Central European Economic Journal

Janusz Kudła¹, Katarzyna Kopczewska², Agata Kocia³, Robert Kruszewski⁴,
Konrad Walczyk⁵

Dynamic Fiscal Solvency with Consumption and Capital Taxes

¹ Faculty of Economic Sciences, University of Warsaw, ORCID ID: 0000-0003-2485-6877, e-mail: jkudla@wne.uw.edu.pl.

² Faculty of Economic Sciences, University of Warsaw, ORCID ID: 0000-0003-1065-1790.

³ Faculty of Economic Sciences, University of Warsaw, ORCID ID: 0000-0001-7978-7747.

⁴ Warsaw School of Economics.

⁵ Warsaw School of Economics.

Abstract: To finance public expenditure a government needs to raise revenue, which mainly comes from taxes and borrowings. During a financial crisis, however, financing of budget deficit is particularly difficult because of a rise in debt servicing costs that crowd out other expenses and raise the concern for government solvency. In extreme cases, governments are constrained to tax, as borrowing opportunities are strictly limited or unavailable. Still, governments can choose from tax menu options (income and consumption taxes), given the flexibility of the tax mix. This article presents a long-term dynamic model of fiscal solvency that shows the equilibrium the revenue maximising government can obtain with reasonable tax rates when capital income can be shifted and there are constraints on the consumption tax. Specifically, the solution predicts a positive level of bonds in the long-term equilibrium and the tax rates dependent positively on the abundance of the tax bases.

Keywords: dynamic modelling, capital tax, consumption tax, fiscal solvency

JEL Codes: C61, H21, H26

Acknowledgements

The article dedicated to prof. Krzysztof Opolski on the occasion of the 70th birthday anniversary

1 Introduction

To finance public expenditure a government needs to raise revenue, which mainly comes from taxes and borrowings. During a financial crisis, however, financing of budget deficit is particularly difficult because of a rise in debt servicing costs that crowd out other expenses and raise the concern for government solvency. In extreme cases, governments are constrained to tax, as borrowing opportunities are strictly limited or unavailable. Still, governments can choose from tax menu options (income and consumption taxes), given the flexibility of the tax mix, that is, the substitutability of one tax for another as a revenue source. However, any two taxes are not perfectly substitutable with each other, but they do differ in their impact on capital and labour supply, output, prices, risk aversion, income distribution and so on. Moreover, taxation is often motivated by non-fiscal reasons, such as redistributive objectives or curbing production and consumption of good with negative external effects. All of these make designing the optimal tax system a challenging task.

Although the theoretical literature has presented a number of arguments for differential taxation, we are still far from *fully understanding what the main determinants of the direct to indirect tax mix are* (Martinez-Vazquez, Vulovic and Liu 2011, 38). So far, however, the main focus of the literature has been put on the effects of the structures composed of labour and consumption taxes, and on a possibility to substitute one for another in order to increase welfare. Yet, to be efficient, an increase in the labour tax requires a low elasticity of labour income and remains unpopular for social and political reasons. It seems, therefore, more practical to consider capital taxes as a complement to consumption taxes when a government strives to remain solvent in the long term. This point is of particular interest as the capital – consumption tax mix is a relatively neglected area of research.

The model presented hereafter takes into account two different taxes, on capital and consumption but not on labour, and concentrates on the fiscal solvency without detailed specification of the labour market. This implies one production factor (capital) and two sources of budgetary revenues: capital income and consumption. Consumption is only indirectly linked to capital by its net remuneration. The government budget constraint is complemented with bonds revenue. We analyse the problem of determining the capital-consumption tax mix with the possibility of public debt issuance. It means

that the optimisation problem has to be set by the government, not by economic agents. The government's choice is limited by the fiscal solvency rule and possible income shifting hindering tax collection. We strive to determine some features of the long-term equilibrium, solving dynamic optimisation problem with continuous time. In other words, we are looking for optimal long-term tax policy of the government aiming to sustain fiscal solvency, contingent on the values of chosen parameters. We assume capital income to be partially or fully shifted abroad as a consequence of tax avoidance strategies of economic agents. We believe that this better reflects the limited mobility of physical capital and the treat of possible income shifting. For example, one can expect the latter to intensify during an insolvency crisis when fast liquidation of physical investments is not feasible but the possibility of capital income withdrawal is not affected. It should be noted that taxation of consumption depends greatly on the values of external parameters because only part of consumption is explicit and can be taxed.

