Unification on strongly θ -continuous functions

Bishwambhar Roy and T. Noiri

Abstract. We introduce a new type of functions called strongly μ - θ -continuous functions which unifies some weak forms of strongly θ -continuous functions and investigate their properties.

AMS Subject Classification (2000). 54C08; 54C10

Keywords. $\mu(\theta)$ -open set, $\mu(\theta)$ -closed, strong μ - θ -continuity

1 Introduction

In [16], T. Noiri introduced the concept of strongly θ -continuous functions. Some properties of strongly θ -continuous functions are also studied by Long and Herrington in [12]. Recently, some weaker forms of strongly θ -continuous functions are investigated in [10, 17, 18, 21]. The aim of this paper is to unify different weak forms of strongly θ -continuous functions by using the generalized topology introduced by Á. Császár [3].

Throughout this paper, we shall denote the closure and the interior of a set A by clA and $intA$, respectively. A subset A of a topological space (X, τ) is said to be preopen [13] (resp. semiopen [11], β -open [1], b -open [2], e -open [7]) if $A \subseteq int(cl(A))$ (resp. $A \subseteq cl(int(A))$, $A \subseteq cl(int(clA))$, $A \subseteq cl(int(A)) \cup int(cl(A))$, $A \subseteq int(cl_s(A)) \cup cl(int_s(A))$). The semi-closure (resp. pre-closure, β -closure, b -closure) of a set A is denoted by $sclA$ (resp. $pclA$, βclA or $spclA$, $bclA$) and is defined by the intersection of all

The author acknowledges the financial support from UGC, New Delhi.

semiclosed (resp. preclosed, β -closed, b -closed) sets containing A . A point $x \in X$ is called a θ -cluster [24] (resp. semi- θ -cluster [8], pre- θ -cluster [9], semi-pre θ -cluster [18], b - θ -cluster [21]) point of a set A if $clU \cap A \neq \emptyset$ (resp. $sclU \cap A \neq \emptyset$, $pclU \cap A \neq \emptyset$, $\beta clU \cap A \neq \emptyset$, $bclU \cap A \neq \emptyset$) for each open (resp. semiopen, preopen, β -open, b -open) set U containing x . The set of all θ -cluster (resp. semi- θ -cluster, pre- θ -cluster, semi-pre θ -cluster, b - θ -cluster) points of A is denoted by $cl_\theta(A)$ (resp. $scl_\theta(A)$, $pcl_\theta(A)$, $\beta-cl_\theta(A)$, $bcl_\theta(A)$). If $A = cl_\theta(A)$ (resp. $A = scl_\theta(A)$, $A = pcl_\theta(A)$, $A = \beta-cl_\theta(A)$, $A = bcl_\theta(A)$), then A is known as θ -closed (resp. semi- θ -closed, pre- θ -closed, semi-pre- θ -closed, b - θ -closed) and the complement of a θ -closed (resp. semi- θ -closed, pre- θ -closed, semi-pre- θ -closed, b - θ -closed) set is called a θ -open [24] (resp. semi- θ -open [8], pre- θ -open [9], semi-pre- θ -open [18], b - θ -open [21]) set. For any topological space (X, τ) , we shall denote the collection of all open (resp. preopen, semi-open, β -open, b -open) sets by τ (resp. $PO(X)$, $SO(X)$, $\beta O(X)$, $BO(X)$ or $\gamma O(X)$).

We now recall some notions defined in [3]. Let X be a non-empty set and $expX$ denote the power set of X . We call a class $\mu \subseteq expX$ a generalized topology [3] (briefly, GT) if $\emptyset \in \mu$ and union of elements of μ belongs to μ . A set X with a GT μ on it is called a generalized topological space (briefly, GTS) and is denoted by (X, μ) . Throughout the paper we shall use μ to mean GT on X while the collection of all μ -open sets containing x is denoted by $\mu(x)$.

