Unified approach to the sliding-mode control and state estimation
– application to the induction motor drive

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Abstract. In this paper a generalized design procedure of the sliding mode systems is described. A unified approach is applied to control and state variables estimation algorithms. Selected solutions are then applied in the induction motor drive system. An identical design procedure is used to design the speed control and MRAS-type speed estimator. Presented algorithms are verified using experimental tests performed on the 3 kW laboratory setup.

Key words: induction motor, sliding mode control, motor torque control, chattering, state estimation, model reference adaptive system (MRAS).

Notation

- estimation error,
- stator voltage frequency, [Hz],
- rotor current vector,
- stator current vector,
- control signals / estimated variables vector,
- Lyapunov function,
- electromagnetic and load torques,
- Laplace operator,
- stator and rotor winding resistances,
- switching functions vector,
- time constant, [s],
- mechanical time constant of drive, [s],
- time constant, connected with the per unit system introduction, [s],
- stator voltage vector,
- state variables vector,
- main reactance ,
- rotor reactance,
- stator reactance,
- stator and rotor winding leakage reactance,
- gain matrix,
- angle of the stator/rotor flux vector,
- leakage coefficient,
- rotor flux vector,
- stator flux vector,
- amplitude of stator/rotor flux vector,
- mechanical speed.

All variables, written in small letters, are transferred to the per-unit system [p.u.]. Base values, necessary for the transition, can be found in the Appendix. Bold font indicates vector values.

Sub- and superscripts

- Nominal value,
- Maximum value,
- Reference value,
- Estimated value,
- Desired dynamics.

1. Introduction

Induction motor (IM) drives, due to their operational reliability, small size and low cost, are continuously becoming more and more popular among the electrical motors applied in the industry [1–3]. However, the most complicated mathematical model entails that the most complex control structure must be used.

One of the control method groups, which ensure excellent dynamics of the IM drive, is the Sliding Mode Control (SMC) [4]. It can be ranked as the vector control method. These control methods require that the nonmeasurable variables, such as the rotor flux vector, electromagnetic torque or stator current vector components in a synchronous frame, are estimated. Similarly, the sliding modes can be successfully used to design the state estimator, insensitive to some motor parametric changes.

The sliding mode control of the IM main three variables: torque, speed and position was proposed for the first time in [5]. The relay method is natural for the inverter – sliding mode algorithm defines the transistor on/off signals directly. Unfortunately, the proposed method introduce large acoustic noise and mechanical oscillations of the drive, caused by the chattering in regulated variables. Therefore, many papers tried...
The designing procedure is first described for a general plant, and next applied to the induction motor drive control and estimation. In the first place, the speed control structure, taking the advantage of the equivalent control method is presented. Then, the sliding mode estimator, designed using the Model Reference Adaptive System technique is described [30]. The estimator utilizes the relay method.

This paper is organized as follows: first, the whole designing procedure of the sliding mode system is presented and its particular steps are listed. Then, these steps are applied to the induction motor speed control and speed estimation. The presented algorithms are verified using experimental results. Finally, there are some conclusions at the end of the paper.

2. Sliding mode systems design

Sliding mode systems design can be divided into several steps. Every step of the procedure is strictly connected with all the other steps. These steps are identical for both control and estimation.

2.1. Determination of the mathematical model of the plant and identification of its parameters. To design the sliding mode system, a mathematical model of the plant, together with its parameters must be determined. This phase of the design process is important especially when the state variables estimation is taken into consideration. However, the parameters of the plant are also used in the control algorithm.

2.2. Choice of the control signals / vector of estimated variables. On the basis of the mathematical model and the topology of the structure of the control system or the state estimator, the following vector has to be defined:

$$ k = \begin{bmatrix} k_1 & k_2 & \ldots & k_n \end{bmatrix}^T, $$

(1)

where $n$ – number of available signals.