The proposed approach exhibits several derogations from the standard analyses. First, we assume maximisation of budget revenues instead of utility (or welfare) maximisation, that is, the government is of the Leviathan type. Therefore, the objective function does not require the determination of a specific utility form and better fits to the governmental behaviour during financial distress when constraints on borrowing are binding. Government revenues come from the two tax instruments (capital and consumption taxes) and bond issuance. Second, the analysis concentrates on the values of capital, consumption, bonds and the respective tax rates in equilibrium. Next, the effects of parameter deviations from equilibrium are calculated with the procedure originally proposed by Boadway (1979).

Moreover, we propose three more extensions to the typical multi-tax dynamic model. First, the bond issuance is positive in the steady state but its level is limited by the ratio of tax revenues to the product of labour and capital. Fulfilling the constraint ensures fiscal solvency of the country. This is a simplifying assumption that allows for introducing the solvency rule into the model without detailed specification of the financial market. The idea stems from creditworthiness and measures the capacity of the market to accept the given level of governmental bond issue. Hence, the bonds are not a transitory phenomenon of budgetary financing, and they remain positive in the equilibrium simply because they provide less distortion to the economy

than taxes (on capital and consumption). At the same time, there is an upper limit on the tax burden and debt financing that does not violate the fiscal solvency postulate. This assumption better fits to the persistent nature of debt and prevents the debt to be zero in the long term. Second, taxing of capital income can be partially avoided by shifting it abroad. The shifting possibility is exogenous because it depends on the existing ‘tax shifting technology’ and the level of foreign ownership of capital. The tax shifting technology includes the existence of tax havens and a group of controlled enterprises that may generate tax savings. At a given moment, it is determined by local tax regulations and tax enforcement efficiency. Third, the consumption tax can be raised but there are some external factors limiting its maximum rate as consumption tax burden may be partly avoided. They are not directly modelled but stay linked with legislative or competitive reasons such as cross-border purchases or shadow economy expansion. It rules out the possibility of fully taxing consumption as an equivalent of imposing distortionary taxes on capital.

There exists equilibrium in the model for the sufficiently small shifting parameter and time preference parameter. The obtained equilibrium has one internal (non-boundary) solution with positive bond issuance and two positive tax rates. Despite the fact that the model cannot be solved analytically, it allows for testing the impact of some parameter changes on the tax rates, bonds and the value of state variables (capital and consumption) in the equilibrium. The numerical simulation includes the impact of four important parameters on the state variables and two tax rates. The parameters cover capital income shifting, the market reaction on the indebtedness of a given country to the proxy of production, the explicit consumption rate (the part of consumption which is taxed) and intertemporal preferences.

The article is structured as follows. First, a dynamic model of government revenue maximization in continuous time is described. In the next step, we present the sketch of dynamic optimisation procedure involving the description of differential equations of control variables. This lets us carry out numerical analyses curbed with control variables (tax rates) bounded to the range 0–1. The analysis concentrates on the impact of the four parameters affecting consumption, capital, the capital tax, the consumption tax and the issuance of bonds. Then the time-varying rates of change are provided according to the procedure originally proposed by Boadway (1979). The article ends with conclusions.

2 The model

Government maximises the revenues consisting of tax receipts of consumption c , capital k and bonds issue b_t :

$$a\tau_c + \tau_k srk + b_t. \quad (1)$$

The control variables in the model are the tax rates on consumption τ_c and tax rate on capital τ_k . We allow only for solutions with the tax rates satisfying the following condition:

$$0 < \tau_c, \tau_k < 1.$$

This excludes the possibility of consumption or capital subsidisation sometimes postulated in optimal taxation models.