For a GTS (X, μ) , the elements of μ are called μ -open sets and the complement of μ -open sets are called μ -closed sets. For $A \subseteq X$, we denote by $c_\mu(A)$ the intersection of all μ -closed sets containing A , i.e., the smallest μ -closed set containing A ; and by $i_\mu(A)$ the union of all μ -open sets contained in A , i.e., the largest μ -open set contained in A (see [3, 5]). Obviously in a topological space (X, τ) , if one takes τ as the GT, then c_μ becomes the usual closure operator. Similarly, c_μ becomes pcl (resp. scl , βcl , bcl) if μ stands for $PO(X)$ (resp. $SO(X)$, $\beta O(X)$, $BO(X)$ or $\gamma O(X)$). We note that each of these collections is a GT on X .

It is easy to observe that i_μ and c_μ are idempotent and monotonic, where $\gamma : expX \rightarrow expX$ is said to be idempotent if $A \subseteq X$ implies $\gamma(\gamma(A)) = \gamma(A)$ and monotonic if $A \subseteq B \subseteq X$ implies $\gamma(A) \subseteq \gamma(B)$. It is also well known from [5, 6] that if μ be a GT on X , $x \in X$, and $A \subseteq X$, then $x \in c_\mu(A)$ if and only if $M \cap A \neq \emptyset$ for every $M \in \mu(x)$ and $c_\mu(X \setminus A) = X \setminus i_\mu(A)$.

2 $\mu(\theta)$ -open sets

Definition 2.1. The $\mu(\theta)$ -closure [14] of a subset A in a GTS (X, μ) is denoted by $\mu(\theta)\text{-cl}A$ and is defined to be the set of all points x of X such that for each $U \in \mu(x)$, $c_\mu(U) \cap A \neq \emptyset$.

Remark 2.1. Let (X, τ) be a topological space. Then the definition of the θ -closure [24] (resp. pre- θ -closure [9], semi- θ -closure [8], b - θ -closure [21], semi-pre- θ -closure [18]) can be obtained by taking $\mu = \tau$ (resp. $PO(X)$, $SO(X)$, $BO(X)$, $\beta O(X)$).

Definition 2.2. A subset A in a GTS (X, μ) is said to be $\mu(\theta)$ -closed if $\mu(\theta)\text{-cl}A = A$. The complement of a $\mu(\theta)$ -closed set is called a $\mu(\theta)$ -open set.

Remark 2.2. (i) Let (X, τ) be a topological space. Then the definition of a θ -closed [24] (resp. pre- θ -closed [9], semi- θ -closed [10], b - θ -closed [21], semi-pre- θ -closed [18]) set can be obtained by taking $\mu = \tau$ (resp. $PO(X)$, $SO(X)$, $BO(X)$, $\beta O(X)$).

(ii) It follows from the definition of $\mu(\theta)$ -open sets that a set A is $\mu(\theta)$ -open if and only if for each $x \in A$, there exists $U \in \mu(x)$ such that $c_\mu(U) \subseteq A$. Also, clearly every $\mu(\theta)$ -open (resp. $\mu(\theta)$ -closed) set is μ -open (resp. μ -closed).

Example 2.1. Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a\}, \{a, c\}, \{b, c\}, X\}$. Then (X, μ) is a GTS. It is easy to see that $\{a, c\}$ is a μ -open set which is not $\mu(\theta)$ -open.

Theorem 2.1. For any μ -open set A in a GTS (X, μ) , $c_\mu(A) = \mu(\theta)\text{-cl}A$.

Proof. It is clear that for any subset A of X , $c_\mu(A) \subseteq \mu(\theta)\text{-cl}A$. Next, let A be any μ -open subset of X and $x \notin c_\mu(A)$. Then there exists $V \in \mu(x)$ such that $A \cap V = \emptyset$. Then $c_\mu(V) \subseteq c_\mu(X \setminus A) = X \setminus A$. So that $x \notin \mu(\theta)\text{-cl}A$. $\square$

Theorem 2.2. For any subset A of a GTS (X, μ) , $\mu(\theta)\text{-cl}A = \bigcap \{c_\mu(U) : A \subseteq U \in \mu\}$.

