Introduction

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The semi-Lagrangian method is a well established technique for the numerical solution of partial differential equations that model, possibly among others, an advection process. It is well known that the solution of the advection equation is constant along the characteristic curves of the flow. The semi-Lagrangian method exploits this property of advection in an essential way and allows to work on a fixed computational mesh, a main difference with respect to fully Lagrangian methods. Employing an approximation of the characteristics as a key ingredient of the numerical method results in very mild stability restrictions, in contrast to the typical Courant - Friedrichs - Lewy stability conditions for explicit time discretizations in finite difference, finite volume and finite element schemes. This higher level of efficiency is achieved in a rather local way, without having to solve global algebraic problem as in typical implicit methods. This also entails that space and time discretizations are inherently intertwined, as opposed to method of lines approaches where the two discretisations are considered as two conceptually different steps.

Semi-Lagrangian methods were originally proposed in the meteorological literature, aimed at computing potential vorticity advection in simplified models of large scale atmospheric flow. Similar methods were then developed, in some cases independently, in other modelling communities and are often known under different names, such as for example the modified method of characteristics, the Eulerian-Lagrangian method and the

Lagrange-Galerkin method. A comprehensive review of the theoretical aspects of semi-Lagrangian methods is presented in [1], while earlier reviews of their applications in different areas of computational fluid dynamics have appeared for example in [2], [3].

This special issue, which includes, among others, papers presented in a minisymposium devoted to semi-Lagrangian methods organized within the 2014 congress of the Italian Applied and Industrial Mathematics Society SIMAI, we have tried to collect contributions that address recent developments of semi-Lagrangian methods, ranging from the more classical applications in the field of numerical weather prediction to areas that are very different from those in which semi-Lagrangian methods were first proposed.

Concerning numerical weather prediction, the developments of semi-Lagrangian methods at the Met Office over the last 20 years have been reviewed by T. Benacchio and N. Wood, and the impact of some discretization choices has been analyzed in a simplified setting. Several contributions are devoted to the development of conservative semi-Lagrangian methods. In particular, two high order, conservative, spectral element semi-Lagrangian methods have been proposed and discussed by C. Erath, M. Taylor and R. Nair, along with their parallel scaling properties. A conservative extension of a full second order, semi-implicit, semi-Lagrangian and p -adaptive DG method has been studied by G. Tumolo, while A. Hamiaz, M. Mehrenberger, H. Sellama and E. Sonnendrücker have proposed conservative semi-Lagrangian methods on curvilinear grids. A conservative, flux form semi-Lagrangian method for diffusion problems in divergence form has been proposed by L. Bonaventura and R. Ferretti. R. Bermejo and L. Saavedra have reviewed the development and application of Lagrange-Galerkin methods for the incompressible Navier Stokes equations. Finally, semi-Lagrangian schemes for the BGK equation have been extended to treat boundary conditions, in particular diffusive ones, by M. Groppi, G. Russo and G. Stracquadanio.

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