Generalized Jensen functional equation on restricted domain

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Abstract. We prove the Hyers-Ulam stability on restricted domains of generalized Jensen functional equation

$$\sum_{k=0}^{m-1} f(x + b_k y) = m f(x), \quad x, y \in E,$$

where $b_k = \exp(\frac{2i\pi k}{m})$ for $0 \leq k \leq m-1$. These results are applied to study of an asymptotic behavior of these functional equation.

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1 Introduction

The question concerning the stability of functional equations has been first raised by S. M. Ulam in 1940 [38]. S. M. Ulam started the stability by the following question

Given a group G , a metric group (G', d) , a number $\delta > 0$ and a mapping $f: G \longrightarrow G'$ which satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$ for all

$x, y \in G$, does there exist an homomorphism $h: G \longrightarrow G'$ and a constant $\gamma > 0$, depending only on G and G' such that $d(f(x), h(x)) \leq \gamma$ for all x in G ?

In 1941, Ulam's problem for the case of approximately additive mappings was solved by D. H. Hyers [17] on Banach spaces. In 1950 T. Aoki [2] provided a generalization of the Hyers theorem for additive mappings and in 1978 Th. M. Rassias [32] generalized the Hyers theorem for linear mappings by considering an unbounded Cauchy difference. For more information on the concept of the stability of functional equations see for example [7],[12],[16],[18],[19],[20],[23],[26].

The first stability theorem for the K-quadratic functional equation was proved in 1941 by Hyers-Ulam [17] and in 1978 by Th.M. Rassias [32] for $K = \{id\}$ then by F. Skof in 1983 [37] for $K = \{-id, id\}$ in Banach spaces. In 1984, P.W. Cholewa [11] extended Skof's result to an abelian group. In 1992, S. Czerwik [13], in the spirit of Hyers-Ulam- Rassias generalized the Skof's theorem.

Recently, the stability problem of the K-quadratic functional equation has been investigated by a number of mathematicians, the interested reader should refer to Ait Sibaha , Bouikhalene and Elqorachi [1], B.Bouikhalene, et al. [5], A.Charifi, et al. [10], Ab.Chahbi, et al. [9] and R. Łukasik [27], see also [6],[20],[22]-[25] and [32].

In 1998 Jung [22] investigated the Hyers-Ulam stability for additive and quadratic mappings on restricted domains. In 2002, J. M. Rassias [36] improved the bounds and thus the stability results obtained by Jung. Besides, he establish the Ulam stability for more general equations of two types on a restricted domain. Finally, he apply our recent results to the asymptotic behavior of functional equations of different types.

The stability problems of several functional equations on a restricted domain have been extensively investigated by a number of authors, for example ([8], [15], [30] and [31]).

Throughout this paper, let E, F be vector spaces over $K = \mathbb{Q}$ or $\mathbb{R}$ and $2 \leq m \in \mathbb{N}$. Our aim is to prove the Hyers-Ulam stability on restricted domains of generalized Jensen functional equation

$$\sum_{k=0}^{m-1} f(x + b_k y) = m f(x), \quad x, y \in E, \quad (1.1)$$

where $b_k = \exp(\frac{2i\pi k}{m})$ for $0 \leq k \leq m - 1$. These results are applied to study of an asymptotic behavior of these functional equation.

2 Notations and Preliminary results

In this section, we need to introduce some notions and notations.

A function $A : E \rightarrow F$ between two vector spaces E and F is said to be additive provided if $A(x + y) = A(x) + A(y)$ for all $x, y \in E$. In this case, it is easily seen that $A(rx) = rA(x)$ for all $x \in E$ and all $r \in \mathbb{Q}$.

Let $k \in \mathbb{N}$ and $A : E^k \rightarrow F$ be a function, then we say that A is k -additive provided if it is additive in each variable. In addition, we say that A is symmetric provided if

$$A(x_{\sigma(1)}, x_{\sigma(2)}, \dots, x_{\sigma(k)}) = A(x_1, x_2, \dots, x_k)$$

whenever $x_1, x_2, \dots, x_k \in E$ and σ is a permutation of $(1, 2, \dots, k)$.

Let $k \in \mathbb{N}$ and $A : E^k \rightarrow F$ be symmetric and k -additive and let $A_k(x) = A(x, x, \dots, x)$ for $x \in E$ and note that $A_k(rx) = r^k A_k(x)$ whenever $x \in E$ and $r \in \mathbb{Q}$.

In this way a function $A_k : E \rightarrow F$ which satisfies for all $\lambda \in \mathbb{Q}$ and $x \in E$, $A_k(\lambda x) = \lambda^k A_k(x)$ will be called a rational-homogeneous form of degree k , (assuming $A_k \neq 0$).

