

Simulating heart rate kinetics during incremental and interval training

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Summary

Study aim: To apply a recently proposed dynamical systems model to simulate, for the first time, the heart rate (*HR*) response to exercise of time-dependent intensity.

Material and methods: The applied model is expressed in the form of two coupled ordinary differential equations, one giving the rate of change of *HR* and the other providing the time dependency of exercise intensity. According to the model, the *HR* is assumed to depend on time, velocity, lactate accumulation and the subject's overall cardiovascular condition. For the purposes of the present work, training sessions were simulated, consisting of exponentially and step-wise increasing intensities, as well as interval training.

Results: Training sessions of exponentially and step-wise increasing velocity, as well as an interval training session, have been simulated. Successful simulations of the corresponding *HR* response were achieved.

Conclusions: The present work successfully demonstrates the model's excellent performance in simulating the *HR* response to exercises of time-dependent intensity. The applied model has been shown to correctly simulate the heart rate response also during exercises of complicated intensity patterns, such as the interval training session. The study confirms the ability of the implemented mathematical model not only to simulate and predict heart rate kinetics for any given exercise intensities but also to provide important information regarding an individual's cardiovascular condition. This is of vast importance, not only in the area of fitness and sport, where it can serve as a fundamental tool for the design of efficient training sessions, but also in the areas of cardiovascular health, prediction and rehabilitation.

Keywords: Modelling – Simulation – Exercise physiology

Introduction

Based on ventricle contractions, heart rate is defined as the number of heart beats per minute. The temporal evolution of the heart rate (heart rate kinetics) of a person who starts moving from rest depends on the intensity of the exercise, as well as on a number of other factors, such as temperature, heat, fatigue, age, over-training, nutrition and hydration, altitude, medication, infectious disease or even mental activity [1, 8, 24].

The analysis of the heart rate response to exercise is a very important tool in the areas of physiology, cardiovascular health, prediction and rehabilitation, as it can detect hidden physiological responses or abnormalities and also provide important information regarding cardiovascular condition (see also [1, 2, 3, 9, 11, 13, 22]). In the area of fitness and sport it can also serve as a fundamental tool for the design of efficient training sessions.

Two important values are associated with heart rate: its maximum value HR_{\max} , which is the highest value that can be achieved in an all-out effort to the point of exhaustion, and its resting value HR_{\min} , which is measured in the absence of any movement while the subject is relaxed but awake, in a neutrally temperate environment, not having recently exerted themselves or having been subject to stress or surprise [8, 14, 17, 24]. Professional athletes have been reported to have a resting heart rate as low as 35 beats/min, while for a normal person of age 10 years or more it is normally $60 \text{ beats/min} < HR_{\min} < 100 \text{ beats/min}$ [1, 9, 24, 27]. Men have been shown to have a lower resting heart rate than women, with a reported difference of about 5 beats/min in athletes [24]. Regarding its maximum value HR_{\max} , it should be noted that this is a highly reliable value that remains constant for a particular subject [25] and changes only slightly with age (a slight but steady decrease of about one beat/min per year, beginning at 10 to 15 years of age, has been observed).

The heart rate of an individual increases or decreases between these two extreme values in response to the body's need to maintain an equilibrium between requirement and delivery of oxygen and nutrients. Changes in heart rate can be provoked by physical exercise, fatigue (lactate accumulation in the blood), changes in temperature, sleep, nutrition, hydration, altitude, medication, diseases, anxiety or stress [1, 8, 24].

When all other parameters that can affect the heart rate during exercise are kept constant, then the heart rate response can be assumed to be a function of exercise intensity and the individual's cardiovascular condition. The "heart rate demand" D , which is the value the heart rate is called to meet, depends in general on time and exercise intensity. Always $D \geq HR_{\min}$. For low intensity exercises the demand D is only intensity-dependent and $D < HR_{\max}$. For severe or very high intensity exercise $D \geq HR_{\max}$ and an abrupt rise in heart rate towards HR_{\max} is observed [24]. For such exercise intensities the achievement of a steady state in the heart rate values is delayed and a slow increase in heart rate can be observed, a phenomenon that has been referred to as the slow component [21, 24].

