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Topology of Complex Networks and Demand of Intraday Liquidity: Based on the Real-Time Gross Settlement System³

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Abstract: Based on the payments and settlement system, the influence of the topology of capital flow networks on the extra short-term liquidity demand is investigated. Through modelling the circulating mechanism of liquidity in a network, its different influencing factors are analysed. The factors relating to the strength of nodes and leakage of liquidity that influences the liquidity demands of real-time settlements are studied from the perspective of both the system and members, using different simulation methods. The results show that strength will lead the member’s liquidity demand to increase but the strength distribution will lead the system’s liquidity demand to decrease, in cases with no leakage effect or unchanged leakage effect. The liquidity demand of the entire system is positive compared to the amount of leakage effect but uncorrelated to the distribution of the leakage effect among members, if the effect of strength distribution is unchanged. If the effects of strength, strength distribution and leakage are changed together, the latter is the dominant factor that influences the liquidity demand of both system and members. The above findings are useful for the management and supervision of short-term liquidity demand in complex financial systems, and for liquidity risk management and liquidity rescue policymaking.

Keywords: network structure, intraday liquidity, real-time settlement, liquidity management.

JEL Codes: G21, G32, C15

1 Introduction

The liquidity flow in the financial system will change rapidly in direction or/and volume in a very short time frame. This characteristic of liquidity flow could make risk management and supervision more difficult. A significant number of studies in the literature have attempted to research how the topology of complex financial networks contributes to the expanding and shrinking of liquidity in a system. These studies include work done on the topology of complex financial networks and their formation (e.g., Boss et al. 2004; Garratt et al. 2011; Silva et al. 2016), the robustness of systems and the importance of nodes under different network topologies (e.g., Iori et al. 2006; Soramäki et al. 2013; Liu et al. 2016), and the contagiousness of systemic risk in a financial network (e.g., Imakubo and Soejima 2010; Lenuz and Tedeschi 2012; Hoffman et al. 2015). In the field of intraday liquidity, recent papers focus mainly on settlement mechanisms and their involvement (e.g., Bech and Soramäki 2005; Martin and McAndrews 2010; Tsuchiya 2013), the behaviour of members (e.g., Bech and Garratt 2003; Mills and Nesmith 2008; Abbink et al. 2010; Galbiati and Soramäki 2013), systemic externality (e.g., Bedford et al. 2004; Bech and Garratt 2012) and policies of central banks (e.g., Ball et al. 2011; Zhang 2012; Munoz and Gonzalez 2013). However, there are few papers that attempt to investigate the implications of network topology on the liquidity demand of individual members or on whole systems by analysing how the liquidity circulates through a complex network.

This paper aims to contribute to this understanding by offering a model of intraday liquidity demand and circulation in a complex network, and then by studying how the topology of network affects the intraday liquidity demand in an RTGS system through simulations. The paper is organized as follows. Section 2 provides a model to show how the liquidity of a member circulates through the network to settle the payments in one day. Section 3 describes the design of the network and the details of the simulation. Section 4 presents the results and Section 5 concludes.

2 A model of liquidity circulation in complex networks

In an RTGS system, the members settle their obligations using their intraday liquidity every trading day. Let the matrix \( P_{i\rightarrow j} \) denote this directed and weighted intraday payment network, and let us suppose that every member in the system will have nonzero inflows or outflows or both in a trading day. Let the \( p_{ij} \) of \( P_{i\rightarrow j} \) denote the total value of payment from member \( i \) to \( j \) in day, and, namely, the directed and weighted linkage between node \( i \) and node \( j \). This yields \( p_{ij} = 0, \forall i \neq j \) and \( p_{ii} \geq 0, \forall i \neq j \). Using the terms of a complex network, the out-strength and in-strength of \( i \) equals the total outflow and inflow if \( i \) in day, respectively, i.e., \( OS_i = \Sigma_p p_i \) and \( IS_i = \Sigma_p p_i \). Thus, the total strength of the network is the total value of payments in the system, i.e., \( TS = \Sigma_i OS_i = \Sigma_i IS_i = \Sigma_i \Sigma_j p_{ij} \).
The real-time gross settlement is driven by the intraday liquidity held by members. The initial liquidity held by members at the beginning of a day is denoted as the vector \( L_{\text{ini}} \) \((l_i \geq 0 \forall i)\). We suppose that no additional liquidity can be acquired by any members during the day for settlement. \( L_{\text{ini}} \) will be injected into the network through the settlement of payments by members, and will circulate in the network to settle yet more payments.

