η -Ricci Solitons on Quasi-Sasakian Manifolds

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Abstract. The object of the present paper is to study η -Ricci solitons in a 3-dimensional non-cosymplectic quasi-Sasakian manifolds. We study a particular type of second order parallel tensor in this manifold. Beside this we consider this manifold satisfying some curvature properties of Ricci tensor.

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1 Introduction

Ricci soliton is a natural generalization of Einstein metrics on a Riemannian manifold, being generalized fixed points of Hamilton's Ricci flow $\frac{\partial}{\partial t}g = -2S$ [14]. The evaluation equation defining the Ricci flow is a kind of nonlinear diffusion equation, an analogue of the heat equation of the metrics. Under the Ricci flow a metric can be improved to evolve into a more canonical one by smoothing out its irregularities, depending on the Ricci curvature of the manifold: it will expand in the directions of the negative Ricci curvature and

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shrink in the positive case.

A more general notion of Ricci soliton named η -Ricci soliton was introduced by J. T. Cho and M. Kimura [8], which was treated by C. Calin and M. Crasmareanu on Hopf hypersurfaces in complex space forms [7]. An η -Ricci soliton is a quadruple (g, V, λ, μ) , where V is a vector field on M , λ and μ are constants and g is a Riemannian (or pseudo-Riemannian) metric satisfying the equation

$$\mathcal{L}_V g + 2S + 2\lambda g + 2\mu\eta \otimes \eta = 0, \quad (1)$$

where S is the Ricci tensor associated to g and η is an one form. In this connection we mention the works of Blaga ([2], [3]), Majhi et al. [15] and Prakasha and Hadimani [17]. In particular, if $\mu = 0$, then the notion of η -Ricci soliton (g, V, λ, μ) reduces to the notion of Ricci soliton (g, V, λ) .

An important geometrical object in studying Ricci solitons is well-known to be a symmetric $(0, 2)$ -tensor field which is parallel with respect to the Levi-Civita connection, some of its geometric properties being described in ([1], [10]) etc. In [3], the author studied the second order parallel tensor field in the context of para-Kenmotsu manifolds admitting η -Ricci solitons. In [17] authors studied non-existence of certain kinds of para-Sasakian manifolds admitting η -Ricci soliton. Beside these Ricci solitons on a three dimensional quasi-Sasakian manifolds have been studied by U. C. De and A. K. Mondal in [11]. η -Ricci solitons on Sasakian 3-manifolds have been studied in [15].

Quasi-Sasakian manifold is a natural generalization of Sasakian manifold. Motivated by the above studies, in this paper we consider the η -Ricci solitons in a 3-dimensional quasi-Sasakian manifold.

The present paper is organized as follows:

After preliminaries in section 2, section 3 is devoted to study η -Ricci solitons in a 3-dimensional non-cosymplectic quasi-Sasakian manifold and we prove that the manifold is an η -Einstein β -Sasakian manifold with $\lambda + \mu = -2\beta^2$. Here we construct an example to verify the result. In the next part of this section we prove that if the symmetric second order tensor field $\alpha = \mathcal{L}_\xi g + 2S + 2\mu\eta \otimes \eta$ is parallel with respect to the Levi-Civita connection of the metric tensor, then the manifold admits an η -Ricci soliton. Section 4 deals with the study η -Ricci solitons satisfying some curvature properties of Ricci tensor in 3-dimensional non-cosymplectic quasi-Sasakian manifolds. In the first part of this section we prove that an η -Ricci soliton in non-cosymplectic quasi-

Sasakian manifolds with Ricci tensor of Codazzi-type becomes a Ricci soliton. We also prove that in an η -Ricci soliton in non-cosymplectic 3-dimensional quasi-Sasakian manifolds the Ricci tensor is cyclic parallel. Beside these, we prove a corollary in this section. Finally, in section 5, we prove that ϕ -Ricci symmetric η -Ricci soliton on 3-dimensional non-cosymplectic quasi-Sasakian manifolds becomes a Ricci soliton.