Considering the induction motor drives, this vector (1) can contain control signals, such as on/off signals of the voltage source inverter transistors or can become a scalar when a cascade control structure is taken into account. On the other hand, it can consist of estimated signals, such as speed and stator current vector components.

2.3. Definition of the switching functions vector. The next step is to select so-called switching functions – the purpose of the sliding mode system is to ensure a zero value of these functions. They can be gathered in the following vector:

$$ s = \begin{bmatrix} s_1 & s_2 & \ldots & s_n \end{bmatrix}^T. $$

(2)

The maximum dimension of the above vector (2) corresponds with the size of the vector (1). The form of switching functions depends strictly on the application. The switching functions can be both stationary in time [4] and time-varying [31]. Most often they are linear, however, they can be nonlinear as well: parabolic [32], elliptical [33] or the so-called terminal attractor [34], which ensures a finite reaching
time of the switching function. The terminal attractor can also be time-varying [35].

The forms of the switching functions, in the estimator application, are most often the estimation errors. However, they can be also a combination of state variables and estimation errors; it will be presented in the case of the induction motor speed estimation.

Knowledge of the derivative of the switching functions vector is required in the design process:

\[
\dot{s} = \left[ \dot{s}_1 \ \dot{s}_2 \ \ldots \ \dot{s}_n \right]^T.
\] (3)

The switching functions vector (2) must be chosen in a way that allows the decomposition of (3), described by the following equation:

\[
\dot{s} = f + Dk,
\] (4)

where \( D \) – matrix dependent on the control signal vector \( k \) and \( f \) – column vector independent on \( k \). Additionally, vector \( f \) can be decomposed into:

\[
f = f_1 + f_2,
\] (5)

where \( f_1 \) – part that can be calculated using measured or estimated signals, \( f_2 \) – part which depends on the nonmeasurable variables, such as external disturbances.

2.4. Sliding mode control / estimation law. The most important part of the design process is the choice of the sliding mode algorithm, which ensures the zero value of all switching functions (2). Three chosen structures are presented in Fig. 1 [36]. This stage of the design is strictly connected with the chosen control and switching functions vectors. All of the presented methods can be applied in control and estimation systems as well.

The relay method (Fig. 1a) is most often used sliding mode control/estimation method. In this method, the control signals take only two opposite values:

\[
k = -\Gamma \text{sign}(s^*)^T,
\] (6)

where \( \Gamma \) – gain matrix:

\[
\Gamma = \begin{bmatrix}
\gamma_1 & 0 & \ldots & 0 \\
0 & \gamma_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \gamma_n
\end{bmatrix}
\] (7)

with positive parameters \( \gamma_1, \gamma_2, \ldots, \gamma_n > 0 \). Modified switching functions vector:

\[
s^* = s^TD = \begin{bmatrix}
s_1^* \\
s_2^* \\
\vdots \\
s_n^*
\end{bmatrix}.
\] (8)

In order to reduce the oscillations in the control and estimated variables signals the equivalent control method (Fig. 1b) can be applied. In this method, vector \( k \) consists of two different parts: continuous \( k^{eq} \) and discontinuous \( k^d \):

\[
k = k^{eq} + k^d,
\] (9)

where \( k^{eq} \) can be calculated from the following expression:

\[
\dot{s} = f + Dk^{eq} = 0
\] that yields:

\[
k^{eq} = -D^{-1}f.
\] (11)

However, since vector \( f \) consists of the part, which is nonmeasurable, the continuous component of the control signal becomes:

\[
k^{eq} = -D^{-1}f_1.
\] (12)

The discontinuous component is to overcome the nonmeasurable and unknown signals in \( f_2 \):

\[
k^d = -\Gamma^d \text{sign}(s^*)^T,
\] (13)

where, similarly, the gain matrix:

\[
\Gamma^d = \begin{bmatrix}
\gamma^d_1 & 0 & \ldots & 0 \\
0 & \gamma^d_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \gamma^d_n
\end{bmatrix}
\] (14)

with positive parameters \( \gamma^d_1, \gamma^d_2, \ldots, \gamma^d_n > 0 \).

