OPTIMIZATION OF THE SANDWICH COLUMN WITH THE TRUSS CORE WHICH IS SUBJECTED TO THE COMPRESSIVE LOADING

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Abstract

The sandwich structures have multifold advantages with respect to other types of structures. Besides the architectural possibilities due to their appearance, those structures can carry the same or even higher loads than some other similar structures. Optimization of the sandwich columns with the truss core, subjected to the compressive axial load, is presented in this paper. The two types of optimization were performed: the three-parameter and the four-parameter optimization - the so called full optimization. The optimization of the column geometry (face thickness, core member height and core member diameter and core height) was performed, from the aspect of the minimal weight of the structure in terms of the load index. It was performed for four types of restrictions imposed by the corresponding column failure modes: column buckling, truss macro-buckling, local buckling of the face and face wrinkling. The three-parameter optimization resulted in somewhat larger weight of the column than the full, four-parameter optimization.

Keywords:

Sandwich column;
Truss core;
Optimization;
Load;
Minimal weight.

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1. Introduction

The sandwich panels consist of the two rigid plates (faces) and the core; they are made in such a way to possess high bending stiffness and strength and a small mass. They are intensively applied in various types of transportation vehicles: cars, ships or airplanes. Roughly speaking, the sandwich structures' faces are carrying the bending and planar loads, while the core is carrying the transversal load. The most frequently used materials for manufacturing the sandwich panels are metals or composites for the faces; metals, wood and polymers for the core.

The sandwich plates are structural elements that have multifunctional application. Due to their structure, they are very interesting for applications where the high loading capacity is required simultaneously with the small weight of the structure. A large number of researchers were dealing with analysis of the sandwich plates exposed to various types of loads and straining. Allen (1969) has defined the methods for design and calculations of the sandwich panels. In paper by Evans et al. (1999) is presented a survey of mechanical and thermal characteristics of the cellular metal systems with comparison to other cellular and dense materials. An analysis was also presented of the prototype systems, which enable implementation with other construction concepts. Brittain et al. (2001) were dealing with estimates of the mechanical characteristics and structural efficiency of the truss beam subjected to the four-point bending. The behavior of that beam was compared to behavior of the beam with the box cross-section via the dimensionless parameters for stiffness, mass and carrying capacity. An optimization of the sandwich plates with the truss core, subjected to the bending moment and the transversal force, from the aspect of the minimal mass, was presented in paper by Wicks and Hutchinson (2001). They analyzed two types of the sandwich plates – plates that consist of the two flat faces and the truss core and plates with two truss faces with the truss core. Behavior of the truss core under the compressive loading conditions was analyzed, as well. Wicks and Hutchinson (2004) have analyzed performances of the sandwich plates with the truss core, subjected to the bending moment and the transversal force, through the comparison to the plates with the honeycomb

core. Valdevit et al. (2004) performed the structural optimization of the sandwich plates with the prismatic core. They performed the estimate of the optimal dimensions and the minimal mass of the sandwich plates with the prismatic core when the load is acting in the longitudinal and the lateral directions. A comparative analysis was presented of the structural performances of the sandwich plates with the prismatic core to plates with the honeycomb core. Dharmasena et al. (2011) have analyzed behavior and response of the sandwich plates with the truss core to impact load of the high intensity in the air.

One of the interesting problems related to the sandwich structures is behavior of the sandwich column subjected to compressive load. This problem was the subject of study in an article by Fleck and Sridhar (2002). They analyzed the sandwich column, which consisted of the glass reinforced by the epoxy fibers and the core made of the polymer foam and which was subjected to the compressive load. They constructed the fracture map to present the dependence of the fracture modes on geometry and relative density of the core. In addition, they determined the minimal mass in terms of the load index. Douville and Le Grogne (2013) presented analytical solutions for the local and the global buckling of the sandwich beams and columns for various types of loading. They considered the classical sandwich beam - column (with the core made of the homogeneous material) and conducted an analysis of buckling within the elastic domain, in the sense that they determined the critical values and corresponding branching for different loadings (compressive axial and pure bending). Wadee et al. (2010) have solved analytically the problem of the sandwich struts buckling and their response to post buckling, using two models for the core bending and two different beam theories.