From \( P_{\text{ini}} \) we can generate the transition matrix \( Z_{\text{ini}} \), the element of which is

\[
z_{ij} = \frac{p_{ij}}{\text{OS}_i}.
\]

The transitional probability \( z_{ij} \) is the chance that a payment from \( i \) is paid to \( j \) exactly, such that \( z_i \) also shows the distribution of the total out-payment of \( i \) among other members (or nodes). Let \( d(L) \) denote the function that transforms a liquidity vector to a diagonal matrix, with that vector as the main diagonal element. Let \( L_{\text{ini}} \) denote a \( n \times n \) identity matrix. Thus, the process of circulation of liquidity in the payment network can be expressed as:

1. **Smooth Circulation.** If there is no liquidity leakage or stopping mechanism, equation (4) will not converge, i.e., every nonzero initial liquidity will circulate in the network forever, and therefore, an infinitive value of payments can be settled in the system.

2. **Effect of Strength and Effect of Strength Distribution.** The elements of matrix \( P \) restrict the total value of the payment between every pair of nodes, i.e., \( x_{ij} \leq p_{ij} \) and \( \sum_j x_{ij} \leq \sum_j p_{ij} \). If the value of the payment settled by the node equals its out-strength, the liquidity of this node will not enter into the network and circulate again. This effect of strength exists at the member level and the system level. In addition, the distribution of total out-strength of a node may have systemic effect with the same total out-strength. This effect is called an effect of strength distribution and it exists only on the level of the system.

3. **Effect of Leakage.** If the network is asymmetric, i.e., there are some nodes with in-strengths greater than their out-strengths, or \( IS_i > OS_i \), some liquidity will sink to these nodes and the circulation process will be broken. These kinds of nodes are called leak nodes for the purposes of this paper. Of course, must be spillover nodes with \( IS_i < OS_i \) if there are any leak nodes in the network. While the leak nodes rely more on inflow to settle their out-payments and therefore have less demand for initial liquidity, they also provide less liquidity to the system. In contrast, spillover nodes have more demand for initial liquidity, and provide more liquidity to the system in general. If there are liquidity leakages, equation (4) will converge. In particular, if there are some nodes with nonzero in-strength and zero out-strength\(^1\), equation (4) will converge to:

\[
x_{ni} = \sum_{r} X_{ni}^{r} = d(L)(I - Z)^{-1}.
\]

4. **Effect of Timing.** The liquidity from one node arrives at other nodes concurrently, in equations (1) to (5). However, in practice, the payments are settled one by one. Thus, the timing of payments will also affect the liquidity circulation, especially when there are liquidity leakages or gridlocks.

5. **Settlement and Real-Time Settlement.** In contrast to normal settlement, real-time settlement requires matching the liquidity demand to supply not only in terms of total value, but also in terms of timing. Liquidity demand for real-time settlement cannot be described by equations (1) to (5), and must be calculated by simulations.

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\(^1\) For example, in the case of default.
Let $\mathcal{T} = \{0, 1, \ldots, T\} \in \mathbb{N}$ be the set of trading time in a trading day, $P^t_i$ be the set of payments of member $i$ at the specific time $t$ and let $t \in \mathcal{T}$ and $P^t = \bigcup_{i=1}^{n} P^t_i$ be the set of payments of the entire system. Similarly, let $S^t = \bigcup_{i=1}^{n} S^t_i$ be the set of settled payments and $Q^t = \bigcup_{i=1}^{n} Q^t_i$ be the queued payments of the system at time $t$. This gives us $Q^t_i \cup S^t_i = P^t_i$.

According to the general rules of RTGS, the settlement of payment $p^t_{ij}$ will be attempted at time $t$ following the time order, and will be settled successfully if and only if $l^t_i \geq p^t_{ij}$ and $Q^t \neq \emptyset$. Then the payment $s^t_{ij} = p^t_{ij}$ will be added to $S^t$. Otherwise, if $l^t_i < p^t_{ij}$ or $Q^t = \emptyset$, $p^t_{ij}$ will be queued into $Q^t$, denoted by $q^t_{ij}$. When $l^t_i > l^{t-1}_i$, the orders in $i$’s queue will be settled according to FIFO. The condition for which payment $q^t_{ij}$ can be settled is $l^t_i \geq q^t_{ij}$, $\exists q^t_{ij} \in Q^t_i$ and $\exists q^t_{ij} \in Q^t_i$, $v < t \leq t \in \mathcal{T}$.