The first two elements of equation (1) describe the tax revenues. Consumption is divided into two sets: explicit (taxable) and implicit (untaxed). The share of these two sets of consumption is specified by the parameter a , which is defined as the size of a taxable part of consumption. The consumption tax rate is τ_c whilst $1 - a$ is the part of consumption done in the shadow economy or abroad. For the purpose of optimisation, we assume that a is constant and has a value in the range from 0.5 to 1 to ascertain the value of τ_c in the range 0–1. In the further numerical analysis, changes in this parameter will allow us to determine how different opportunities of consumption taxation (e.g. tax base broadening or shadow economy limitation) can affect the other rates of taxation and the bond level in the equilibrium. Income from capital is taxed at the rate τ_k . Alike in the case of consumption, only part of the income is taxed because the rest can be hidden or transferred abroad. The size of the transferred or concealed income is described by the shifting parameter s . The s remains in the range of 0–1, where 1 means no income shifting and 0 a full shifting. In practice, this covers the broad range of phenomena such as hiding income, limitations imposed on income transfer abroad, tax exemptions or legal forms of income tax reduction.

Apart from the taxes, the government can issue bonds. By and large, debt financing is superior to tax revenues because it does not generate distortions to an economy such as taxes levied on the factors of production or consumption. However, the size of bond issuance is not arbitrary. We assume that the level of new bonds is determined on the financial market and requires meeting a pre-specified ratio of tax revenues to

the output. The fulfilment of this ratio means fiscal solvency. As the production function in the equilibrium is not known (it depends, amongst others, on the relation between capital and technological progress), we assume that it could be substituted by the product of capital as a proxy of the output. Accordingly, the bond issuance is the greater, the higher is the tax revenues in relation to the product of capital. In other words, the new debt level depends positively on the relation similar to the rate of fiscalism, that is, the ratio of tax revenues to the output. Nevertheless in the contrast to the rate of fiscalism, we augment the formula with non-negative parameter p , defining the sensitivity of the financial market to the fiscal solvency condition. The amount of this parameter determines the bond issuance accepted by the investors under current circumstances. In such a way, we avoid the necessity of modelling the market interest rate and the capacity of the financial market. Finally, the restriction on the new debt (bond issuance) takes the following form:

$$b_t = p \frac{a\tau_c c + \tau_k srk}{k}, p \geq 0 \quad (2)$$

where $c, k > 0$, which enforces a positive amount of consumption and capital. This constraint precludes the situation when the level of bonds in the equilibrium tends to infinity or zero. Both the cases seem to be unpractical, and they would mean either a lack of proper risk assessment by the investors or the infinite capacity of the financial market. Simultaneously, equation (2) is consistent with the observation that poor countries have difficulty in obtaining high tax revenues and, therefore, are characterised by less indebtedness than wealthy countries.

There are two state variables: consumption and capital. Each of them is dependent on taxation. Specifically, changes in consumption over time are driven by two tax rates, and changes in capital depend on capital tax rate. Differential equations for the state variables are written as follows:

$$\frac{dc}{dt} = (a(1 - \tau_c) - (1 - a))[(1 - \tau_k)srk - b_t] \quad (3)$$

where $a \in [0.5; 1]$ and $s \in [0; 1]$.

Consumption is determined by disposable income (the expression in the square brackets of equation (3), the proportion of taxed and untaxed consumption, a , and the tax burden, τ_c).

$$\frac{dk}{dt} = (1 - \tau_k)srk - c \quad (4)$$

Capital is created from capital income, net of tax and gross consumption. The government does not create capital in the economy. The depreciation of capital (similar to that in Wildasin, 2011) is omitted in the model merely because we find it less important for modelling the decision taken by the government striving for the long-term solvency.

Taking the objective function and the differential equations for the state variables, we can formulate the dynamic maximisation problem:

$$\max_{0 \leq \tau_c, \tau_k \leq 1} \int_0^\infty (a\tau_c c + \tau_k srk + b_t) e^{-\rho t} dt \quad (5)$$

where the constraints on the state variables are given by equations (3) and (4). ρ is the rate of time preference and can be considered as a positive number, $\rho > 0$. Then, we proceed according to the well-established methodology of dynamic optimisation (Pontryagin et al., 1962; Seierstad, Sydsæter, 1987; Léonard, Long, 1992; Schättler, Ledzewicz, 2012).