Proof. For any $U \in \mu$ with $A \subseteq U$, we have $\mu(\theta)\text{-cl}A \subseteq \mu(\theta)\text{-cl}U = c_\mu(U)$ (by Theorem 2.1). So L. H. S. $\subseteq$ R. H. S.

Next, let $x \notin \mu(\theta)\text{-cl}A$. Then for some $V \in \mu(x)$, $c_\mu(V) \cap A = \emptyset$. So that $A \subseteq X \setminus c_\mu(V) = U$ (say) $\in \mu$ such that $c_\mu(U) = c_\mu(X \setminus c_\mu(V)) \subseteq X \setminus V$ and hence $x \notin c_\mu(U)$. Hence R. H. S. $\subseteq$ L. H. S. $\square$

Definition 2.3. [22] A GTS (X, μ) is said to be μ -regular if for each μ -closed set F of X with $x \notin F$, there exist disjoint μ -open sets U and V such that $x \in U$ and $F \subseteq V$.

It is known from [22] that a GTS (X, μ) is μ -regular iff for each $x \in X$ and each $U \in \mu$ containing x there exists $V \in \mu$ containing x such that $x \in V \subseteq c_\mu(V) \subseteq U$.

Theorem 2.3. *In a μ -regular GTS (X, μ) , a subset A of X is $\mu(\theta)$ -open if and only if A is μ -open.*

Proof. In view of Remark 2.2(ii), it is sufficient to show that a μ -closed set A is $\mu(\theta)$ -closed. Infact, if $x \notin A$, then by μ -regularity of X , there exists a μ -open set V such that $x \in V \subseteq c_\mu(V) \subseteq X \setminus A$, i.e., $c_\mu(V) \cap A = \emptyset$. Hence $x \notin \mu(\theta)\text{-cl}A$. Therefore, A is $\mu(\theta)$ -closed. $\square$

3 Strong θ - μ -continuity

Definition 3.1. *A function $f : (X, \mu) \rightarrow (Y, \sigma)$, where (X, μ) is a GTS and (Y, σ) is a topological space, is said to be strongly μ - θ -continuous (briefly, st. μ - θ -continuous) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \mu(x)$ such that $f(c_\mu(U)) \subseteq V$.*

Remark 3.1. If (X, τ) is a topological space and $\mu = \tau$ (resp. $PO(X)$, $SO(X)$, $\beta O(X)$, $BO(X)$), then strong μ - θ -continuity reduces to a strongly θ -continuous [12, 16] (resp. strongly- θ -pre-continuous [?Nori01], strongly θ -semi-continuous [10], strongly θ - β -continuous [18], strongly θ - b -continuous [21]) function.

Remark 3.2. In a topological space (X, τ) , $\tau \subseteq SO(X) \subseteq BO(X) \subseteq \beta O(X)$ and $\tau \subseteq PO(X) \subseteq BO(X) \subseteq \beta O(X)$ and hence the following implications hold :

$$\begin{array}{ccc} \text{strong } \theta\text{-continuity} & \Rightarrow & \text{strong } \theta\text{-pre-continuity} \Rightarrow \text{strong } \theta\text{-}e\text{-continuity} \\ \Downarrow & & \Downarrow \end{array}$$

$$\text{strong } \theta\text{-semi-continuity} \Rightarrow \text{strong } \theta\text{-}b\text{-continuity} \Rightarrow \text{strong } \theta\text{-}\beta\text{-continuity}$$

None of the implications are reversible as shown by the following examples.

Example 3.1. (a) Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then (X, τ) is a topological space. It is easy to see that $SO(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. The identity function $f : (X, \tau) \rightarrow (X, \tau)$ is not strongly θ -pre-continuous but it is strongly θ -semi-continuous.