2 Preliminaries

A $(2n+1)$ -dimensional differentiable manifold M is said to admit an almost contact structure if it admits a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form η satisfying ([4], [5])

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0 \quad \text{and} \quad \eta \circ \phi = 0, \quad (2)$$

where $X \in TM$.

Let g be the compatible Riemannian metric with an almost contact structure (ϕ, ξ, η) , that is,

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad (3)$$

for $X, Y \in TM$. Then M becomes an almost contact metric manifold equipped with an almost contact metric structure (ϕ, ξ, η, g) . From (2) and (3) it can be easily seen that for $X, Y \in TM$

$$g(X, \phi Y) = -g(\phi X, Y), \quad g(X, \xi) = \eta(X). \quad (4)$$

An almost complex structure J can be defined on the product $M \times \mathbb{R}$ of M and the real line $\mathbb{R}$ by $J(X, \nu \frac{d}{dt}) = (\phi X - \nu \xi, \eta(X) \frac{d}{dt})$, where ν is a scalar field on $M \times \mathbb{R}$. If the structure J is complex analytic, the almost contact metric structure (ϕ, ξ, η, g) is said to be normal. A necessary and sufficient condition of an almost contact metric manifold to be normal is that the Nijenhuis tensor field N vanishes on the manifold ([4], [18]). A normal almost contact metric manifold is quasi-Sasakian if the fundamental 2-form Φ , defined by $\Phi(X, Y) = g(\phi X, Y)$ is closed ([6], [19]). Beside this, Olszak [16] proved that a 3-dimensional almost contact metric manifold M is a quasi-Sasakian manifold if and only if

$$\nabla_X \xi = -\beta \phi X, \quad (5)$$

where $X \in TM$ and β is some function on M , such that $\xi\beta = 0$, ∇ being the operator of the covariant differentiation with respect to the Levi-Civita connection of M . Hence a 3-dimensional quasi-Sasakian manifold is cosymplectic if and only if $\beta = 0$. For $\beta = \text{constant}$, the manifold reduces to a β -Sasakian manifold and $\beta = 1$ gives the Sasakian structure.

In a non-cosymplectic 3-dimensional quasi-Sasakian manifold the following relations hold ([12], [16]):

$$(\nabla_X \phi)Y = \beta[g(X, Y)\xi - \eta(Y)X], \quad (6)$$

$$R(X, Y)\xi = -(X\beta)\phi Y + (Y\beta)\phi X + \beta^2[\eta(Y)X - \eta(X)Y], \quad (7)$$

$$\begin{aligned} S(X, Y) = & \left(\frac{r}{2} - \beta^2\right)g(X, Y) + \left(3\beta^2 - \frac{r}{2}\right)\eta(X)\eta(Y) \\ & - \eta(X)d\beta(\phi Y) - \eta(Y)d\beta(\phi X), \end{aligned} \quad (8)$$

$$(\nabla_X \eta)(Y) = -\beta g(\phi X, Y), \quad (9)$$

where $X, Y \in TM$, R , S and r being the curvature tensor of type $(1, 3)$, Ricci tensor of type $(0, 2)$ and the scalar curvature of the manifold respectively.

A manifold M is said to be an η -Einstein manifold if the Ricci tensor S is of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y), \quad (10)$$

and Einstein if

$$S(X, Y) = \kappa g(X, Y) \quad (11)$$

where a and b are smooth functions on the manifold and κ is a constant.