The necessary condition for the problem (5) under the constraints (3) and (4) can be expressed using the current-value Hamiltonian function H . Let Φ_1 and Φ_2 denote the right-hand sides of equations (3) and (4), respectively. Then,

$$H = a\tau_c c + \tau_k srk + b_t + \mu_1 \Phi_1 + \mu_2 \Phi_2. \quad (6)$$

By maximising H due to the control variables, the following conditions must be met (we are looking for solutions that belong to the interior of the set of admissible controls, because the optimum solution may also lie at the boundary of the control region):

$$\begin{aligned} \frac{\partial H}{\partial \tau_c} &= ac + \frac{\partial b_t}{\partial \tau_c} + \mu_1 \frac{\partial \Phi_1}{\partial \tau_c} = 0, \\ \frac{\partial H}{\partial k} &= srk + \frac{\partial b_t}{\partial k} + \sum_{i=1}^2 \mu_i \frac{\partial \Phi_i}{\partial k} = 0. \end{aligned} \quad (7)$$

Then, the equations of motion for the co-state variables can be written as follows:

$$\begin{aligned} \mu_1 &= -a\tau_c - a \frac{\partial b_t}{\partial c} (2 - \tau_c) + \mu_2 + \rho \mu_1, \\ \mu_2 &= -sr\tau_k - \frac{\partial b_t}{\partial k} - \mu_1 (a(1 - \tau_c) - (1 - a)) \end{aligned} \quad (8)$$

$$\left[(1 - \tau_k)sr - \frac{\partial b_t}{\partial k} \right] - \mu_2 ((1 - \tau_k)sr - \rho).$$

Equation (7) defines the control variables as functions of the state and co-state

$$\begin{aligned}\tau_c &= \Psi_1(k, c, \mu_1, \mu_2), \\ \tau_k &= \Psi_2(k, c, \mu_1, \mu_2),\end{aligned}\quad (9)$$

and co-state variables as functions of the state and control variables:

$$\begin{aligned}\mu_1 &= \Psi_1(k, c, \tau_1, \tau_2), \\ \mu_2 &= \Psi_2(k, c, \tau_1, \tau_2),\end{aligned}\quad (10)$$

Taking the time derivatives of equation (9) and substituting equations (3), (4), (8) and (10), we get the differ-

ential equations that describe the dynamics of the control variables (see Appendix for discussion):

$$\begin{aligned}\dot{c} &= \Theta_1(k, c, \tau_c, \tau_k), \\ \dot{k} &= \Theta_2(k, c, \tau_c, \tau_k),\end{aligned}\quad (11)$$

Finally, we should find the solutions of the four-dimensional system of nonlinear differential equations describing the dynamics of state and control variables (equations (3), (4) and (11)). Unfortunately, this problem cannot be solved analytically, but we can formulate the following proposition:

Proposition: For sufficiently large s and small p , the model has the unique equilibrium:

$$\begin{aligned}k &= \frac{\sqrt{p^2[wr^2s^2(-1+a^2-\rho)-8(-1+a)^2rs(-2-2a+4a^2-3p)\rho+\rho^2(-2+6a-4a^2+\rho)^2]}+pq}{2(-2+2a-\rho)(-rs+\rho)}, \\ c &= -\frac{(-1+a)[p(-4(-1+a)ars+2(1+a(-3+2a)+rs)\rho-\rho^2)+\sqrt{p^2(w(-2+2a-\rho)(-rs+\rho)^2+q^2)}}{(2-2a+\rho)^2}, \\ \tau_k &= -\frac{p[4(-1+a)rs(-1+a-\rho)+(2-6a+4a^2-\rho)\rho]+\sqrt{p^2(w(-2+2a-\rho)(-rs+\rho)^2+q^2)}}{4(-1+a)prs(2-2a+\rho)}, \\ \tau_c &= 2-\frac{1}{a},\end{aligned}$$