(b) Consider $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, X\}$. It is easy to check that $PO(X) = \{\emptyset, \{a\}, \{b\}, \{a, c\}, \{a, b\}, \{b, c\}, X\}$. Then the identity function $f : (X, \tau) \rightarrow (X, \tau)$ is strongly θ -pre-continuous but it is not strongly θ -semi-continuous.

(c) Consider $\mathbb{R}$ with the usual topology τ_u and $\sigma = \{\emptyset, \mathbb{R}, [0, 1] \cap Q\}$, where Q denotes the set of rational numbers. Then the identity function $f : (\mathbb{R}, \tau_u) \rightarrow (\mathbb{R}, \sigma)$ is strongly θ - β -continuous but it is not strongly θ - b -continuous.

(d) Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, X\}$ and $\sigma = \{\emptyset, X, \{b, c, d\}\}$. Then the identity function $f : (X, \tau) \rightarrow (X, \sigma)$ is strongly θ - b -continuous but it is not strongly θ - e -continuous.

(e) Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, X\}$ and $\sigma = \{\emptyset, X, \{d\}\}$. Then the identity function $f : (X, \tau) \rightarrow (X, \sigma)$ is strongly θ - e -continuous but it is not strongly θ - β -continuous so it is not strongly θ - b -continuous.

Theorem 3.1. For a function $f : (X, \mu) \rightarrow (Y, \sigma)$, where (X, μ) is a GTS and (Y, σ) is a topological space, the following properties are equivalent :

- (i) f is st. μ - θ -continuous;
- (ii) $f^{-1}(V)$ is $\mu(\theta)$ -open in X for each open set V of Y ;
- (iii) $f^{-1}(F)$ is $\mu(\theta)$ -closed in X for each closed set F of Y ;
- (iv) $f(\mu(\theta)\text{-cl}A) \subseteq \text{cl}(f(A))$ for all $A \subseteq X$;
- (v) $\mu(\theta)\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}B)$ for all $B \subseteq Y$.

Proof. (i) $\Rightarrow$ (ii) : Let V be any open set of Y . Suppose that $x \in f^{-1}(V)$. Then there exists $U \in \mu$ containing x such that $f(c_\mu(U)) \subseteq V$. Therefore we have $x \in U \subseteq c_\mu(U) \subseteq f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\mu(\theta)$ -open in X .

(ii) $\Rightarrow$ (iii) : This is trivial.

(iii) $\Rightarrow$ (iv) : Let A be any subset of X . Since $\text{cl}(f(A))$ is closed in Y , by (iii), $f^{-1}(\text{cl}(f(A)))$ is $\mu(\theta)$ -closed in X and we have $\mu(\theta)\text{-cl}A \subseteq \mu(\theta)\text{-cl}(f^{-1}(\text{cl}(f(A)))) \subseteq \mu(\theta)\text{-cl}(f^{-1}(\text{cl}(f(A)))) = f^{-1}(\text{cl}(f(A)))$. Thus we obtain $f(\mu(\theta)\text{-cl}(A)) \subseteq \text{cl}(f(A))$.

(iv) $\Rightarrow$ (v) : Let B be any subset of Y . Then by (iv), we obtain $f(\mu(\theta)\text{-cl}(f^{-1}(B))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}B$ and hence $\mu(\theta)\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}B)$.

(v) $\Rightarrow$ (i) : Let $x \in X$ and V be any open set in Y containing $f(x)$. Since $Y \setminus V$ is closed in Y , by (v) we have $\mu(\theta)\text{-cl}(f^{-1}(Y \setminus V)) \subseteq f^{-1}(Y \setminus V)$. Thus $f^{-1}(Y \setminus V)$ is $\mu(\theta)$ -closed in X and hence $f^{-1}(V)$ is a $\mu(\theta)$ -open set containing x . Thus there exists $U \in \mu(x)$ such that $c_\mu(U) \subseteq f^{-1}(V)$; hence $f(c_\mu(U)) \subseteq V$. This shows that f is st. μ - θ -continuous. $\square$

Definition 3.2. [19] A topological space (X, τ) with a GT μ is said to be μ_g -regular if for each closed set F and each point $x \notin F$, there exist disjoint μ -open sets U and V such that $x \in U$ and $F \subseteq V$.