Control signal in the third control method, the linear feedback control with switched gains (Fig. 1c) is as follows:

\[
k = \Gamma(x)x.
\] (15)

The gain matrix is switched depending on the state variables vector:

\[
\Gamma(x) = \begin{cases}
\Gamma_1 \text{ when } s(x)x \geq 0 \\
\Gamma_2 \text{ when } s(x)x < 0
\end{cases}
\] (16)

Fig. 1. Three chosen sliding mode control topologies: a) relay control, b) equivalent control method, c) linear feedback control with switched gains.
and two different gain matrices:

\[
\Gamma^d = \begin{bmatrix}
\gamma_1^d & 0 & \ldots & 0 \\
0 & \gamma_2^d & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \gamma_n^d \\
\end{bmatrix}, \\
\Gamma^2 = \begin{bmatrix}
\gamma_1^2 & 0 & \ldots & 0 \\
0 & \gamma_2^2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \gamma_n^2 \\
\end{bmatrix}
\]  

(17)

Due to the large number of the parameters in (17) this control method and its application to the induction drive control will not be considered in the following part of the paper. However, this control method has been applied with the time-varying switching line in the position control of the induction motor drive [14].

2.5. Selection of control / estimation gains. All parameters included in the gain matrices: \( \Gamma, \Gamma^d, \Gamma^1, \Gamma^2 \) must be determined properly. The parameters in the \( \Gamma \) matrix (7), in the relay control, are often assumed as maximum available values, e.g. \( \pm 1 \), which indicate the on/off state, or a maximum acceptable torque value in the cascade speed control. However, this method and its large gains leads to an undesired phenomenon called chattering, which induces oscillations in controlled variables, acoustic noise and mechanical vibrations of the real system.

As it will be shown, the parameters included in the \( \Gamma^d \) matrix can be set to much lower value in comparison with the relay control method. This helps to reduce the oscillations effectively.

This point of the design process is strictly connected with the following stage – the parameters in every method must be large enough to ensure the stability of the proposed system.

2.6. Stability analysis of the proposed system. The last step of the design should be the verification of the proposed system stability. This step proves that the designed algorithm ensures a zero value of the switching functions. This task can be made using the Lyapunov method. The positive-defined Lyapunov function can be defined as:

\[
L = \frac{1}{2} s^T \dot{s} = \frac{1}{2} (s_1^2 + s_2^2 + \ldots + s_n^2) > 0.
\]

(18)

The system is asymptotically stable if the derivative of the function (18):

\[
\dot{L} = s^T \dot{s} = s_1 \dot{s}_1 + s_2 \dot{s}_2 + \ldots + s_n \dot{s}_n
\]

(19)

is negative.

In the case of the relay method, equation (19) becomes:

\[
\dot{L} = s^T \dot{s} = s^T (f + Dk) = s^T f - s^T \Gamma \text{sign}(s^*).\]

(20)

There are defined two vectors:

\[
|a| = \begin{bmatrix} |a_1| & |a_2| & \ldots & |a_n| \end{bmatrix}
\]

and a unit vector:

\[
I = \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix}.
\]

(22)

Taking into account (21) and (22) equation (20) yields:

\[
\dot{L} = s^T f - \Gamma |s^T D|^T.
\]

(23)

Negative value of (23) is given if:

\[
s^T f < \Gamma |s^T D|^T
\]

(24)

that is equivalent to:

\[
\Gamma D^T > |D^{-1} f|,
\]

(25)

for each element of the above vectors, respectively.

The parameters included in the \( \Gamma \) matrix must be positive and large enough to fulfil the above inequality (25).

