lines. The model accurately predicts ventilation for small puffs of long duration, and it correctly predicts trends in ventilation with smoking conditions; but it does not exactly predict ventilation for short, large puffs. Because the error in model predictions increased with increasing puff velocity, it is likely that distortion of the puff shape was the cause of the model's failure. A syringe-type smoking machine was used for these experiments, and significant deviations from the assumed sinusoidally shaped puff have, under certain conditions, been observed for these machines (8). If the actual puff shape had been measured directly, the ventilation could probably have been predicted much more accurately.

Part 4: Effect of Vent Design on Flow Dependence

Plug wrap and tipping papers are the least linear elements of the cigarette, and their properties largely determine how filter ventilation changes with flow rate. In general, the less linear the flow through these pa-

Figure 5. Relationship between linearity and flow exponent. *



* Numbers represent multiple unresolvable measurements.

pers, the greater the dependence of filter ventilation on flow rate. The linearity of the flow-pressure drop relationship for these papers varies considerably, depending on their porosity, thickness, and perforation size. In order to quantitatively describe the linearity of the flow-pressure drop relationship for the plug wrap and tipping papers, the mathematical definition shown in equation 4 was used to calculate a numerical indicator of flow linearity:

$$\text{linearity} = k_l/(k_l + k_i) \quad . \qquad [4]$$

Figure 6. Factors affecting linearity.

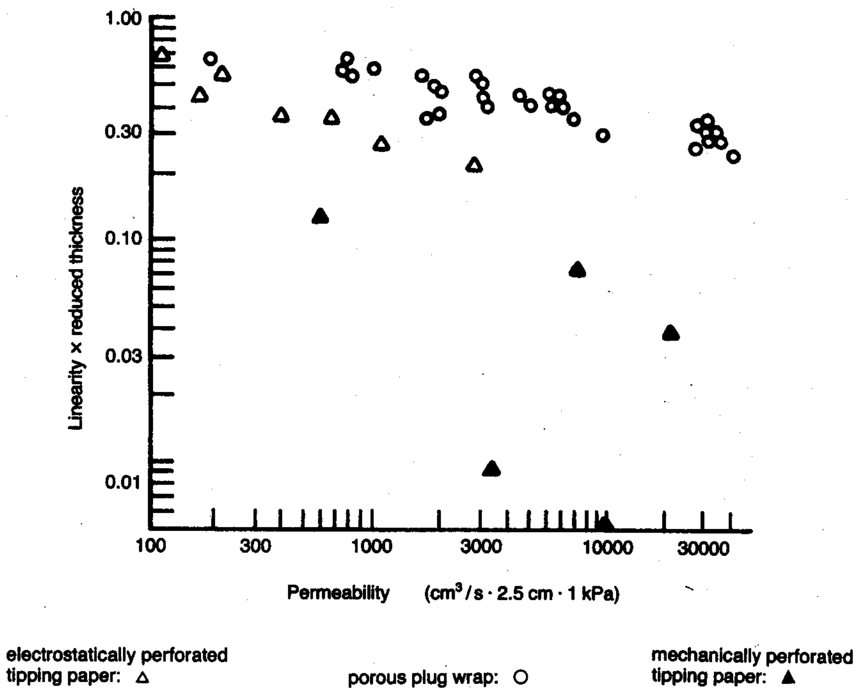
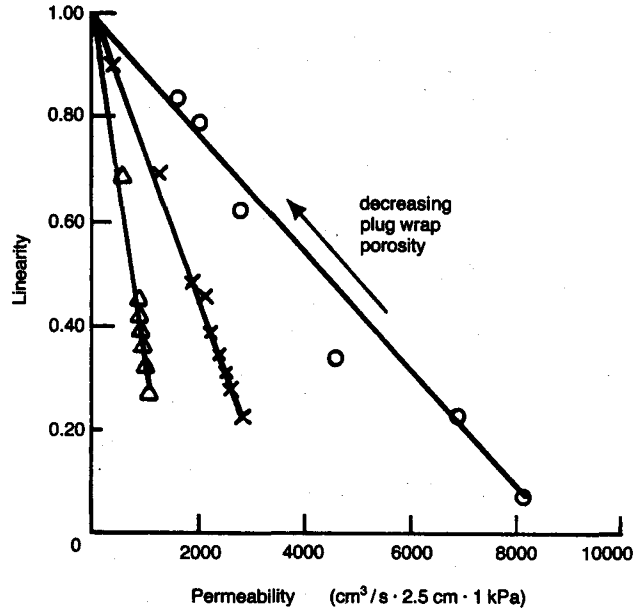


Figure 7. Vent system linearity.



symbol	*perforation type*	*permeability (cm/(kPa · s))*	*flow linearity*
Δ	electrostatic	1082	0.27
×	electrostatic	2759	0.24
○	mechanical	8209	0.08

Values of linearity, calculated from the flow coefficients (k_l and k_i) defined in Appendix 1, vary from 0 to 1, where 0 represents completely inertial flow and 1 is completely laminar flow. This description of flow linearity is analogous to the flow exponent in equation 1 which varies from 0.5 to 1.0. The relationship between linearity and flow exponent appears in Figure 5, which is a plot of the linearity values and flow exponents calculated for all the plug wraps, tipping papers, and combinations of the two that were studied. The solid line shown in Figure 5 was calculated from equation 5:

$$\text{linearity} = 2 \times \text{flow exponent} - 1.0 \quad . \qquad [5]$$

The solid line demonstrates that, although equation 5 approximates the relationship between linearity and the flow exponent, the actual relationship between the two quantities is significantly more complicated.