such that $0 < \tau_c, \tau_k < 1$ and $c, k > 0$, where

$$w = 16(-1+a)^2$$

and

$$q = -4(-1+a)^2rs + p(6+2a(-5+2a)+p).$$

3 Numerical analysis

The described model cannot be solved analytically, so we need to run numerical simulation to discover some features of the obtained solution. The value of parameters used in the numerical simulation should help to describe the behaviour of the two control variables: the level of bond issues and state variables. We found a and s as the most interesting parameters because they represent the shifting activity and the tax avoidance. The level of interest income from capital is set at 5%, so $r = 0.05$, and the tolerance for debt is high, $p = 4$. These values are constant for all simulations. In addition, we

require a to be >0.5 . This means that more than half of the total consumption is taxed. This condition precludes the taxation of consumption to be outside the range 0–1. Time preference rate is set at 0.5. Moreover, we check for the intertemporal preference rate and the rate of market reaction on the indebtedness of a given country to the proxy of production. For that reason, there are four scenarios.

The first scenario examines the impact of a parameter a (the explicit rate of consumption). One can see the taxation of capital is growing with parameter a . However, with different rate of adjustment, for low a , the dynamic is decreasing and, for high a , the dynamic is increasing. The taxation of consumption is also increasing with the share of explicit consumption but the speed of this adjustment is roughly constant. Higher taxation involves higher bond issuance because the credibility of government is increasing. Unfortunately, the obtained result indicates that if the level of hidden consumption is high, then neither of tax instruments can improve the budgetary situation of the government. We can say that for solvency reasons, the government should

Fig. 1. Equilibrium tax rates as a function of explicit consumption share ($p = 0.03$, $s = 0.8$)

Fig. 2. Equilibrium volume of consumption capital as a function of explicit consumption share ($p = 0.03$, $s = 0.8$)

Fig. 3. Equilibrium volume of bonds as a function of the shifting activity ($p = 0.03$, $s = 0.8$)

ensure that the consumption will not be tax avoided. The consumption increases at first and then decreases at the equilibrium as the a becomes larger. The increas-

ing dynamic of capital tax is probably responsible for the consumption drop. Despite the rising taxation, the capital is growing all the time together with explicit consumption share (Fig. 1–3).

The second scenario describes the impact of parameters referring to shifting (or hiding) capital income. The high s means low shifting. The most pronounced effect is linked to the capital income taxation, which is strongly decreasing with the increase in s . This differs significantly from the reaction of consumption tax that stays constant. The two tax rates seem to be independent. The level of debt is growing because tax revenues on consumption are increasing, which is due to the growth of consumption. However, the capital invested in the country tends to be lower if the shifting possibilities diminish. We can say that low income shifting spurs consumption but deteriorates the capital accumulation. In this situation, the government should prefer bond financing and stable taxation of consumption. This is

Fig. 4. Equilibrium tax rates as a function of the tax avoidance ($p = 0.03$, $a = 0.9$)

Fig. 5. Equilibrium volume of consumption capital as a function of the tax avoidance ($p = 0.03$, $a = 0.9$)

Fig. 6. Equilibrium volume of bonds as a function of the tax avoidance ($p = 0.03$, $a = 0.9$)

interesting result because, on the one side, it advocates for high taxation of capital when capital is abundant in the country irrespective of the shifting possibilities.

On the other side, the high s provides the low level of capital, so the high shifting possibilities are beneficial for the capital accumulation and the low ones are destructive (Fig. 4–6).

The third scenario involves the behaviour of intertemporal preferences. Higher preferences for the future require the increase in capital and consumption but the proportion of these two state variables is changing differently with ρ . The speed of capital increase is accelerating whilst the consumption is slowing down. At the same time, tax rate on consumption stays unaffected but the tax rate on capital is increasing. The latter improves solvency making bonds level to be rising. The optimal policy (in the case of different intertemporal preferences) requires the constant taxation of consumption and taxation of capital dependent on its level but not proportional. The tax revenues should be augmented with bond revenues according to the current 'credibility' level (Fig. 7–9).