Various models of the cardiovascular response to exercise have been proposed, mainly based on the statistical goodness of fit of the model curves to heart rate data, starting from the mono-exponential model for moderate constant exercise intensity [11] and continuing to the three-phase model [2, 3, 9, 13, 18, 20, 22], which gives the heart rate response to constant intensity exercise by means of three exponential functions. Other models using first order linear approximations have also been proposed (see [15] and the references therein); however, the implementation of these models is limited, as they can only be applied to exercises of constant intensity.

The dynamical systems model presented in [20], which is based on a similar model of oxygen uptake kinetics [19, 21], simulates and predicts the heart rate response to exercise via numerical solution of a system of coupled differential equations, one giving the rate of change of heart rate and the other giving the rate of change of exercise velocity. This model can be implemented for exercise of linear or non-linear intensities, of any functional form. Another recently proposed model of heart rate dynamics during cycling [15] is based on the dynamical systems model of [20] and simulates heart rate dynamics across non-linear exercise intensities by introducing a differential equation for the rate of change of heart rate demand. According to this model, the rate of change of D is assumed to depend on the current value of D , as well as the power output and cadence, and two more model parameters are introduced.

Based on the assumption that a model should include as few parameters as possible, an improved and advanced model of [20] has also been recently proposed [24]. This

model assumes only one parameter, which is related to the subject's overall cardiovascular condition and takes into account the dynamics of the body's physiological responses, i.e. lactate accumulation and the kinetics near the extremes, the maximum and minimum heart rate values.

The aim of the present study is to apply this recently proposed dynamical systems model [24] to simulate, for the first time, the heart rate (HR) response to exercise of time-dependent intensity.

Materials and methods

Modelling heart rate kinetics as a dynamical system

This section presents a brief introduction to the dynamical systems model herein implemented for the simulation of heart rate kinetics. For more details on this model please refer to [24].

The model is expressed in the following form of a system of coupled ordinary differential equations [24], one regarding the rate of change of heart rate HR and the other regarding the rate of change of exercise intensity (velocity v):

$$\dot{HR} = -f_{\min} f_{\max} f_D \quad (1)$$

$$\dot{v} = I(t) \quad (2)$$

where:

- t is the time elapsed from the beginning of the data recording session,
- $\dot{HR} = \frac{dHR}{dt}$,
- v is the exercise velocity, which is in general time-dependent,
- $\lambda \in [0, 1]$ is a global parameter of the model that defines fitness level, such that $\lambda \approx 1$ refers to an excellent cardiovascular condition,
- $HR \equiv HR(HR(0), \lambda, v, t)$ is the subject's current heart rate, which is assumed to depend on time, as well as the initial heart rate $HR(0)$ (as measured at the beginning of the data recording session), the subject's current cardiovascular condition (as defined by the value of λ) and on the velocity v of the exercise,
- $I(t)$ is assumed to be only time-dependent,
- the repelling term $f_{\max}(HR)$ controls the dynamics near the maximum heart rate HR_{\max} ,
- the repelling term $f_{\min}(HR, \lambda)$ controls the dynamics near the minimum heart rate HR_{\min} and is assumed to depend on λ ,
- the attracting term $f_D(HR, HR(0), v, \lambda, t)$ controls the overall heart rate dynamics near the attractor of the heart rate demand D and is assumed to depend on time, velocity v , the initial heart rate value $HR(0)$ and the subject's current cardiovascular condition.

Regarding the minimum heart rate value, it is assumed that $HR_{\min}(\lambda = 1) = 35$ beats/min for males and $HR_{\min}(\lambda = 1) = 40$ beats/min for females [24]. It is also assumed that, for $\lambda < 1$,

$$HR_{\min}(\lambda) \equiv 35/\lambda \text{ beats/min}$$

for males and

$$HR_{\min}(\lambda) \equiv (35/\lambda + 5) \text{ beats/min}$$

for females [24].