A function $p : E \rightarrow F$ is called a generalized polynomial (GP) function of degree $m \in \mathbb{N}$ if there exist $a_0 \in F$ and a rational-homogeneous form $A_k : E \rightarrow F$ (for $1 \leq k \leq m$) of degree k , such that

$$p(x) = a_0 + \sum_{k=1}^m A_k(x)$$

for all $x \in E$.

Let F^E denote the vector space, over a field K , consisting of all maps from E into F . For each $h \in E$, define the linear difference operator Δ_h on F^E by

$$\Delta_h f(x) = f(x + h) - f(x)$$

for all $f \in F^E$ and all $x \in E$. Notice that these difference operators commute ($\Delta_{h_1} \Delta_{h_2} = \Delta_{h_2} \Delta_{h_1}$ for all $h_1, h_2 \in E$) and if $h \in E$ and $n \in \mathbb{N}$, then Δ_h^n the n -th iterate of Δ_h satisfies

$$\Delta_h^n f(x) = \sum_{k=0}^n (-1)^{n-k} \binom{n}{k} f(x + kh)$$

for all $f \in F^E$ and all $x, h \in E$.

The following theorem is proved by Mazur and Orlicz [28],[29] then in greater generality by Djokovic [14].

Theorem 2.1. *Let $n \in \mathbb{N}$ and $f : E \rightarrow F$ be a function between two vector spaces E and F , then the following assertions are equivalent.*

1. $\Delta_h^n f(x) = 0$ for all $x, h \in E$.
2. $\Delta_{h_n} \dots \Delta_{h_1} f(x) = 0$ for all $x, h_1, \dots, h_n \in E$.
3. f is a generalized polynomial (GP) of degree at most $n - 1$.

3 Main results

Lemma 3.1. *Suppose that $2 \leq m \in \mathbb{N}$, $b_k \in K$ for $0 \leq k \leq m$, F is a real (or complex) Banach space, $\delta \geq 0$ and $f_k : E \rightarrow F$ for $0 \leq k \leq m$, be mappings fulfilling*

$$\left\| \sum_{k=0}^m f_k(x + b_k y) - f_k(x) \right\| \leq \delta \quad (3.1)$$

for all $x, y \in E$ and assume that $b_k - b_j \neq 0$ and $b_0 \neq 0$ whenever $0 \leq j < k \leq m$. Then,

$$\left\| \Delta_{h_{m+1}} \dots \Delta_{h_1} f_0(x) \right\| \leq 3^m \delta \quad (3.2)$$

for all $x, h_1, \dots, h_{m+1} \in E$.

Proof. For $0 \leq k \leq m$ let $d_{jk} = b_k - b_j$ so that $d_{jk} = 0$ if $j < k$ and $d_{kk} = 0$. For $0 \leq k \leq m$ and $x, y, h_1 \in E$,

$$(x - b_m h_1) + b_k(y + h_1) = x + b_k y + d_{km} h_1.$$

By using these quality and (3.1), we find that

$$\begin{aligned} & \left\| \sum_{k=0}^m (f_k(x + b_k y + d_{km} h_1) - f_k(x + b_k y)) - \sum_{k=0}^m (f_k(x + d_{km} h_1) - f_k(x)) \right\| \\ & \leq \left\| \sum_{k=0}^m f_k(x + b_k y + d_{km} h_1) - f_k(x - b_m h_1) \right\| + \left\| \sum_{k=0}^m f_k(x + d_{km} h_1) - f_k(x - b_m h_1) \right\| \\ & \quad + \left\| \sum_{k=0}^m f_k(x + b_k y) - f_k(x) \right\| \leq 3\delta \end{aligned}$$

and thus, since $d_{mm} = 0$,

$$\left\| \sum_{k=0}^{m-1} \Delta_{d_{km}h_1} f_k(x + b_k y) - f_k(x) \right\| \leq 3\delta \quad (3.3)$$

for all $x, y, h_1 \in E$. Repeating the argument that led from (3.1) to (3.3), we find that

$$\left\| \sum_{k=0}^{m-2} \Delta_{d_{k,m-1}h_2} \Delta_{d_{km}h_1} f_k(x + b_k y) - f_k(x) \right\| \leq 3\delta \quad (3.4)$$

for all $x, y, h_1, h_2 \in E$. Applying this reasoning $m - 2$ more times we are inclined to admit that

$$\|\Delta_{d_{01}h_m} \dots \Delta_{d_{0m}h_1} f_0(x + b_0 y) - f_k(x)\| \leq 3^m \delta \quad (3.5)$$

for all $x, y, h_1, \dots, h_m \in E$. Consequently,

$$\|\Delta_x \Delta_{d_{01}h_m} \dots \Delta_{d_{0m}h_1} f_0(b_0 y)\| \leq 3^m \delta. \quad (3.6)$$

Since $d_{0k} \neq 0$ for $1 \leq k \leq m$, and $b_0 \neq 0$ the last inequality simplify asserts that

$$\|\Delta_{h_{m+1}} \dots \Delta_{h_1} f_0(z)\| \leq 3^m \delta \quad (3.7)$$

for all $z, h_1, \dots, h_{m+1} \in E$.