Fig. 7. Equilibrium volume of capital and consumption as a function of the intertemporal preferences ($s = 0.8, a = 0.9$)

Fig. 8. Equilibrium tax rates as a function of intertemporal preferences ($s = 0.8, a = 0.9$)

Fig. 9. Equilibrium volume of bonds as a function of the intertemporal preferences ($s = 0.8, a = 0.9$)

4 Variational equations

Let $x(t) = [k(t), c(t), \tau_c(t), \tau_k(t)]$ denote the vector of the state and control variables, and let

$$F(x, \alpha) = [\Phi_1(x, \alpha), \Phi_2(x, \alpha), \Theta_1(x, \alpha), \Theta_2(x, \alpha)]$$

where $\alpha = s, a$, denote the vector of functions that describe dynamics of the state and control variables. In order to see how changes in the shifting activity (parameter a) and the tax avoidance (parameter s) affect the solution of the four-dimensional dynamic system,

$$\dot{x}(t) = F(x, \alpha) \quad (12)$$

it is necessary to derive the ‘variational equations’ (see: Boadway 1979, Wildasin 2011). If $y = \xi(t, \alpha)$ denotes

Fig. 10. The time-varying rate of change in the tax rates and state variables ($a = 0.9$)

Fig. 11. The time-varying rate of change in the tax rates and state variables ($s = 0.8$)

Fig. 12. The time-varying rate of change in the tax rates and state variables ($\rho = 0.03$)

the solution vector to (12), then variational equation is given by

$$\dot{y} = F_x(x^*, \alpha)y + F_\alpha(x^*, \alpha) \quad (13)$$

where $F_x(x^*, \alpha)$ is the Jacobian matrix of F and $F_\alpha(x^*, \alpha)$ is a vector of partial derivatives of F calculated with respect to the chosen parameter α evaluated at the equilibrium x^* .

To see how changes in the chosen parameter affect the solution to equation (12), we assume that the dynamic system (12) is in the equilibrium. Then we increase the chosen parameter (initial value of the parameter that is perturbed is indicated in braces) and we observe how this perturbation evolves over time. This allows for numerical simulation. The parameters of the simulations are set to $r = 0.05$, $p = 4$, $\rho = 0.03$, $s = 0.8$, $a = 0.9$.

The time-varying rate of change in the solution to the system (12) with respect to the parameter a (which is assumed to be equal to 0.9) is shown in Fig. 10.

One can observe that change in a provides for the decrease in consumption and consumption tax rate over time. This is accompanied by increasing taxation

of capital and initially increasing and finally decreasing level of capital.

The time-varying rate of change in the solution to the system (12) with respect to the parameter s (which is assumed to be equal to 0.8) is shown in Fig. 11.

The change in s from equilibrium increases the capital and consumption as well as the consumption tax over time. The taxation of capital is increasing first and then decreasing (Fig. 12).

The change in ρ from equilibrium increases capital but decreases consumption. The effect on taxation is ambiguous. At the beginning, both taxes decrease but, after some time, the taxation of capital starts growing.

5 Conclusions

There are some interesting numerical results referring to the equilibrium. First, the higher the explicit part of consumption (the taxed consumption), the higher are the taxation of capital, taxation of consumption and the level of debt. The latter is a consequence of dynamic constraint on bond issuance. The level of bond indebted-

ness is positively affected by the level of tax revenues. In other words, higher revenue allows for higher level of debt. But for very high level of capital taxation, we can expect rapid fall of the consumption.

If consumption tax is not efficient in generation of revenue (because of high level of untaxed consumption), then there is no policy providing for the high level of revenues. The consumption achieves the maximum value for some level of explicit consumption, and for higher values of explicit, consumption is decreasing. The latter effect is triggered by very high capital taxation. However, the tax rates on capital and consumption are independent in many cases (for s and ρ). Higher capital income tax shifting parameter (low s) induces the accumulation of capital and encourages the increase in capital tax. The intertemporal preferences do not affect the consumption tax and the relation between capital, and capital tax is increasing but not proportional.