The functions f_{\max} and f_{\min} are defined as follows (for more details and an extended study on these functions please refer to [24]):

$$f_{\max(HR)} = -\left\{1 - e^{-\left(\frac{HR - HR_{\max}}{\alpha_1}\right)^2}\right\}$$

$$f_{\max(HR\lambda)} = -\left\{1 - e^{-\left(\frac{HR - HR_{\min\lambda}}{\alpha_2}\right)^2}\right\}$$

where $\alpha_1 = \alpha_2 = 10$ beats/min (these optimal numerical values were found by trial and error during a thorough study on fitting the model to time series data [24]).

The function f_D is defined as [24]

$$f_D(HR, HR(0), v, \lambda, t) \equiv -\alpha\lambda[HR - D(HR(0), \lambda, v, t)],$$

where the optimal value of the parameter α was found to be, by trial and error as with the other parameters of the model, $\alpha = 0.08 \text{ s}^{-1}$. Assuming that the heart rate demand is affected by the value of $HR(0)$, the total heart rate demand D can be expressed as the sum of a term describing the intensity-dependent heart rate demand D_d and a term that simulates the heart rate demand “memory” effects [24]:

$$D(HR, HR(0), v, \lambda, t) = D_d(v, \lambda, t) + \{HR(0) - D_d(v, \lambda, t)e^{-\sigma t^2}\}.$$

The standard deviation of the Gaussian-like curve that describes the memory effects on the heart rate demand is modelled [24] by the relation with optimal parameter values $\alpha_4 = 0.003 \text{ s}^{-2}$ and $\alpha_5 = 4$.

$$\sigma \equiv \alpha_4\lambda\left(\frac{HR_{\max} - HR_{\min}}{HR(0) - HR_{\min}}\right)^{\alpha_5}$$

In the absence of any effects of lactate accumulation (low exercise intensity), the constant value of the heart rate demand D_{ss} is the steady state that the heart rate values reach after a few minutes of exercise. For higher exercise intensities the effects of blood lactate accumulation are apparent a few minutes after the beginning of the exercise and add a “slow component” effect as the exercise continues. This lactate-induced heart rate demand $D_{La}(\lambda, v, t)$ is assumed to be added as an extra component to D_{ss} [24]:

$$D_d(\lambda, v, t) \equiv D_{ss}(\lambda, v, t) + D_{La}(\lambda, v, t).$$

For more details on how $D_{La}(\lambda, v, t)$ is modelled please refer to [24].

For low to moderate intensity exercises, the constant heart rate demand D_{ss} is modelled as follows (see also [12]):

$$D_{ss} = \frac{v}{v_{\max}} (HR_{\max} - HR_{\min}) + HR_{\text{rest}}$$

where the value HR_{rest} refers to the resting heart rate (idling, pre-exercise heart rate for $v = 0$) and, as discussed in [24], the velocity value $v_{\max} \equiv v_{\max}(\lambda)$ refers to the maximum exercise velocity achievable by an individual of cardiovascular condition λ . It has been assumed that $v_{\max}(\lambda) = 27\sqrt{\lambda}$ (km/h) [24].

Intensity pattern simulation

The simulations were carried out using the mathematical package Maple, version 10 professional.

For the simulations a hypothetical male runner was assumed, with $\lambda = 0.85$, $HR_{\max} = 185$ beats/min, $HR_{\min} = 45$ beats/min, a 800 m running velocity of 25 km/h and a velocity $vVO_{2\max} = 20$ km/h (see also [6]).

Let us denote as t_0 the exercise (or data recording) starting time.

Aiming to demonstrate the validity as well as the ability of the model [24] to simulate heart rate kinetics, the following different intensity patterns were selected, as examples:

1. incremental exercise, strictly increasing velocity,
2. incremental exercise, sequence of steps,
3. interval training [4, 5].