Figure 6 shows the relationship between the quantity, linearity times the reduced thickness, and permeability for porous plug wrap and electrostatically and mechanically perforated tipping papers. The reduced thickness is defined as the ratio of the average thickness of the paper samples in the experimental population divided by the thickness of the paper being measured. For each paper type, the linearity of the flow-pressure drop curve decreases with increasing paper permeability. However, for a given paper permeability, the flow-pressure drop relationship is most linear for plug wrap and becomes progressively less linear for electrostatically and mechanically perforated tipping papers. This accounts for the observation that cigarettes made from tipping papers with the same permeabilities and ventilations, but different perforation types, have different performances. Because tipping papers have different linearities and because their permeability is measured at a flow which differs from the flow in the final cigarette, the actual filter ventilation obtained with these papers will vary even though they have the same permeability. Plotting the product of the linearity and the reduced thickness simplifies the graph by compensating for differences in paper thickness. This is because thick papers are more linear than their thinner counterparts even when they have the same permeabilities; significant thickness variations were observed only for plug wraps.

For filter vent systems composed only of tipping paper, the measured linearity values indicate that the dependence of ventilation on flow rate should increase as the size of the perforation increases. However, vent systems generally consist of both tipping and plug wrap papers. The performance of these combinations can differ significantly from that of tipping paper.

Figure 7 shows the dependence of vent system linearity on the permeability of combinations of plug wrap and tipping paper. As was observed for single papers, linearity increases with decreasing permeability, with isolated tipping papers being the most non-linear. Flow through the combinations of plug wrap and tipping paper becomes more linear as plug wrap porosity decreases.

A combination of mechanically perforated tipping paper and porous plug wrap with a permeability of 2000 $cm^3/2.4$ cm · s · kPa is significantly more linear than electrostatically perforated tipping paper and porous plug wrap with the same permeability. However, if the permeabilities of the two tipping papers had been equal, the paper combination with the more laminar, electrostatically perforated tipping paper would have been more linear. In general, vent systems which use high permeability tipping and which control flow largely with the plug wrap are more linear than those which use very porous plug wrap and which control flow largely with the tipping paper. It is, therefore, possible to manipulate the dependence of ventilation on flow rate at a target ventilation by the selection of plug wrap and tipping papers.

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Appendix 1.

DERIVATION OF A FLOW DEPENDENT VENTILATION EQUATION.

Mathematical statement	*Assumption*
1. $P_u = P_v$	basic assumption
2. $k_u F_u = P_v$	$P_u = k_u F_u$
3. $k_u F_t - k_u F_v = P_v$	$F_t = F_u + F_v$
4. $k_u F_t - k_u F_v = k_l F_v + (k_i F_v)^2$	$P_v = k_l F_v + (k_l F_v)^2$
5. $k_l^2 F_v^2 + (k_l + k_u) F_v - k_u F_t = 0$	algebra
6. $F_v = k' (\sqrt{1 + k'' F_t} - 1)$	$k' = (k_l + k_u) / 2 k_l^2$ $k'' = 4 k_l^2 k_u / (k_l + k_u)^2$
7. $V \quad 100 k' (\sqrt{1 + k'' F_t} - 1) / F_t$	$V = 100 F_v / F_t$

symbol	*definition of symbol*	*units*
P_u	pressure drop upstream of the vents	kPa
P_v	pressure drop across the vents	kPa
F_u	flow upstream of the vents	cm^3/s
F_v	flow through the filter vents	cm^3/s
F_t	total flow through the cigarette	cm^3/s
V	fractional filter ventilation	(none)
k_u	linear coefficient of upstream flow	$kPa \cdot s/cm^3$
k_l	linear coefficient of vent flow	$kPa \cdot s/cm^3$
k_i	quadratic coefficient of vent flow	$\sqrt{kPa} \cdot s/cm^3$

Appendix 2.

MODELLING THE EFFECTS OF PUFF VARIABLES ON VENTILATION.

1. Specify puff volume, duration, and shape.

example: $$F_t = 27.5 \sin(\pi T[s] / 2); \quad 0 < T < 2 \quad . \qquad [a]$$

2. Determine vent flow versus time for the specified puff.

example: $$F_v = k' (\sqrt{1 + k'' \, 27.5 \sin(\pi T / 2)} - 1) \quad . \qquad [b]$$

3. Determine total vent flow for the puff.

example: $$F_v(\text{tot}) = \int_{T=0}^{T=2} k' (\sqrt{1 + k'' \, 27.5 \sin(\pi T / 2)} - 1) \, dT \quad . \qquad [c]$$

4. Determine the ventilation for the puff.

example: $$\text{ventilation } [\%] = 100 \, F_v(\text{tot}) / F_t(\text{puff volume}) \quad . \qquad [d]$$