The changes in parameters at the equilibrium trigger the adjustment over time. In most cases, this adjustment was monotonic but sometimes (for capital or capital tax) the adjustment process involves the changes in both directions. It can indicate that taxation of capital is very vulnerable to the changes in parameter, and it is difficult to obtain sustainable solution after such a change.

References

- [1] Boadway, Robin. 1979. "Long-Run Tax Incidence: A Comparative Dynamic Approach". *Review of Economic Studies* (46)3: 505–511.
- [2] Léonard, Daniel and Ngo van Long. 1992. *Optimal Control Theory and Static Optimization in Economics*. Cambridge: Cambridge University Press.
- [3] Martinez-Vazquez, Jorge, Violeta Vulovic and Yongzheng Liu. 2011. "Direct versus indirect taxation: trends, theory, and economic significance". In: *The Elgar Guide to Tax Systems*, eds. Emilio Albi and Jorge Martinez-Vazquez, 37–92. Edward Elgar Publishing.
- [4] Pontryagin, Lev S., Vladimir G. Boltyanskii, Revaz V. Gamkrelidze and Evgenii F. Mishchenko. 1962. *The Mathematical Theory of Optimal Processes*. New York: Interscience Publishers.
- [5] Seierstad, Atle and Knut Sydsæter. 1987. *Optimal Control Theory with Economic Applications*. New York: Elsevier North-Holland.
- [6] Schättler, Heinz and Urszula Ledzewicz. 2012. *Geometric Optimal Control. Theory, Methods and Examples*. New York: Springer-Verlag.
- [7] Strulik, Holger and Timo Trimborn. 2012. "Laffer Strikes Again: Dynamic Scoring of Capital Taxes". *European Economic Review* (56)6: 1180–1199.
- [8] Wildasin, David E. 2011. "Fiscal Competition for Imperfectly-Mobile Labor and Capital: A Comparative Dynamic Analysis". *Journal of Public Economics* (95)11: 1312–1321.

Appendix

First-order condition for the maximisation of the Hamiltonian function (7)

$$\frac{\partial H}{\partial \tau_c} = ac + \frac{\partial b_t}{\partial \tau_c} + \mu_1 \frac{\partial \phi_1}{\partial \tau_c} = 0$$

$$\frac{\partial H}{\partial \tau_k} = srk + \frac{\partial b_t}{\partial \tau_k} + \mu_1 \frac{\partial \phi_1}{\partial \tau_k} + \mu_2 \frac{\partial \phi_2}{\partial \tau_k} = 0$$

is a system of two linear equations in μ_1 and μ_2 with solution:

$$\mu_1 = -\frac{ac + \frac{\partial b_t}{\partial \tau_c}}{\frac{\partial \phi_1}{\partial \tau_c}} = \phi_1(k, c, \tau_c, \tau_k),$$

$$\mu_1 = \frac{srk + \frac{\partial b_t}{\partial \tau_k} - \frac{\frac{\partial \phi_1}{\partial \tau_k}}{\frac{\partial \phi_1}{\partial \tau_c}} \left(ac + \frac{\partial b_t}{\partial \tau_c} \right)}{\frac{\partial \phi_2}{\partial \tau_k}} = \phi_2(k, c, \tau_c, \tau_k)$$

where

$$\frac{\partial b_t}{\partial \tau_c} = \frac{ac}{k}, \frac{\partial b_t}{\partial \tau_k} = sr,$$

$$\frac{\partial \phi_1}{\partial \tau_c} = -a[(1 - \tau_k)srk - b_t] - (a(1 - \tau_c) - (1 - a)) \frac{\partial b_t}{\partial \tau_c},$$

$$\frac{\partial \phi_1}{\partial \tau_k} = -(a(1 - \tau_c) - (1 - a)) \left[srk + \frac{\partial b_t}{\partial \tau_k} \right],$$

$$\frac{\partial \phi_2}{\partial \tau_k} = -srk.$$

When we consider (7) as a system with tax rates as variables, then again it is a linear system with solutions given by equation (9).