□

Theorem 3.2. *Suppose that E is a vector space and F is real (or complex) Banach space, $2 \leq m \in \mathbb{N}$, $b_k = \exp(\frac{2ik\pi}{m})$ for $0 \leq k \leq m - 1$ and $\delta \geq 0$. If $f : E \rightarrow F$ satisfies*

$$\left\| \sum_{k=0}^{m-1} f(x + b_k y) - m f(x) \right\| \leq \delta \quad (3.8)$$

for all $x, y \in E$. Then there exists a unique generalized polynomial (GP) $q : E \rightarrow F$ at most $m - 1$ such that $q(0) = 0$ and

$$\|f(x) - f(0) - q(x)\| \leq 3^{m-1} \delta \text{ for all } x \in E.$$

Moreover

$$\sum_{k=1}^m q(x + b_k y) - m q(x) = 0 \text{ for all } x, y \in E.$$

Proof. If we put $f_k = f$ for each k , in (3.8), we get

$$\left\| \sum_{k=0}^{m-1} f_k(x + b_k y) - f_k(x) \right\| \leq \delta \quad (3.9)$$

Then, by Lemma 3.1

$$\|\Delta_{h_m} \dots \Delta_{h_1} f(z)\| \leq 3^{m-1} \delta$$

for all $z, h_1, \dots, h_m \in E$.

Hence, according to (Theorem II [20]), there exists a generalized polynomial (GP) $q : E \rightarrow F$, of degree at most $m - 1$ such that

$$\|f(x) - q(x)\| \leq 3^{m-1} \delta \quad (3.10)$$

and

$$q(x) = f(0) + \sum_{j=1}^{m-1} A_j(x) \quad (3.11)$$

where $A_j : E^j \rightarrow F$ are symmetric, j -additive mappings.

By (3.10) and (3.11), we obtain

$$\left\| \sum_{k=0}^{m-1} (q(x + b_k y)) - m q(x) \right\| \leq \left\| \sum_{k=0}^{m-1} (q(x + b_k y) - f(x + b_k y)) \right\| + \|m f(x) - m q(x)\|$$

so,

$$\left\| \sum_{k=0}^{m-1} (q(x + b_k y)) - m q(x) \right\| \leq 2m \cdot 3^{m-1} \delta \quad (3.12)$$

for all $x, y \in E$.

Now (3.12) says, in light of (3.11), that

$$\left\| \sum_{j=1}^{m-1} \left(\sum_{k=0}^{m-1} A_j(x + b_k y) - m A_j(x) \right) \right\| \leq 2m \cdot 3^{m-1} \delta \quad (3.13)$$

for all $x, y \in E$. In (3.13), replace x by rx and y by ry ($r \in \mathbb{Q}$) to conclude that, for all $x, y \in E$ and all $r \in \mathbb{Q}$,

$$\left\| \sum_{j=1}^{m-1} r^j \left(\sum_{k=0}^{m-1} A_j(x + b_k y) - m A_j(x) \right) \right\| \leq 2m \cdot 3^{m-1} \delta \quad (3.14)$$

By continuity, (3.14) holds for all real r , and all $x, y \in E$. Now suppose that $\phi : F \rightarrow \mathbb{R}$ is a continuous linear functional. Then

$$\left\| \sum_{j=1}^{m-1} r^j \phi \left(\sum_{k=0}^{m-1} (A_j(x + b_k y)) - mA_j(x) \right) \right\| \leq \|\phi\| 2m \cdot 3^{m-1} \delta \quad (3.15)$$

for all $x, y \in E$ and all $r \in \mathbb{R}$. Since a real polynomial function is bounded if and only if it is constant, from the last inequality we surmise that, for $1 \leq j \leq m-1$,

$$\phi \left(\sum_{k=0}^{m-1} A_j(x + b_k y) - mA_j(x) \right) = 0 \quad (3.16)$$

for all $x, y \in E$. Since this is so for every continuous linear functional $\phi : F \rightarrow \mathbb{R}$, by the Hahn- Banach theorem,

$$\sum_{k=0}^{m-1} A_j(x + b_k y) - mA_j(x) = 0 \text{ for } x, y \in E \text{ and } 1 \leq j \leq m-1. \quad (3.17)$$