Similar methodology can be applied to the equivalent control method. The derivative of the Lyapunov function becomes:

\[
\dot{L} = s^T \dot{s} = s^T (f_1 + f_2 + Dk) = s^T (f_1 + f_2 - DD^{-1} f_1 - DT^d \text{sign}(s^*))
\]

(26)

Its negative value is ensured when:

\[
\Gamma^d D > |D^{-1} f|
\]

(27)

similarly as for (25).

Comparing (25) and (27) it is clear that the values of the parameters in the \( \Gamma^d \) matrix can be remarkably reduced, in relation to the relay control method.

3. Application of the sliding modes to the induction motor control and state estimation

3.1. Mathematical model of the controlled object. The design process presented in the previous chapter can be successfully applied to create both control and estimation algorithms. In the following part of the article, the mentioned steps will be taken to design the equivalent speed control method and relay speed estimator.

The first stage of the designing process should be the determination of the mathematical model of the plant and identification of its parameters. Both control and estimation algorithms utilize the same mathematical model of the squirrel-cage induction motor. The model can be derived using commonly known simplifications, in the \( \alpha - \beta \) frame and per unit system [1, 3]:

- Voltage equations:

\[
u_a = r_a i_a + T_N \frac{d}{dt} \psi_a,
\]

(28)

\[
\psi_a = x_a i_a + x_m i_r,
\]

(30)

- Current-flux equations:

\[
0 = r_y i_r + T_N \frac{d}{dt} \psi_r - j\omega_m \psi_r,
\]

(29)
The first order inertial element: torque regulator is any regulator, which acts approximately to the inverter directly [37]. The direct method does not guarantee the netic torque follows the reference value. It is assumed that the control signals vector 

\[ \mathbf{v}_r = x_r + x_m i_m, \]  

(31)

Equations (28)–(31), after some algebraic operations, will be used to create a model of the sliding mode estimator:

Equations (28)–(31), after some algebraic operations, will be used to create a model of the sliding mode estimator:

\[ \frac{d\omega_m}{dt} = \frac{1}{T_{me}} (m_e - m_o), \]  

(32)

\[ m_e = \text{Im} (\psi_s^* i_s) = \psi_{s\alpha} i_{s\beta} - \psi_{s\beta} i_{s\alpha}, \]  

(33)

where \( \text{Im}(x) \) – imaginary part of a complex value.

The value of the parameters should differ according, among others, to the power of the motor.

In order to prevent the above mentioned negative effects, the cascade control structure (a series connection of two regulators) will be the point of interest of the following part of this paper. The outer speed regulator produces a reference torque value, hence the control signals vector (1) becomes a scalar:

\[ \mathbf{k} = [m_e^{ref}], \]  

(36)

The inner torque controller ensures that the electromagnetic torque follows the reference value. It is assumed that the torque regulator is any regulator, which acts approximately to the first order inertial element:

\[ \frac{m_e(p)}{m_e^{ref}(p)} = \frac{1}{T_{me} p + 1}, \]  

(37)

where \( T_{me} \) – substitute time constant of the torque regulation loop.

Similarly, the switching function vector becomes a scalar as well:

\[ s = |s_\omega| = \omega_m^{ref} - \omega_m - T_{c} \dot{\omega}_m, \]  

(38)

where \( T_{c} \) – a time constant that defines the desired dynamics of the speed. If the control purpose is met, i.e. zero value of the switching function (38), the speed control structure has the dynamics of the first order inertial block with \( T_{c} \) time constant.