At the end of a day, $T$, member $i$ has settled all his payments if $Q^t_i \neq \emptyset$, and has settled in real time only if $Q^t_i = \emptyset$, $\forall t \in \mathcal{T}$. The minimum liquidity requirement to settle all payments of a member in real time is called upper bound liquidity (UBL) and is given by:

$$\text{UBL}_i = \max \left( \max_{t} (P^t_i - S^t_i), 0 \right) \quad (6)$$

where $P^t_i$ is the entire required payment value of $i$ for unit of time $t$, and $S^t_i$ is the whole settled payment value of $i$ until time $t$, respectively. Similarly, the UBL of a system is defined by the sum of UBLS of all members as:

$$\text{UBL} = \sum_{i} \text{UBL}_i \quad (7)$$

According to (6), we also have:

$$\max(\text{OS}_i - \text{IS}_i, 0) \leq \text{UBL}_i \leq \text{OS}_i \quad (8)$$

$$\sum_{i} \max(\text{OS}_i - \text{IS}_i, 0) \leq \text{UBL} \leq \text{TS}. \quad (9)$$

The relative UBL of member or system (RUBL) is defined as:

$$0 \leq \frac{\text{UBL}_i}{\text{OS}_i} \leq 1 \quad (10)$$

$$0 \leq \frac{\text{UBL}}{\text{TS}} \leq 1. \quad (11)$$

This paper intends to study how the structural effect, i.e., the effect of strength, strength distribution or leakage, will affect the intraday liquidity demand of the entire system or a member to settle the payments in real time. Other effects, such as the effect of gridlock or timing, will be controlled during the simulation.

### 3 Network topologies and simulations

At first, a directed and weighted globally coupled network with nodes is constructed as a bench network, denoted by FE. In FE, there is a direct link between every pair of nodes with equal weight $\bar{w}$. Thus, the total strength of FE is $TS = n(n - 1)\bar{w}$. In FE, there is no effect of strength distribution nor of leakage, because all nodes are homologous and their out-strength is equal to their in-strength. All other types of networks mentioned in this paper are derived from this bench network with an unchanged total strength of the system, $TS$, to eliminate the effect of strength in the system.

1. Distribution of total strength among the nodes. There are many complex financial networks for which the degree of distribution follows a power law. According to the BA model, we construct a scale-free network with the same number of nodes and total strength as the FE network, denoted by BS. The BS network is symmetric, i.e., the out-strength of every node is equal to its in-strength. In BS, there are not only effects of strength at the node level but also effects of strength distribution at the system level. The coefficient of variation of out-strength of all nodes, $\text{VOS}$, is calculated to denote the effect of strength distribution at the level of the system. $\text{OS}_i$ is used to denote the effect of strength at the level of nodes.

2. Asymmetry. Based on the symmetric networks FE and BS, two types of asymmetric networks, ES and BA, are constructed with equal total strength. At first, all nodes are sorted in descending order of strength. Then, the first $j$ ($j$ is a predefined parameter) columns of the matrix lower triangle are deducted by a random proportion. Finally, the deducted values are added to the corresponding elements of the matrix upper triangle. The asymmetry of a network is denoted by the relative level of leakage of the network:

$$\text{TL} = \frac{\sum \max(\text{IS}_i - \text{OS}_i, 0) \text{OS}_i}{\text{TS}}. \quad (12)$$
And the level of leakage of a node is denoted by:

\[ TS_i = IS_i - OS_i \]  

(13)

A node is a leak node if \( TL_i \) is positive, and a spillover node if \( TL_i \) is negative.

The networks are also transposed to study whether the distribution of leakage among nodes has a systemic effect. A superscript \( j \) is added to denote the network with a different \( j \) and a superscript \( T \) is added to denote the transposed network. The total strength of the nodes are the same in ES but they differ significantly in BA. To show this difference, the coefficient of variation of the total strength of all nodes, \( VTS \), is also calculated for all networks.

3. Number of nodes. Two different number of nodes, \( n_1 = 30 \) and \( n_2 = 58 \), are set for every type of network (denoted by a subscript). The average weight of a link of the two types of networks is set to \( w_1 = 38 \) and \( w_2 = 10 \), respectively, to keep the total strength of the networks remain unchanged.