Incremental exercise, strictly increasing velocity

A velocity function that follows an exponential rise was chosen as an example for this subsection. Denoting as τ the time constant of the exponential and v_m as the maximum velocity value of the training session, then the func-

$$v(t) = v_m\left(1 - e^{-\frac{t-t_0}{\tau}}\right)$$

tional dependence of the velocity on time, for this specific case, can be expressed as:

For the simulation of this example exercise it was assumed that $t_0 = 0$, $v_m = 26$ km/h and $\tau = 12$ s. The initial HR value was set equal to $HR(0) = 60$ beats/min and the total exercise time was set equal to 60 s.

Incremental exercise, sequence of steps

Let us assume that the velocity pattern in this case consists of a sequence of N steps and let us denote as T_{si} and v_i the time duration and the velocity of the i -th step respectively, where $v_i < v_{i+1} \forall i = 1, \dots, N-1$.

$$v(t) = \frac{v_N}{2} \left\{1 + \frac{1}{N} \sum_{i=1}^N \tanh\left(\frac{(t-t_0) - \sum_{j=1}^{i-1} T_{sj}}{\sigma_s}\right)\right\},$$

The step velocity pattern can be simulated in this case by means of a sum of hyperbolic tangents as follows: where the parameter σ_s is to be numerically chosen so as to simulate the step-wise change in v . For the simulations we assumed an example exercise starting from

$t_0 = 0$, with $v_i = 5i$ km/h, $i = 1, \dots, 5$, $T_{S_i} = 20$ s and total exercise time 100 s. The initial HR value was set equal to $HR(0) = 60$ beats/min. The parameter σ_s was chosen to be $\sigma_s = 0.5$.

Interval training

The simulated interval training pattern follows the recommendations presented in [4] and [5]. Let us assume that the velocity pattern consists of a sequence of N sets of exercise intervals and their subsequent recoveries. Let us also denote as v_i the velocity and T_i the time duration of each exercise interval, and similarly v_{Ri} the velocity and T_{Ri} the time duration of each recovery. If $v_i \equiv v_I$ and $v_{Ri} \equiv v_R$, $\forall i = 1, \dots, N$ then, using a combination of hyper-

$$v(t) = v_R + \frac{v_I - v_R}{2} \sum_{i=1}^N \tanh \left\{ \frac{(t - t_0) - \sum_{j=1}^i (T_j + T_{Rj})}{\sigma} \right\}, \\ + \frac{v_I - v_R}{2} \sum_{i=1}^N \tanh \left\{ \frac{-(t - t_0) + \sum_{j=1}^{i-1} (T_j + T_{Rj}) + T_i}{\sigma} \right\},$$

bolic tangents as before, the velocity pattern of interval training can be simulated by a function given as follows:

As mentioned in the previous subsection, the parameter σ is to be numerically chosen. For the simulations presented in the following section, $\sigma = 0.5$.

For the purposes of the present subsection, an example anaerobic training session was chosen, consisting of 6 intervals at 125% vVO_{2max} , i.e. $v_I = 25$ km/h, $v_R = 0$ km/h and $T_i = T_{Ri} = 30$ s; see [5, 6].

The training session was assumed to start at $t = 0$ with a 200 s warm-up exercise at constant velocity of 14 km/h, from $HR(0) = 55$ beats/min, followed by a 30 s rest before

the interval training. The 30-30 sessions were thus assumed to start at $t_0 = 230$ s.

Details of the numerical procedure

The resulting simulated heart rate response is provided, in general, via the numerical solution of the two coupled equations of the model [24], i.e. the simultaneous solution of equations (1) and (2).

There are cases, as in the previous sections, where for reasons of simplicity the exercise velocity is provided as an explicit function of time $v(t)$. In such cases the solution of equation (2) is given ab initio and therefore the simulated heart rate time series are given by numerical solution of equation (1) only, where $v \equiv v(t)$.

The numerical solution of equation (1) was carried out in the present study by use of the Runge-Kutta-Fehlberg (RKF45) method [10], a numerical differential equation solver of order $O(h^4)$ with an error estimator of order $O(h^5)$. The algorithm's step size was set equal to $h = 0.01$.