A. Gray [13] introduced the notion of cyclic parallel Ricci tensor and Codazzi-type of Ricci tensor. A Riemannian manifold is said to have cyclic parallel Ricci tensor if the Ricci tensor of the manifold satisfies the following:

$$(\nabla_X S)(Y, Z) + (\nabla_Y S)(Z, X) + (\nabla_Z S)(X, Y) = 0. \quad (12)$$

Again a Riemannian manifold is said to have Codazzi-type of Ricci tensor if the Ricci tensor S is non-zero and satisfies the following:

$$(\nabla_X S)(Y, Z) = (\nabla_Y S)(X, Z). \quad (13)$$

3 η -Ricci soliton on 3-dimensional quasi-Sasakian manifolds

Let M be a 3-dimensional quasi-Sasakian manifold. We write (1) as

$$2S(X, Y) = -(\mathcal{L}_\xi g)(X, Y) - 2\lambda g(X, Y) - 2\mu\eta(X)\eta(Y), \quad (14)$$

for $X, Y \in TM$. In a quasi-Sasakian manifold, ξ is a Killing vector field [6]. Therefore $(\mathcal{L}_\xi g)(X, Y) = 0$. Hence (14) takes the form

$$S(X, Y) = -\lambda g(X, Y) - \mu\eta(X)\eta(Y). \quad (15)$$

In view of (15) we state the following:

Proposition 3.1. *A 3-dimensional quasi-Sasakian manifold admitting η -Ricci soliton is an η -Einstein manifold.*

Comparing the equations (8) and (15), we have

$$\begin{aligned} (\lambda + \frac{r}{2} - \beta^2)g(X, Y) + (\mu + 3\beta^2 - \frac{r}{2})\eta(X)\eta(Y) \\ = \eta(X)d\beta(\phi Y) + \eta(Y)d\beta(\phi X). \end{aligned} \quad (16)$$

Replacing X by ξ in (16) and using (2), we get

$$(\lambda + \mu + 2\beta^2)\eta(Y) = d\beta(\phi Y). \quad (17)$$

Replacing Y by ϕY in (17) and using (2) yields

$$d\beta(\phi^2 Y) = 0. \quad (18)$$

Since in a 3-dimensional quasi-Sasakian manifold $\phi^2 Y \neq 0$, in general, therefore (18) gives $d\beta = 0$ i.e., $\beta = \text{constant}$. Again using $d\beta = 0$ and in virtue of the fact $\eta(Y) \neq 0$, in general, (17) gives $\lambda + \mu = -2\beta^2$. Therefore we state the following:

Theorem 3.1. *A 3-dimensional non-cosymplectic quasi-Sasakian manifold admitting an η -Ricci soliton is an η -Einstein β -Sasakian manifold with $\lambda + \mu = -2\beta^2$.*

Example:

Here we construct an example [11] of a 3-dimensional non-cosymplectic quasi-Sasakian manifold which verifies the above theorem. We consider the 3-dimensional manifold $M = \{(x, y, z) \in \mathbb{R}^3, (x, y, z) \neq (0, 0, 0)\}$, where (x, y, z) are the standard coordinates of $\mathbb{R}^3$. We consider the linearly independent vector fields

$$e_1 = \frac{\partial}{\partial x} - y \frac{\partial}{\partial z}, e_2 = \frac{\partial}{\partial y}, e_3 = \frac{\partial}{\partial z}$$

at each point of M . Let g be a Riemannian metric given by

$$\begin{aligned} g(e_i, e_i) &= 1 \\ g(e_i, e_j) &= 0, \quad \text{for } i \neq j. \end{aligned}$$

Let η be the 1-form defined by $\eta(Z) = g(Z, e_3)$, for any $Z \in TM$.

Let ϕ be the $(1, 1)$ -tensor field defined by

$$\phi e_1 = -e_2, \quad \phi e_2 = e_1, \quad \phi e_3 = 0.$$

Then using the linearity of ϕ and g , we have

$$\begin{aligned} \eta(e_3) &= 1, \phi^2 Z = -Z + \eta(Z)e_3, \\ g(\phi Z, \phi W) &= g(Z, W) - \eta(Z)\eta(W), \end{aligned}$$

for $Z, W \in TM$. Thus for $e_3 = \xi$, (M, ϕ, ξ, η, g) becomes an almost contact metric manifold.