It is known from [19] that a topological space (X, τ) with a GT μ is μ_g -regular if and only if for each $x \in X$ and each $U \in \tau$ with $x \in U$, there exists $V \in \mu$ such that $x \in V \subseteq c_\mu(V) \subseteq U$.

Theorem 3.2. *Let X be μ_g -regular, where μ is a GT on X , then a continuous function $f : (X, \tau) \rightarrow (Y, \sigma)$ is st. μ - θ -continuous.*

Proof. Suppose $f : (X, \tau) \rightarrow (Y, \sigma)$ is continuous and X is μ_g -regular. For any $x \in X$ and any open set V of Y containing $f(x)$, $f^{-1}(V)$ is an open set of X containing x . Since X is μ_g -regular, there exists $U \in \mu(x)$ such that $x \in U \subseteq c_\mu(U) \subseteq f^{-1}(V)$. Thus $f(c_\mu(U)) \subseteq V$. This shows that f is st. μ - θ -continuous. $\square$

Definition 3.3. *A GTS (X, μ) is said to be μ - T_2 [23] (resp. μ -Urysohn) if for each pair of distinct points x and y of X , there exist $U \in \mu(x)$ and $V \in \mu(y)$ such that $U \cap V = \emptyset$ (resp. $c_\mu(U) \cap c_\mu(V) = \emptyset$).*

Theorem 3.3. *If a function $f : (X, \mu) \rightarrow (Y, \sigma)$ is a st. μ - θ -continuous injection and Y is T_0 (resp. T_2) then X is μ - T_2 (resp. μ -Urysohn).*

Proof. (i) Suppose Y is T_0 . Let x and y be any two distinct points of X . Since f is injective, $f(x) \neq f(y)$ and there exists either an open set V containing $f(x)$ but not containing $f(y)$ or an open set W containing $f(y)$ but not containing $f(x)$. If the first case holds, then there exists $U \in \mu(x)$ such that $f(c_\mu(U)) \subseteq V$. Thus $f(y) \notin f(c_\mu(U))$. So $X \setminus c_\mu(U) \in \mu(y)$. If the second case holds, then we obtain a similar result. Therefore in any case X is μ - T_2 . (ii) Suppose Y is T_2 . Let x and y be any two distinct points of X . Then $f(x) \neq f(y)$. Since Y is T_2 , there exist disjoint open sets V and W containing $f(x)$ and $f(y)$, respectively. Since f is st. μ - θ -continuous, there exist μ -open sets G and H containing x and y , respectively, such that $f(c_\mu(G)) \subseteq V$ and $f(c_\mu(H)) \subseteq W$. It thus follows that $c_\mu(G) \cap c_\mu(H) = \emptyset$. This shows that X is μ -Urysohn. $\square$

Theorem 3.4. *If $f : (X, \mu) \rightarrow (Y, \sigma)$ is a st. μ - θ -continuous function and Y is Hausdorff, then the subset $E = \{(x, y) : f(x) = f(y)\}$ is $\mu(\theta)$ -closed in $X \times X$.*

Proof. Suppose that $(x, y) \notin E$. It follows that $f(x) \neq f(y)$. Since Y is Hausdorff, there exist disjoint open sets V and W in Y containing $f(x)$ and $f(y)$, respectively. Since f is st. μ - θ -continuous, there exist $U \in \mu(x)$ and $G \in \mu(y)$ such that $f(c_\mu(U)) \subseteq V$ and $f(c_\mu(G)) \subseteq W$. Let $D = U \times G$. It then follows that $(x, y) \in D \in \mu \times \mu$ and $c_\mu(U \times G) \cap E \subseteq [c_\mu(U) \times c_\mu(G)] \cap E = \emptyset$. Therefore E is $\mu(\theta)$ -closed in $X \times X$. $\square$

Definition 3.4. The graph $G(f)$ of a function $f : (X, \mu) \rightarrow (Y, \sigma)$ is said to be strongly μ -closed if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \mu(x)$ and an open set V in Y containing y such that $[c_\mu(U) \times V] \cap G(f) = \emptyset$.