Results

Incremental exercise, strictly increasing velocity

Figure 1 shows the simulated heart rate time series for the 60 s incremental exercise of strictly increasing velocity. In the same figure the velocity values are also shown for each time t . As can be seen in figure 1, the smooth, slow increase in HR, that follows the increase in v , was

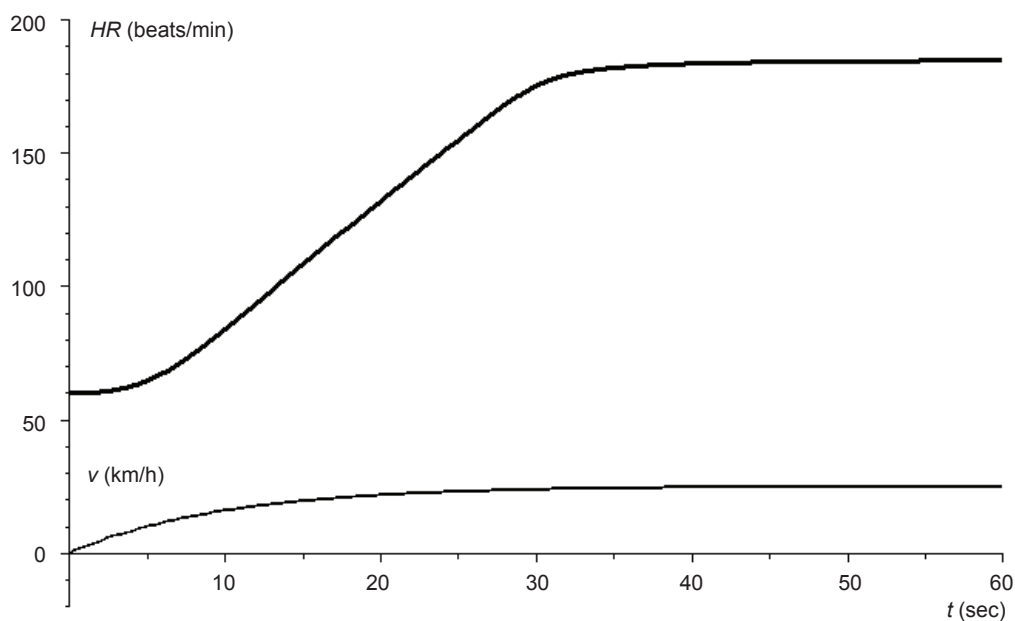


Fig. 1. Incremental exercise, exponentially increasing velocity. On the vertical axis the time series of both the HR and the v are shown (thick and thin black lines respectively)

successfully simulated. For the interpretation of the plateau *HR* values towards the end of the exercise it should be remembered that the subject's maximum heart rate was assumed to be $HR_{max} = 185$ beats/min and that his 800 m running velocity was assumed to be 25 km/h.

Incremental exercise, sequence of steps

Figure 2 shows the simulated heart rate time series for the 100 s incremental exercise which consisted of a sequence of steps.

As in the previous case, the *HR* values follow the step-wise changes in v until HR_{max} is reached and a plateau in *HR* values is achieved. By careful observation of figure 2 a short duration "slow component" effect can be seen in the *HR* time series during the penultimate step, as the *HR* values approach HR_{max} .

Interval training

Figure 3 shows the simulated heart rate time series for the 590 s interval training session.