Derivative of the switching function (38) can be calculated as:

\[ \dot{s}_\omega = f_{1\omega} + f_{2\omega} + d_\omega m_e^{ref}, \]  

(39)

where respectively:

\[ f_{1\omega} = \frac{\omega_m^{ref}}{T_c} - \frac{T_m}{T_{me} T_c} m_e, \]  

(40)

\[ f_{2\omega} = \frac{T_c}{T_M} \dot{m}_o + \frac{1}{T_M} m_o. \]  

(41)

According to the methodology given in previous chapter, the equivalent sliding mode control law becomes as follows:

\[ m_e^{ref} = m_e^{ref, eq} + m_e^{ref, d}, \]  

(42)

where the continuous part:

\[ m_e^{ref, eq} = \frac{T_M T_{me}}{T_c} \left( \omega_m^{ref} - \frac{T_c - T_{me}}{T_M T_{me}} m_e \right), \]  

(43)

and the discontinuous part:

\[ m_e^{ref, d} = \frac{\Gamma_d}{T_{me}} \frac{T_M T_{me}}{T_c} \text{sign}(s_\omega). \]  

(44)

There are two parameters that have to be selected – the time constant \( T_{c} \) and the discontinuous part gain \( \Gamma_d^{me} \). The value of the parameters should differ according, among others, to the power of the motor.

As it was mentioned above, the control gain \( \Gamma_d^{me} \) should be determined taking into consideration the stability analysis of the proposed system. The Lyapunov function can be defined as:

\[ L = \frac{1}{2} s_\omega^2 > 0 \]  

(45)

and its derivative, *

\[ \dot{L} = s_\omega \dot{s}_\omega = s_\omega f_{2\omega} - \Gamma_d^{me} |s_\omega|. \]  

(46)

The derivative (46) is negative, and the system is asymptotically stable, when

\[ \Gamma_d^{me} > \left| \frac{T_c}{T_M} \dot{m}_o + \frac{1}{T_M} m_o \right|. \]  

(47)

According to (47), the gain \( \Gamma_d^{me} \) should be selected to compensate the load torque and its changes.

The block diagram of the sliding mode equivalent control method for induction motor drive is presented in Fig. 2. The inner torque regulator relies on the electromagnetic torque and rotor flux vector – they are estimated using the proper estimator. If it is necessary, the measured stator current vector and DC link voltage are used as well. The second purpose of the inner controller is to stabilize the amplitude of the chosen motor flux – stator or rotor. The control signal, i.e. reference torque, is limited to the maximum available value \( m_e^{\text{max}} \).
Fig. 2. The block diagram of the sliding mode equivalent control method in application to the induction motor drive speed control

3.3. Sliding Mode Model Reference Adaptive System speed estimator – SM-MRAS. The design method of the sliding mode estimator, presented below, will be analogous to the one described in chapter 2. This approach helps to generalize the sliding mode algorithms and indicates the universality of this technique.

The estimator will be designed as a Model Reference Adaptive System (MRAS) estimator. The reference model will be the induction motor itself. The adaptive part of the estimator will be both stator current and rotor flux vectors equations.

In order to start the designing process the estimated variables vector must be selected. The estimated speed and an additional variable \( \mu \), introduced to decrease the sensitivity of the estimator to the rotor time constant changes, form this vector:

\[
k_o = [\hat{\omega}_m, \hat{\mu}]^T.
\]

Selecting such pair of the estimated variables (48), the mathematical model of the estimator is a modification of equations (34)–(35):

\[
T_N \frac{d \hat{i}_s}{dt} = \frac{1}{x_s \sigma} (u_s - \left( r_s + \frac{r_r x_m}{x_r} \right) \hat{i}_s)
+ \frac{x_m}{x_r} \left( \frac{r_r}{x_r + \mu} \right) \hat{\psi}_r - \frac{j x_m}{x_r} \hat{\omega}_m \hat{\psi}_r,
\]

\[
T_N \frac{d \hat{\psi}_r}{dt} = -\left( \frac{r_r}{x_r + \mu} \right) \hat{\psi}_r + \frac{x_m r_r}{x_r} \hat{i}_s + j \hat{\omega}_m \hat{\psi}_r.
\]