The possibility for optimization of the sandwich columns with the truss core, subjected to the compressive axial loading, was considered in this paper.

Optimization assumes determination of the minimal mass, necessary to prevent any type of the structural failure, for the given load index. The dominant form is one with the largest value of the minimal weight, for all the values of the load index. Rathburn et al. (2005) have shown that the three-parameter optimization results in somewhat heavier structures than the full – four-parameter optimization. The full optimization of the sandwich structures implies that four independent geometrical factors are expressed via the four restraints, i.e. the types of failure. Then one determines for which of them is the first derivative of the objective function, in this case – the unit mass in terms of the independent geometrical parameters, equal to zero.

2. Problem formulation

The sandwich column considered, of the height H , whose bottom end is clamped, while the upper end is subjected to axial compressive load P , is presented in Figure 1. The column has the tetragonal truss core (with 120° planar symmetry); each member of the truss has length L_c and circular cross section of radius R_c . The core's height is H_c . The 120° symmetry ensures that the bending and in-plane shear stiffnesses are isotropic, Allen (1990). The angle between the core members and the face is $\theta = \sin^{-1}(H_c / L_c)$, while the thickness of the isotropic plates, constituting the sandwich plate's faces, is t_f . The significantly more efficient structure would be obtained if two different materials were adopted for the faces and the core. In this work, however, the assumption was made that the faces and the core are made of the same material, whose characteristics are: Young's elasticity modulus, Poisson's ratio, yield stress and material density, E , ν , σ_y and ρ , respectively.

Unit mass per the sandwich column with the truss core area is

$$W = 2\rho \left[t_f + \frac{\pi}{\sqrt{3}} \frac{L_c R_c^2}{L_c^2 - H_c^2} \right]. \quad (1)$$

Four types of failure exist for the sandwich column subjected to axial compressive load – column buckling, core macro-buckling, face local buckling (micro-buckling) and face wrinkling, Fleck and Sridhar (2002), presented in Figure 2.

Fig. 1: The sandwich column with the truss core: loading and geometry.

Fig. 2: Failure modes of the sandwich column due to the compressive loading: (a) column buckling, (b) buckling of the truss, (c) local buckling of the face and (d) face wrinkling.

When the sandwich column is subjected to axial compressive loading, the faces of the unbuckled column are carrying the complete load, so the forces in the core elements are equal to zero. However, in calculating the Euler critical force for the column buckling, the shear stiffness of the core elements is taken into account, together with the flexural stiffness of the faces. That critical force amounts to, Allen (1969):

$$P_{cr} = \frac{\pi^2 E t_f R_c^2 H_c^2 (H_c + t_f)^2}{2 H^2 R_c^2 H_c^2 + \pi \sqrt{3} t_f (H_c + t_f)^2 L_c^3} . \quad (2)$$

Restraining, dictated by the column buckling, is:

$$P \leq P_{cr} , \quad (3)$$

Restraining due to core buckling is:

$$P \leq \sigma_Y t_f , \quad (4)$$

while the restraining due to micro-buckling of the faces is defined by expression, Wicks and Hutchinson (2001):

$$P \leq \frac{49}{216} \frac{\pi^2}{(1-\nu^2)} \frac{Et_f^2}{(L_c^2 - H_c^2)}, \quad (5)$$

The last form of failure is the face wrinkling, which occurs when the shear stiffness is very small, thus the restraining condition against wrinkling is, Allen (1969):

$$P \leq \frac{\pi ER_c^2 H_c^2}{\sqrt{3} L_c^3}. \quad (6)$$

Optimization is performed in order to determine the minimal weight (mass) of the sandwich column which can still carry the prescribed load P . All the geometrical variables are normalized by the column height H . The independent, dimensionless parameters, which define the column geometry, are thus: $x_1 \equiv t_f/H$, $x_2 \equiv R_c/H$, $x_3 \equiv H_c/H$, and $x_4 \equiv d/H$ where $d = \sqrt{L_c^2 - H_c^2}$.