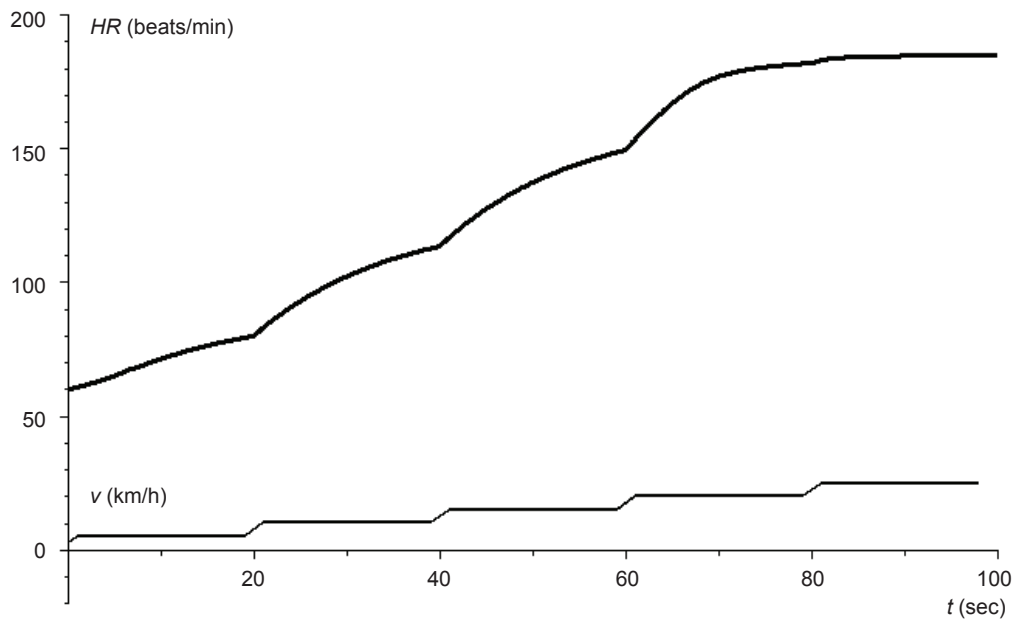


Fig. 2. Incremental exercise, velocity increasing as a sequence of steps. On the vertical axis the time series of both the *HR* and the v are shown (thick and thin black lines respectively)

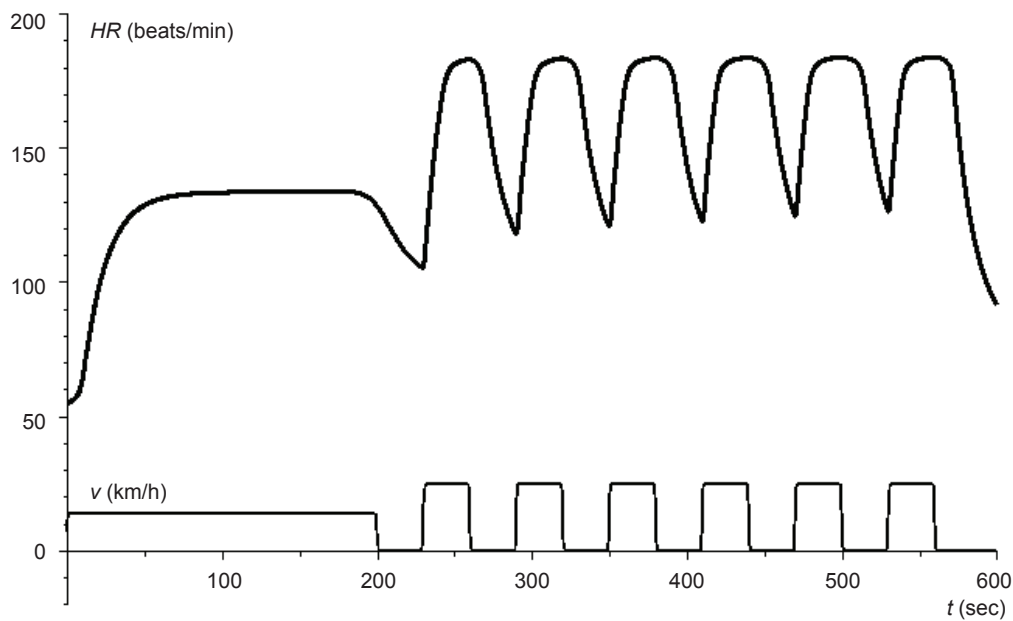


Fig. 3. 30-30 interval training session, following a 200 sec constant velocity warm-up. On the vertical axis the time series of both the *HR* and the v are shown (thick and thin black lines respectively)

The success of the model regarding the simulation of such a training session is obvious. Figure 3 reveals slowly increasing *HR* recovery values, a phenomenon that can be due to the lactate accumulation/“slow component” effect.

