

## A note on the irrelevance of unit root tests and cointegration tests

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### SUMMARY

We show that, in practice, the standard unit root tests, cointegration tests, and similar tests are unreliable. This conclusion is more generally applicable to other related regression-based tests. In particular, these tests attempt to solve a problem by creating another problem.

**Key words:** unit root test, Dicky–Fuller test, ADF test, cointegration test, non-stationarity

### 1. Introduction

In this article, we show that the cointegration and unit root tests (most notably the Dicky–Fuller test) and similar tests are based on circular reasoning (an obvious logical fallacy). In particular, they attempt to solve a problem by creating another similar or even identical problem. Therefore, in practice, these tests are futile attempts to identify or solve statistical problems associated with regression analysis.

Consequently, it is surprising that these tests are widely used, given their lack of reliability. Thousands of academic papers have used these tests without questioning their reliability. These numerous papers offered unwarranted or false conclusions based on these tests.

Our conclusion is more generally applicable to other related regression-based tests, such as the popular Johansen test. Furthermore, this conclusion is even applicable to some statistical tests outside the topic of unit roots and cointegration. Consequently, an alternative methodology of testing should be used.

## 2. Discussion

To show this, consider, for example, the original Dicky–Fuller test that starts with the untested (unverified) assumption of an AR(1) process

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t,$$

then the following associated testing regression

$$\Delta y_t = (\beta_1 - 1) y_{t-1} + \varepsilon_t.$$

Firstly, before using the above regression, we need to know that the error  $\varepsilon$  is stationary (among other properties). But in order to do so, we need to obtain another unit root as follows

$$\Delta \varepsilon_t = (\beta_2 - 1) \varepsilon_{t-1} + u_t.$$

But once again, we need to know that the error  $u$  is stationary, and the cycles continue. So we will be moving in cycles infinitely.

Even if we assume, for sake of argument, that we are clairvoyant and know that  $|\beta_1| < 1$ , this does not necessarily imply that the variable is stationary (since the error properties are not known). This is another fallacy. Therefore, at the practical level, these tests are unreliable (at best) and typically misleading. Other similar econometric/statistical tests suffer the same problem.

Even primitive, naive methods of identifying non-stationarity (such as data plots) are far more reliable. Alternatively, the problem of non-stationarity can be avoided altogether by using the methods suggested by Alghalith (2018) and Alghalith (2019).

In sum, at the practical level, these tests attempt to solve a problem by creating another problem of equal degree. For other legitimate criticisms of econometrics (especially cointegration), see, for example, Luitel and Mahar (2015), Moosa (2011) and Moosa (2017).

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