Then, each q is a generalized polynomial (GP) of degree at most $m-1$ and from (3.17) we find that

$$\sum_{k=0}^{m-1} q(x + b_k y) - mq(x) = 0 \text{ for } x, y \in E \text{ and } 1 \leq j \leq m-1. \quad (3.18)$$

Finally, let p be another generalized polynomial (GP) solution of (1.1) of degree at most $m-1$ such that

$$\|f(x) - f(0) - p(x)\| \leq 3^{m-1} \delta, \quad x \in E,$$

and $p(0) = 0$. Then, from (3.18) and (3.10) we get that $q - p$ is generalized polynomial (GP) of degree at most $m-1$ such that $\|q(x) - p(x)\| \leq 2 \cdot 3^{m-1}$, $x \in E$. Thus, necessarily $q = p + q(0) - p(0)$. Since $q(0) = p(0) = 0$ we get that $q = p$ which ends the proof. $\square$

Theorem 3.3. *Let $d > 0$ and $\delta \geq 0$ be fixed. If a mapping $f : E \rightarrow F$ satisfies the generalized Jensen inequality (3.8) for all $x, y \in E$, with $\|x\| + \|y\| \geq d$, then there exists a unique generalized polynomial (GP) $q : E \rightarrow F$ such that $q(0) = 0$ and*

$$\|f(x) - f(0) - q(x)\| \leq 3^m \delta \text{ for all } x \in E.$$

Moreover

$$\sum_{k=0}^{m-1} q(x + b_k y) - mq(x) = 0 \text{ for all } x, y \in E.$$

Proof. Assume that $\|x\| + \|y\| < d$ and let $t \in E$ such that $\|t\| = d$ where $x = 0$ and

$$t = \left(1 + \frac{d}{\|x\|}\right)x,$$

where $x \neq 0$. We note that $\|t\| = \|x\| + d \geq d$, $\|x\| + \|t\| \geq d$ and

$$\|t\| + \|x + b_k y\| \geq d.$$

We have for all $b_j, 0 \leq j \leq m-1$ is a root of unity, if we put $b_j = e^{i\theta}$, we obtain

$$\|x + b_j t\|^2 = \left(2 \left(1 + \frac{d}{\|x\|}\right) (1 + \cos\theta) + \frac{d^2}{\|x\|^2}\right) \|x\|^2 \geq d^2$$

Thus, from (3.8) and the new functional identity we get

$$\begin{aligned} m \left[\sum_{k=0}^{m-1} f(x + b_k y) - m f(x) \right] &= \left[\sum_{k=0}^{m-1} (m f(x + b_k y) - \sum_{n=0}^{m-1} f(x + b_k y + b_n t)) \right] \\ &+ \left[\sum_{n=0}^{m-1} (-m f(x + b_n t) + \sum_{k=0}^{m-1} f(x + b_n t + b_k y)) \right] \\ &+ m \left[\sum_{n=0}^{m-1} f(x + b_n t) - m f(x) \right], \end{aligned}$$

then

$$m \left\| \sum_{k=0}^{m-1} f(x + b_k y) - m f(x) \right\| \leq 3m\delta.$$

Therefore,

$$\left\| \sum_{k=0}^{m-1} f(x + b_k y) - m f(x) \right\| \leq 3\delta.$$

for all $x, y \in E$. Now, Applying Theorem 3.2 we get the result. $\square$

Corollary 3.4. *A mapping $f : E \rightarrow F$ satisfies the functional equation (1.1) if and only if the asymptotic condition*

$$\left\| \sum_{k=0}^{m-1} f(x + b_k y) - m f(x) \right\| \rightarrow 0, \text{ as } \|x\| + \|y\| \rightarrow \infty. \quad (3.19)$$

holds true.

By the asymptotic condition (3.19), there exists a sequence δ_n monotonically decreasing to 0 such that

$$\left\| \sum_{k=0}^{m-1} f(x + b_k y) - mf(x) \right\| \leq \delta_n \quad (3.20)$$

for all $x, y \in E$ with $\|x\| + \|y\| \geq n$. Hence, it follows from Theorem 3.3 that there exists a unique generalized polynomial $q_n : E \rightarrow F$ such that $q_n(0) = 0$ and

$$\|f(x) - f(0) - q_n(x)\| \leq 3^m \delta_n \text{ for all } x \in E. \quad (3.21)$$

for all $x \in E$. Since δ_n is a monotonically decreasing sequence, the generalized polynomial q_n satisfies (3.21) for all $N \geq n$. The uniqueness of q_n implies for all $N \geq n$, $q_N = q_n$. Hence, by letting $n \rightarrow \infty$, in (3.21) we conclude that $f = f(0) + q_N$ is generalized polynomial of degree at most $m - 1$, and it satisfied the functional equation (1.1).

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