Let ∇ be the Levi-Civita connection with respect to the metric g . Then we have

$$[e_1, e_2] = e_3, \quad [e_1, e_3] = 0 \quad \text{and} \quad [e_2, e_3] = 0.$$

The Riemannian connection ∇ of the metric g is given by

$$\begin{aligned}
2g(\nabla_X Y, Z) = & Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) \\
& -g(X, [Y, Z]) + g(Y, [Z, X]) + g(Z, [X, Y]),
\end{aligned} \tag{19}$$

which is known as Koszul's formula.

Using (19) we calculate the following:

$$\begin{aligned}
\nabla_{e_1} e_3 &= -\frac{1}{2}e_2, \nabla_{e_1} e_2 = \frac{1}{2}e_3, \nabla_{e_1} e_1 = 0, \\
\nabla_{e_2} e_3 &= \frac{1}{2}e_1, \nabla_{e_2} e_2 = 0, \nabla_{e_2} e_1 = -\frac{1}{2}e_3, \\
\nabla_{e_3} e_3 &= 0, \nabla_{e_3} e_2 = \frac{1}{2}e_1, \nabla_{e_3} e_1 = -\frac{1}{2}e_2.
\end{aligned} \tag{20}$$

Therefore for $X = a_1 e_1 + a_2 e_2 + a_3 e_3$, a_1, a_2, a_3 being functions, we have

$$\nabla_X e_3 = \frac{1}{2}\phi X. \tag{21}$$

Therefore the manifold under consideration is a 3-dimensional non-cosymplectic quasi-Sasakian manifold with $\beta = -\frac{1}{2}$.

It is known that

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z. \tag{22}$$

Using the above results, (22) gives

$$\begin{aligned}
R(e_1, e_2)e_3 &= 0, R(e_2, e_3)e_3 = \frac{1}{4}e_2, R(e_1, e_3)e_3 = \frac{1}{4}e_1, \\
R(e_1, e_2)e_2 &= -\frac{3}{4}e_1, R(e_2, e_3)e_2 = \frac{1}{4}e_3, R(e_1, e_3)e_2 = 0, \\
R(e_1, e_2)e_1 &= \frac{3}{4}e_2, R(e_2, e_3)e_1 = 0, R(e_1, e_3)e_1 = \frac{1}{4}e_3.
\end{aligned}$$

From the above relations we obtain the following non-vanishing Ricci tensors:

$$S(e_1, e_1) = -\frac{1}{2}, S(e_2, e_2) = -\frac{1}{2}, S(e_3, e_3) = \frac{1}{2}.$$

Therefore for $Y = b_1 e_1 + b_2 e_2 + b_3 e_3$ we have

$$S(X, Y) = -\frac{1}{2}a_1b_1 - \frac{1}{2}a_2 + \frac{1}{2}a_3b_3. \quad (23)$$

Hence

$$\begin{aligned} & 2S(X, Y) + 2\lambda g(X, Y) + 2\mu\eta(X)\eta(Y) \\ &= (2\lambda - 1)a_1b_1 + (2\lambda - 1)a_2b_2 + (2\lambda + 2\mu + 1)a_3b_3. \end{aligned} \quad (24)$$

From (24) it is clear that for $\lambda = \frac{1}{2}$ and $\mu = -1$

$$2S(X, Y) + 2\lambda g(X, Y) + 2\mu\eta(X)\eta(Y) = 0$$

i.e.,

$$\mathcal{L}_{e_3}g + 2S + 2\lambda g + 2\mu\eta \otimes \eta = 0, \quad (25)$$

since in 3-dimensional quasi-Sasakian manifold $\mathcal{L}_{e_3}g = 0$.

Therefore $(M, \xi, g, \lambda, \mu)$ is an η -Ricci soliton for $\lambda = \frac{1}{2}$ and $\mu = -1$. Also we see that $S(X, Y) = -\lambda g(X, Y) - \mu\eta(X)\eta(Y)$ and $\lambda + \mu = -\frac{1}{2} = -2\beta^2$. Therefore the manifold is an η -Einstein β -Sasakian manifold with $\lambda + \mu = -2\beta^2$, which verifies the Theorem 3.1.