Next step should be the determination of the switching functions vector. This vector can be a combination of stator current vector estimation errors and rotor flux vector components [26]:

\[
s_o = \begin{bmatrix} s_o \\ s_o \end{bmatrix} = \begin{bmatrix} (i_s \beta - i_s \beta) \hat{\psi}_r - (i_s \alpha - i_s \alpha) \hat{\psi}_r \\ \hat{\mu} \end{bmatrix}.
\]

Derivative of the switching function vector (51) can be decomposed into:

\[
\dot{s}_o = f_o + D_o k_o,
\]

where

\[
f_o = \frac{1}{T_N} \begin{bmatrix} f_{o1} \\ f_{o2} \end{bmatrix},
\]

\[
f_{o1} = -\alpha_1 s_o - \frac{r_r}{x_r} s_o - \frac{x_m}{x_r} \left( e_i s \alpha + e_i s \beta \right) + \alpha_2 \left( e_{\psi r} \hat{\psi}_r - e_{\psi r} \hat{\psi}_r \right) + \alpha_3 \hat{\omega}_m \left( \hat{\psi}_r - \hat{\psi}_r \right),
\]

\[
f_{o2} = -\alpha_1 \frac{r_r}{x_r} s_o - \frac{x_m}{x_r} \left( e_i s \alpha + e_i s \beta \right) + \alpha_2 \left( e_{\psi r} \hat{\psi}_r - e_{\psi r} \hat{\psi}_r \right) + \alpha_3 \hat{\omega}_m \left( \hat{\psi}_r - \hat{\psi}_r \right),
\]

and the estimation errors:

\[
\begin{align*}
& e_i s \alpha = i_s \alpha - \hat{i}_s \alpha, \quad e_i s \beta = i_s \beta - \hat{i}_s \beta, \\
& e_{\psi r} = \hat{\psi}_r - \psi_{\hat{\psi}_r} - \psi_{\hat{\psi}_r}.
\end{align*}
\]

Matrix \( D_o \) is as follows:

\[
D_o = \frac{1}{T_N} \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix},
\]

\[
d_{11} = -\frac{x_m}{x_r} \left( \hat{\psi}_r - e_i s \alpha \hat{\psi}_r - e_i s \beta \right),
\]

\[
d_{12} = -e_i s \beta \hat{\psi}_r - e_i s \alpha \hat{\psi}_r,
\]

\[
d_{21} = e_i s \beta \hat{\psi}_r - e_i s \alpha \hat{\psi}_r,
\]

\[
d_{22} = -e_i s \beta \hat{\psi}_r - e_i s \alpha \hat{\psi}_r.
\]

The relay technique (6)–(8) is applied to design the sliding mode estimation law:

\[
k_o = -\gamma \omega \hat{\mu} s_o^T, \quad s_o^* = s_o^T D_o,
\]

where the gain matrix:

\[
\Gamma_o = \begin{bmatrix} \gamma_\omega & 0 \\ 0 & \gamma_\mu \end{bmatrix}.
\]

Assuming that the estimation errors in (55) can be omitted, equation (56) simplifies to:

\[
k_o = \begin{bmatrix} \hat{\omega}_m \\ \hat{\mu} \end{bmatrix} = \begin{bmatrix} \gamma_\omega \hat{s}_0 \gamma_\omega & -\gamma_\mu \hat{\mu} \gamma_\mu \\ -\gamma_\mu \gamma_\omega & -\gamma_\mu \gamma_\mu \end{bmatrix}.
\]

The chosen estimator gains: \( \gamma_\omega, \gamma_\mu \) must be large enough to ensure the asymptotic stability of the estimator, which can be described by the following formula:

\[
\hat{L} = s_o^T \hat{s}_0 < 0,
\]

that is equivalent to:

\[
\Gamma_o \hat{L} > |D_o^{-1} f_o|,
\]

similarly as for (25).