Accordingly, the objective function (mass per unit area), equation (1), in the dimensionless form can be written as:

$$\frac{W}{\rho H} = 2 \left[x_1 + \pi x_2^2 \frac{\sqrt{x_3^2 + x_4^2}}{x_4^2 \sqrt{3}} \right]. \quad (7)$$

Restraints are dictated by the failure types, i.e. by expressions (3) to (6), and in the dimensionless form, expressed via the four independent geometrical parameters, they read:

$$\left(\frac{P}{EH} \right) \left(\frac{2}{\pi^2} \right) x_1^{-1} x_3^{-2} \left[1 + \frac{\pi \sqrt{3}}{2} x_1 \frac{\sqrt{(x_3^2 + x_4^2)^3}}{x_2^2} \right]^{-1} \leq 1, \quad (8)$$

$$\frac{1}{2} \left(\frac{P}{EH} \right) \left(\frac{E}{\sigma_y} \right) x_1^{-1} \leq 1, \quad (9)$$

$$\frac{216(1-\nu^2)}{49\pi^2} \left(\frac{P}{EH} \right) x_1^{-3} x_4^2 \leq 1, \quad (10)$$

$$\frac{\sqrt{3}}{\pi} \left(\frac{P}{EH} \right) x_2^{-2} x_3^{-2} \sqrt{(x_3^2 + x_4^2)^3} \leq 1. \quad (11)$$

In equations (8) to (11) there are two dimensionless parameters – for material σ_y/E and for loading $P/(EH)$. The value for Poisson's ratio in equation (10) is taken to be $\nu = 1/3$.

3. Results and discussion

The optimization problem was solved numerically, by application of the symbolic programming routine *Mathematica*®. The full optimization was performed, using all the four geometrical variables, as well as the three-parameter optimization for which it was taken that $H_c = 0.8 L_c$. The sandwich material was taken to be aluminum for which the dimensionless material parameter is $\sigma_y/E \cong 0.007$. Variation of the dimensionless mass $W/\rho H$ in terms of the dimensionless load parameter $P/(EH)$ is shown in Figure 3.

Fig. 3: Normalized mass per unit area as a function of the dimensionless load parameter for the optimal sandwich column with the truss core.

From Figure 3 one can notice that the necessary minimal mass for the required loading is increasing with the load increase.

Optimal values of dimensions of the sandwich column with the truss core are presented in Figure 4. Solution for optimization was obtained by application of the symbolic programming routine *Mathematica*® and equations (8) to (11).

From Figure 4 one can notice that all the sandwich column optimal dimensions for the required loading are increasing with the load increase. The core member cross section radius increase with increasing load is somewhat slower than those of the other three dimensions.

The solution of optimization of mass per unit area in terms of the dimensionless load parameter, based on different types of restraints, is presented in Figure 5. For the lesser loads, the three restraints are active – buckling, local face buckling and face wrinkling, while for the bigger loads are active column buckling and the local face buckling.

(b)

(c)

(d)

Fig. 4: Optimal dimensions of the sandwich column with the truss core in terms of the dimensionless load parameter: (a) face thickness, (b) core height, (c) core member cross section radius and (d) core member length.

Fig. 5: Normalized mass in terms of the dimensionless load parameter for optimization based on different restraints.

4. Conclusions

The dimensions optimization, of the sandwich column with the truss core, from the aspect of the minimal mass, is presented in this paper. The optimization was performed for all the four column failure modes: column buckling, macro-buckling of the truss, local (micro) buckling of the face and wrinkling of the face.

All the sandwich column optimal dimensions for the required loading are increasing with the load increase. The core member cross section radius increase with increasing load is somewhat slower than those of the other three dimensions.

For the lesser loads, the three restraints are active – buckling, local face buckling and face wrinkling, while for the bigger loads are active column buckling and the local face buckling.

Based on presented results, one can conclude that selection of the optimal sandwich column structure, minimal mass, and optimal dimensions depend on the index of the applied load.

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