Next part of this section is devoted to the study of second order parallel tensor in a 3-dimensional quasi-Sasakian manifold. Let M be a 3-dimensional quasi-Sasakian manifold. Suppose that M is endowed with a second order covariant tensor field α satisfying $\nabla\alpha = 0$, where ∇ is the Levi-Civita connection on M .

Therefore we have

$$\alpha(R(X, Y)Z, W) + \alpha(Z, R(X, Y)W) = 0. \quad (26)$$

In particular for $W = Z = \xi$, (26) yields

$$\alpha(R(X, Y)\xi, \xi) = 0. \quad (27)$$

In a non-cosymplectic 3-dimensional quasi-Sasakian manifold admitting an η -Ricci soliton, from (7) we have

$$R(X, Y)\xi = \beta^2[\eta(Y)X - \eta(X)Y], \quad (28)$$

since $\beta = \text{constant}$.

Therefore using (28) in (27) yields

$$\beta^2[\eta(Y)\alpha(X, \xi) - \eta(X)\alpha(Y, \xi)] = 0. \quad (29)$$

Since the manifold under consideration is non-cosymplectic, hence for $Y = \xi$ (29) gives

$$\alpha(X, \xi) = g(X, \xi)\alpha(\xi, \xi). \quad (30)$$

Differentiating (30) covariantly along Y , we get

$$\alpha(\nabla_Y X, \xi) + \alpha(X, \nabla_Y \xi) = \alpha(\xi, \xi)[g(\nabla_Y X, \xi) + g(X, \nabla_Y \xi) + 2g(X, \xi)\alpha(\nabla_Y \xi, \xi)]. \quad (31)$$

Using (5) and (30) in (31), we obtain

$$\alpha(X, \phi Y) = [\alpha(\xi, \xi)g(X, \phi Y) + 2\eta(X)\alpha(\phi Y, \xi)], \quad (32)$$

since $\beta \neq 0$.

Now, from (30) we see that $\alpha(\phi X, \xi) = 0$. Therefore from (32), we have

$$\alpha(X, \phi Y) = \alpha(\xi, \xi)g(X, \phi Y). \quad (33)$$

Replacing Y by ϕY in (33) and using (2) and (30), we obtain

$$\alpha(X, Y) = \alpha(\xi, \xi)g(X, Y). \quad (34)$$

As α is a parallel tensor field, $\alpha(\xi, \xi)$ is constant. Therefore in view of (34) we state the following:

Proposition 3.2. *In a 3-dimensional non-cosymplectic quasi-Sasakian manifold, any parallel symmetric $(0, 2)$ -tensor field is a constant multiple of the metric tensor.*

Now, we consider the symmetric parallel tensor field $\alpha = \mathcal{L}_\xi g + 2S + 2\mu\eta \otimes \eta$. Therefore $\alpha(\xi, \xi) = 2S(\xi, \xi) + 2\mu\eta(\xi)\eta(\xi)$.

In view of (15) we have $S(\xi, \xi) = -\lambda - \mu$. Therefore $\alpha(\xi, \xi) = -2\lambda$ and we have $\mathcal{L}_\xi g + 2S + 2\mu = -2\lambda g$. This relation defines an η -Ricci soliton on M . Therefore we state the following:

Theorem 3.2. *Let M be a 3-dimensional non-cosymplectic quasi-Sasakian manifold. If the symmetric second order tensor field $\alpha = \mathcal{L}_\xi g + 2S + 2\mu\eta \otimes \eta$ is parallel with respect to the Levi-Civita connection of the metric tensor, then M admits an η -Ricci soliton.*

4 η -Ricci solitons in quasi-Sasakian manifolds with some curvature properties of Ricci tensor

First part of this section deals with the η -Ricci soliton in 3-dimensional non-cosymplectic quasi-Sasakian manifolds with Ricci tensor of Codazzi-type.