The block diagram of the sliding mode speed estimator is presented in Fig. 3. The estimated speed, obtained using (57), has only two values \( +\gamma_\omega \) – this signal is useless in the control structure – therefore it must be filtered. The simplest low-pass filter and its transfer function are shown in Fig. 3.
Stator flux vector and electromagnetic torque can be calculated using directly the induction motor model:
\[
\hat{\psi}_s = \frac{x_m}{x_r} \hat{\psi}_r + x_s \sigma \hat{i}_s, \\
\hat{m}_e = \hat{\psi}_s \sigma \hat{i}_s - \hat{\psi}_s \sigma \hat{i}_s. 
\] (61)

\[
\hat{m}_e = \hat{\psi}_s \sigma \hat{i}_s - \hat{\psi}_s \sigma \hat{i}_s. 
\] (62)

4. Experimental verification

In order to verify the presented sliding mode algorithms, both simulation and experimental tests were performed. Due to the limited capacity of the paper, only the experimental verification is shown in this chapter. The laboratory setup consisted of 3 kW induction motor and 5 kW DC-motor, acting as a load torque. Parameters of the induction motor are shown in Appendix. Data acquisition, measurement, estimation and control were executed using a digital signal processor – dSpace DS 1103. Integration step was equal to 100 µs (frequency 10 kHz) and data downsampling 1.

4.1. Equivalent sliding mode speed control method.

The performance of the sliding mode equivalent control method applied to the induction motor speed control is shown in Fig. 4. It is assumed that the inner torque regulator is the Sliding-Mode Direct Torque Control structure, described in [26].

During the presented test, the reference speed was a reverse function, from 0.5 to −0.5 of the nominal value; the load torque was set to the nominal value \( m_{N} = 0.67 \) [p.u.], see Appendix). The reference dynamics (marked grey in Fig. 4a) is the response of the first order inertial block with time constant \( T_c = 0.1 \) (95% settling time \( T_s = 3T_c = 0.3 \) s). The real speed follows the reference dynamics very well, without visible oscillations.

Continuous (eq) and discontinuous (d) components of the control signal are placed in Fig. 4b. In order to further reduce the oscillations of the discontinuous part, the sign function in (44) was replaced by its approximation – a saturation function [30]. The resultant control signal, the reference torque and its estimated value are presented in Fig. 4c. It can be seen that the estimated torque follows the reference value, and is constrained on the maximum value. Stator phase currents are limited as well (Fig. 4d). The interior torque controller performs also the stabilization of the stator flux – it can be noticed in Fig. 4e.

4.2. Sliding mode speed estimator.

The sliding mode relay technique can be successfully applied to design the speed estimator, as it was described in the previous chapter, and thus create a speed-sensorless drive. Such operation is presented in Fig. 5. The estimated speed, torque and stator flux vector are used directly in the control structure. Again, the reverse speed operation is illustrated in the figure. Estimated speed (Fig. 5a) follows the real speed with a slight error, which is shown in Fig. 5b. This error becomes larger during the reversions – the dynamical delay (introduced by the low-pass filter) can be seen. However, despite this error, the drive works properly.
T. Orlowska-Kowalska and G. Tarchala

Fig. 5. Performance of the sliding mode speed estimator SM-MRAS in application to induction motor drive: a) real and estimated speed, b) speed estimation error, c) estimated amplitudes of stator and rotor fluxes, d) speed switching function, e) measured and estimated $\alpha$-component of the stator current vector.

The estimated amplitude of the stator flux is stabilized by the control structure (Fig. 5c) on the nominal value. The estimated value of the rotor flux amplitude is also constant.

The speed switching function value is almost zero (Fig. 5d) – it was the purpose of the sliding mode algorithm. Since the switching function equation is based on the estimation error of the stator current vector components, the real and estimated component of the $\alpha$-axis is shown in Fig. 5e. As it can be seen, the difference between them is quite small, and increases during reversions.