Discussion

The study of heart rate kinetics is currently the main area of research in exercise physiology. An understanding of heart rate kinetics is fundamental to training methodology and competitive success in sport, as well as to our knowledge of cardiovascular health. A correct and efficient means of interpreting and analyzing heart rate data is of vast importance, as exercise testing is routinely used in both of these areas [9, 16, 23].

The present work demonstrates the application of a recently proposed dynamical systems model [24] to the simulation of heart rate kinetics during exercises of time-varying intensity. The use of modern mathematical methods of analysis such as the model implemented here would be of much benefit for understanding the relationship between the physiological responses and fitness and/or performance, or for identifying certain features in the physiological time series. As a result, the application of such mathematical methods for analysis and modelling in *HR* kinetics will have a large impact not only in the development and understanding of training methodology and the testing of athletes but also in the area of medicine. Aspects of general health, recuperation and rehabilitation, as well as diagnosis of many abnormalities in the *HR* response to movement, or possible disorders that may cause exercise intolerance, are some of the practical advantages of the application of mathematical techniques that model and analyze heart rate responses. Such techniques could also be applied to the study of the heart rate response of population groups for which direct heart rate recordings in intense exercises are not possible or not allowed, such as elderly or pregnant women.

The mathematical model applied in this study is given as a set of coupled ordinary differential equations with respect to time. Its objective is to describe, simulate, fit existing data and ultimately predict the heart rate response to movement. It is worth emphasizing that the power and novelty of this mathematical model lie in the fact that the model is specifically designed (see [24]) to meet the following requirements:

1. A model's features should reflect the body's physiological features.
2. A model should include as few parameters as possible: the model of [24] assumes only one model parameter

(λ), which reflects the person's overall cardiovascular condition (which improves with appropriate training or rehabilitation). Any changes in this global parameter (λ) will reflect improvements in the overall cardiovascular condition (via appropriate training) or loss (due to lack of training or injury). It should be noted here that $0 < \lambda < 1$, so that the condition of $\lambda = 1$ refers to an excellent cardiovascular condition.

3. The functions forming the model should be smooth functions of time.

Regarding comparison of the model implemented herein with other similar existing models, it should be noted that, although the dynamical systems model presented in [20] can be possibly implemented to exercise of linear or non-linear intensities, its fit to raw heart rate time series data has only been demonstrated for exercises of constant intensity [25, 26, 27]. On the other hand, the model of heart rate dynamics during cycling [15] has been successfully applied to heart rate time series data providing numerical values for the model's parameters; however, two more model parameters have been introduced. Finally, even though the improved and advanced version of the model of [20] is capable of following the heart rate dynamics for any linear or non-linear exercise intensity, its ability to fit heart rate time series data, as well as to simulate and predict heart rate responses, has only been shown for exercises of constant intensity [24].

Regarding the historically first model of cardiovascular kinetics, the 3-phase model, it should be noted that this model emerged as the best fit to the data, from a statistical point of view. Its validity is debatable, as it is very likely that the observed three phases are a figment of the incorrect and overly simple treatment of the data [18]. Furthermore, the parameter values that the model provides can only reflect the body's physiological response to the particular exercise intensity, and no reliable predictions can be made for other exercise intensities. Furthermore, there is a strong interdependency in the model's parameters, which dramatically affects their values and consequently the model's confidence. In comparison, the dynamical systems model applied in the present work is not only able to provide important information regarding the cardiovascular condition of a person, but also to simulate and predict heart rate kinetics for any given exercise intensities.

In the present work, training sessions of exponentially and step-wise increasing velocity have been simulated, as well as an interval training session. The corresponding *HR* response was also successfully simulated, via application of the mathematical model [24]. Special emphasis should be given to the model's excellent ability to simulate the heart rate response during exercises of complicated

intensity patterns, such as the interval training session. The study confirms that the applied mathematical model [24] is able not only to provide important information regarding an individual's cardiovascular condition but also to simulate and predict heart rate kinetics for any given exercise intensities.

Future implementations of the model could include fitting to heart rate data of time-varying exercise intensity, where the time series of either $v(t)$ or $\dot{v} = I(t)$ will be known.

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