Differentiating (15) covariantly along Z and using (9), we have

$$(\nabla_Z S)(X, Y) = \mu\beta[g(\phi Z, X)\eta(Y) + g(\phi Z, Y)\eta(X)]. \quad (35)$$

Using (35) in (13), we get

$$\mu\beta[g(\phi Z, X)\eta(Y) + g(\phi Z, Y)\eta(X)] = \mu\beta[g(\phi Y, Z)\eta(X) + g(\phi Y, X)\eta(Z)]. \quad (36)$$

Replacing Z by ξ in (36) and using (2), we obtain

$$\mu\beta g(\phi Y, X) = 0. \quad (37)$$

Since the manifold under consideration is non-cosymplectic and $g(\phi Y, X) \neq 0$, in general, therefore (36) yields $\mu = 0$. Therefore the η -Ricci soliton becomes Ricci soliton. Hence we state the following:

Theorem 4.1. *An η -Ricci soliton in a non-cosymplectic 3-dimensional quasi-Sasakian manifold whose Ricci tensor is of Codazzi-type becomes a Ricci soliton.*

Again for $\mu = 0$, (15) becomes

$$S(X, Y) = -\lambda g(X, Y). \quad (38)$$

Therefore the manifold becomes an Einstein manifold. Again it is known that [20] a 3-dimensional Einstein manifold is a manifold of constant curvature. Thus we have:

Corollary 4.1. *An η -Ricci soliton in a non-cosymplectic 3-dimensional quasi-Sasakian manifold whose Ricci tensor is of Codazzi-type is a manifold of constant curvature.*

In the next part of this section we prove the following:

Theorem 4.2. *In a non-cosymplectic 3-dimensional quasi-Sasakian manifold admitting an η -Ricci soliton, the Ricci tensor is cyclic parallel.*

Proof. In view of (35) and (4), we have

$$\begin{aligned} & (\nabla_X S)(Y, Z) + (\nabla_Y S)(Z, X) + (\nabla_Z S)(X, Y) \\ = & \mu\beta[\eta(Z)\{g(\phi X, Y) + g(X, \phi Y)\} + \eta(Y)\{g(\phi X, Z) \\ & + g(\phi Z, X)\} + \eta(X)\{g(\phi Y, Z) + g(\phi Z, X)\}] \\ = & 0. \end{aligned}$$

Hence the theorem.

5 ϕ -Ricci symmetric η -Ricci soliton in 3-dimensional quasi-Sasakian manifold

In this section we study ϕ -Ricci symmetric η -Ricci soliton on 3-dimensional quasi-Sasakian manifolds. A quasi-Sasakian manifold is said to be ϕ -Ricci symmetric if

$$\phi^2(\nabla_X Q)Y = 0, \quad (39)$$

for all vector fields X, Y .

In view of (15), the Ricci operator Q can be written as

$$QY = -\lambda Y - \mu\eta(Y)\xi, \quad (40)$$

for $Y \in TM$.

Differentiating (40) covariantly along the vector field X and using (5) and (9), we obtain

$$(\nabla_X Q)Y = \mu\beta[g(\phi X, Y)\xi + \eta(Y)\phi X]. \quad (41)$$

Therefore in view of (2), we get

$$\phi^2(\nabla_X Q)Y = -\beta\mu\eta(Y)\phi X. \quad (42)$$

Here we consider the manifold to be non-cosymplectic. Therefore (42) yields $\mu = 0$. Hence we state the following:

Theorem 5.1. *A ϕ -Ricci symmetric η -Ricci soliton on 3-dimensional non-cosymplectic quasi-Sasakian manifolds becomes a Ricci soliton.*

Again $\mu = 0$ implies that the manifold is an Einstein one and hence the manifold is of constant curvature. Thus we have:

Corollary 5.1. *A ϕ -Ricci symmetric η -Ricci soliton on a 3-dimensional non-cosymplectic quasi-Sasakian manifold is a manifold of constant curvature.*

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