5. Conclusions

This paper deals with the sliding mode systems design and its application to the induction motor drives. The design procedure is unified for both the control system and the estimator. The paper tries to divide the design process into several steps, from the determination of the mathematical model of the plant to the stability analysis of the designed system. Three different sliding mode techniques: relay method, equivalent control method and linear control with switched gains are described.

The equivalent control method is then applied to the induction motor speed control. This method gives an excellent speed control and ensures the required dynamics of the speed.

Finally, the relay technique is used to design the speed estimator. The estimator is the Model Reference Adaptive System type estimator. The speed is estimated properly, with small dynamic error during speed reversions.

The presented algorithms are illustrated with the experimental results obtained using a 3 kW induction motor drive.

Appendix – parameters and nominal values of the induction motor, base values

In the following tables parameters of tested induction motor (Table 1), its nominal parameters (Table 2) and base values, necessary during the transition to the per unit system (Table 3) are presented. The moment of inertia of the drive is $J = 0.0292$ kg m$^2$, that corresponds to the mechanical time constant $T_M = J\Omega_b/(p_0M_b) = 0.15$ s.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Parameters of the induction motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Physical values [Ω]</td>
</tr>
<tr>
<td>Stator resistance $R_s$</td>
<td>7.073</td>
</tr>
<tr>
<td>Rotor resistance $R_r$</td>
<td>7.372</td>
</tr>
<tr>
<td>Main reactance $X_m$</td>
<td>187.8</td>
</tr>
<tr>
<td>Stator leakage reactance $X_{s\sigma}$</td>
<td>9.80</td>
</tr>
<tr>
<td>Rotor leakage reactance $X_{r\sigma}$</td>
<td>9.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Nominal parameters of the induction motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Physical values</td>
</tr>
<tr>
<td>Power $P_N$</td>
<td>3.0 [kW]</td>
</tr>
<tr>
<td>Torque $M_N$</td>
<td>20.46 [Nm]</td>
</tr>
<tr>
<td>Rotational speed $N_N$</td>
<td>1400 [rpm]</td>
</tr>
<tr>
<td>Stator voltage $U_sN$</td>
<td>400 [V]</td>
</tr>
<tr>
<td>Stator current $I_sN$</td>
<td>4.0 [A]</td>
</tr>
<tr>
<td>Frequency $f_sN$</td>
<td>50 [Hz]</td>
</tr>
<tr>
<td>Stator flux $\Psi_sN$</td>
<td>1.65 [Wb]</td>
</tr>
<tr>
<td>Rotor flux $\Psi_rN$</td>
<td>1.54 [Wb]</td>
</tr>
<tr>
<td>Pole pairs $p_b$</td>
<td>2 [-]</td>
</tr>
</tbody>
</table>
Unified approach to the sliding-mode control and state estimation...

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Base values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>Physical values</td>
</tr>
<tr>
<td>Power</td>
<td>$S_b = 3/2U_bI_b$</td>
</tr>
<tr>
<td>Torque</td>
<td>$M_b = p_bS_b/\Omega_b$</td>
</tr>
<tr>
<td>Rotational speed</td>
<td>$N_b = 60I_b/\omega_b$</td>
</tr>
<tr>
<td>Stator voltage</td>
<td>$U_b = \sqrt{2}U_{MN}$</td>
</tr>
<tr>
<td>Stator current</td>
<td>$I_b = \sqrt{2}I_{MN}$</td>
</tr>
<tr>
<td>Frequency</td>
<td>$f_b = f_{MN}$</td>
</tr>
<tr>
<td>Angular velocity</td>
<td>$\Omega_b = 2\pi f_{MN}$</td>
</tr>
<tr>
<td>Flux</td>
<td>$\Psi_b = U_b/\Omega_b$</td>
</tr>
</tbody>
</table>

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REFERENCES

[26] T. Orlowska-Kowalska, G. Tarchala, and M. Dybkowski, “A unified approach to the sliding-mode control and state estimation...