By setting \( j = 1, 2, \ldots, 10 \) and transposing the network, a total of 84 networks (2 FEs, 2 BSs, 40 ESs and 40 Bas) are constructed. The TL and VOS or VTS of these networks are depicted in Fig. 1, as a symmetric network with homogenous nodes, are located at origin. BSs without a leakage effect are located on the horizontal axis. ESs and BAs have effects of both strength distribution and leakage, but both effects are greater in the BAs. Furthermore, the ESs have nonzero VOSs but zero VTSs.

At the level of nodes (Fig. 2), all nodes of FEs are located at the origin, as well. In ESs, the total strengths of all nodes are equal, and VOS is negatively correlated to TL. All nodes of BSs are located on the horizontal axis. BA nodes have greater differences in both VOS, and TL, and there is no linear relationship between them.

For every payment network, data of payment flows over 500 trading days are generated. The value per payment is generalized to one to control the gridlock effect. The timing of payments in one day is evenly distributed. The payment flows are the same among all days with the exception of timing. The simulations are based on the algorithm of unlimited real time gross settlement to get the UBL of every day for every node and for the entire system. The UBL of 500 days is averaged to eliminate the effect of timing.

4 Results of the simulation

4.1 The effect of strength distribution

The distributions of UBL for two types of symmetric networks, FE and BS, are shown in Fig. 3. At the level of systems (in the left subplot of Fig. 3), the distributions of the system’s UBL of four networks are like a normal distribution. A network with a higher VOS has a lower average UBL, but higher volatility of UBL. In contrast, a network with more nodes has a higher average UBL, but lower volatility of UBL. At the level of nodes (in the right subplot of Fig. 3), the distribution of the UBL of FE is more like a normal distribution, but that of BS is more like a lognormal distribution. In general, the effect of strength distribution is significantly negative to the liquidity demand of a system in which there is no effect of leakage.
Fig. 3. System and Member UBLs of Symmetric Networks
Source: Authors’ own calculations.

Fig. 4. Relationship between UBL or RUBL and TS of Members in Symmetric Networks
Source: Authors’ own calculations.
According to the results of the simulation, the out-strengths of nodes in symmetric networks are positively correlated to their UBLs in first-order correlation but negatively correlated in second-order correlation (in the left subplot of Fig. 4), and negatively correlated to their RUBL (in the right subplot of Fig. 4). Thus, a node with a higher out-strength in a symmetric network needs more intraday liquidity to settle payments in real time, but can use its liquidity more efficiently. This is why a network with a higher VOS needs less liquidity to settle payments, i.e., the negative effect of strength distribution.

In general, the number of possible orders of all in- and out-payments of a node with the out-strength of $o$ or the total strength of $2o$ in a symmetric network is $(2o)!/(o!)^2$. By calculating the mean UBL and RUBL of this node under all possible cases, the relationship between the out-strength of a node and its UBL or RUBL can be acquired. Let $o$ be an integer in $[1,12]$. The result is shown in subplot 1 of Fig. 5, which is like that in Fig. 4. As the out-strength and in-strength of a node are simultaneously increased, it is helpful to coincide the inflow and outflow of liquidity for the node, and the efficiency of liquidity is therefore increased for the node and for the entire system. A decrease in the number of nodes will also increase the liquidity efficiency of the system.

We generalize the discussion to take into consideration the constant level of leakage of a node (see subplot 2 and 3 in Fig. 5). The subplots show that the relationship between the out-strength and UBL or out-strength and RUBL of nodes with some constant level of leakage, whether leak nodes or spillover nodes, is identical to that of nodes with zero leakage. However, the magnitude of the strength effect differs significantly across different levels of leakage. In general, the less the value of leakage level of a node is, the more is the effect of strength to liquidity demand of a node. Therefore, the liquidity efficiency of a system can be increased through taking payments from leak nodes to spillover nodes when the leakage level of the nodes is constant.

### 4.2 The effect of strength distribution and leakage in ES

We also get the UBL of the system and of the nodes for ES (an asymmetric network in which all nodes have equal total strength), and compare them with those of FE. The effects of leakage and strength distribution on the liquidity demand of the system in ES are depicted in Fig. 6.

1. Effect of Leakage. The system’s $TL$ is significantly positively correlated with the UBL of the system. The intercept of the fitted line corresponds to FE with zero leakage.
2. Distribution of Leakages. The locations of the transposed networks are the same as those of the original networks in Fig. 6. The distribution of leakage does not influence the liquidity demand of the system, ceteris paribus.

3. Distribution of Strength. In contrast to the negative effect of strength distribution in symmetric networks, that in ES is positive.

4. The Number of Nodes. In contrast to networks with 30 nodes, networks with 58 nodes have a fitted line with a lower slope. That is, the effect of leakage and that of strength distribution are stronger for ES, which has fewer nodes.

In summary, the liquidity demand of the system is positively correlated with the extent of leakage, but not with the distribution of leakage among nodes. The reason for the positive effect of strength distribution to the system may be that the leakage effect is more dominant and that increasing only the out-strength is not enough to increase the efficiency of coordinating inflows and outflows.

At the level of nodes (Fig. 7), the liquidity demand of every leak node is approaching zero, and that of every spillover node is mostly equivalent to the absolute value of the node’s TL. The above relationship is more significant when the absolute value of the node’s TL is greater.
The relationship between the relative liquidity demand of nodes and the nodes’ TL is similar. This is why the effect of leakage on the liquidity efficiency of the system is negative. Moreover, the liquidity efficiency of nodes in networks with 58 nodes is lower than that in networks with 30 nodes, which accords with the results of the previous section.

In general, given a node with a total strength of $z$ and an out-strength of $o$, $0 < o < z$, the number of possible orders of all in- and out-payments of the node is given by $z!/(z-o)!o!$. If $z = 24$, then the relationship between UBL or RUBL and TL or the out-strength of the node is shown in Fig. 8, and is similar to the relationship shown in Fig. 7. In networks with equal total strength of
nodes, a node with a lower out-strength is a leak node, and its UBL and RUBL are approaching zero. The UBL and RUBL of a node increase at a greater rate when the node’s out-strength approaches its in-strength. When the node’s out-strength equals its in-strength, the node will be a spillover node, its UBL will be increased linearly and its RUBL will approach one.

4.3 The effect of strength distribution and leakage in BA

Finally, as shown in Fig. 9, there is significant difference between the effect of strength distribution and that of leakage in BA. As in networks with equal total strength of nodes, the effect of leakage on the liquidity demand of the system is also significantly positive, but with a greater slope and goodness of fit. Again, the distribution of leakage among nodes does not affect the liquid-
ity demand of the system. In BA, the effect of strength distribution is not significant, as only leakages have an effect.

At the level of nodes (Fig. 10), the results are like those in the networks with equal total strength of nodes, i.e., the TL or out-strength of nodes is positively correlated with the liquidity demand of spillover nodes, but is uncorrelated with that of leak nodes. To distinguish the effect of leakage from that of strength, we use the method of a numerical simulation to study it with some given out-strength.

As shown in Fig. 11, the liquidity demand and relative liquidity demand of a node significantly increases with a decreasing level of leakage, as in Fig. 10. Given a certain level of leakage, the higher the out-strength of a node, the higher the liquidity demand of the node is, and the less the relative liquidity demand is, as in Fig. 5. Furthermore, the difference between the liquidity demands (or relative liquidity demands) of nodes with various out-strengths is decreasing (or increasing), as the leakage level of the node is decreasing. This reflects the effect of strength given a certain level of leakage.

5 Conclusions

In this paper, we analyse the different influencing factors of intraday liquidity demand by modelling the circulating mechanism of liquidity in a network, and by examining the different structural effects on the liquidity demand of real-time settlement of both a member and of the whole system using different simulation methods. We find the following robust results:

a. that there is a more efficient liquidity circulation in symmetric network compared to an asymmetric network;

b. that the higher the strength of a node, the more efficiently the node can use its inflow to finance its outflow, if the leakage level is constant and there are no gridlocks in the settlement process. This leads to a reduced liquidity demand on the entire network, although the nodes with higher strength need more liquidity in general;

c. that the liquidity demand of a leak node with higher in-strength is approaching zero as its in-strength become higher, and the liquidity demand of a spillover node with higher out-strength is mostly linearly correlated with its out-strength;

d. that in the case of constant total strength of nodes, the liquidity demand of a node or of the entire network is negative correlated with the out-strength of the node or the variance of the nodes’ out-strength of the network, and is positively correlated with the leakage level of the node or that of the entire network;

e. that the network’s liquidity demand is mostly linearly correlated to the leakage level of the network, but uncorrelated to the distribution of total leakage among all notes;

f. that the lower the leakage of a node, the higher its liquidity demand and the lower relative liquidity demand the node has. With reducing leakage levels, the differences between the liquidity demands of nodes with different out-strengths become smaller, and those between the relative liquidity demands of nodes become greater.

In light of previous findings, our results are useful for the management and supervision of short-term liquidity demand in complex financial systems, and for liquidity risk management and liquidity rescue policymaking.

References