Lemma 3.5. The graph $G(f)$ of a function $f : (X, \mu) \rightarrow (Y, \sigma)$ is strongly μ -closed in $X \times Y$ if and only if for each point $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \mu(x)$ and an open set V in Y containing y such that $f(c_\mu(U)) \cap V = \emptyset$.

Theorem 3.6. If $f : (X, \mu) \rightarrow (Y, \sigma)$ is st. μ - θ -continuous and Y is Hausdorff, then $G(f)$ is strongly μ -closed in $X \times Y$.

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. It follows that $f(x) \neq y$. Since Y is Hausdorff, there exist disjoint open sets V and W in Y containing $f(x)$ and y , respectively. Since f is st. μ - θ -continuous, there exists $U \in \mu(x)$ such that $f(c_\mu(U)) \subseteq V$. Therefore, $f(c_\mu(U)) \cap W = \emptyset$ and hence $G(f)$ is strongly μ -closed in $X \times Y$. $\square$

Acknowledgement

The authors are thankful to the referee for some comments to improve the paper.

References

- [1] M. E. Abd El-Monsef, El-Deeb S. N., and R. A. Mahmoud, β -Open sets and β -continuous mappings, *Bull. Fac. Sci. Assiut Univ.*, **12**, (1983), 77–90
- [2] D. Andrijević, On b -open sets, *Mat. Vesnik*, **48**, (1996), 59–64
- [3] Á. Császár, Generalized topology, generalized continuity, *Acta Math. Hungar.*, **96**, (2002), 351–357
- [4] Á. Császár, Separation axioms for generalized topologies, *Acta Math. Hungar.*, **104**, (2004), 63–69
- [5] Á. Császár, Generalized open sets in generalized topologies, *Acta Math. Hungar.*, **106**, (2005), 53–66
- [6] Á. Császár, δ - and θ -modifications of generalized topologies, *Acta Math. Hungar.*, **120**, (2008), 275–279
- [7] On e -open sets DP^* -sets and DPE^* -sets and decompositions of continuity, *Arabian J. Sci. Eng.*, **33** (2A), (2008), 269–282

- [8] **G. Di Maio and T. Noiri**, Weak and strong forms of irresolute functions, *Rend. Circ. Mat. Palermo*, **18(2)**, (1988), 255–273
- [9] **J. Dontchev, M. Ganster, and T. Noiri**, On p -closed spaces, *Internat. J. Math. Math. Sci.*, **24(3)**, (2000), 203–212
- [10] **S. Jafari and T. Noiri**, On strongly θ -semi-continuous functions, *Indian J. Pure Appl. Math.*, **29(11)**, (1998), 1195–1201
- [11] **N. Levine**, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, **70**, (1963), 36–41
- [12] **P. E. Long and Herrington L. L.**, Strongly θ -continuous functions, *J. Korean Math. Soc.*, **18**, (1981), 21–28
- [13] **A. S. Mashhour, M. E. Abd-El Monsef, and S. N. El-Deeb**, On precontinuous and weak precontinuous mappings, *Proc. Math. Phys. Soc. Egypt*, **53**, (1982), 47–53
- [14] **W. K. Min**, A note on $\theta(g, g')$ -continuity in generalized topological spaces, *Acta Math. Hungar.*, **125**, (2009), 387–397
- [15] **W. K. Min**, A note on δ and θ -modifications, *Acta Math. Hungar.*, **132**, (2011), 107–112
- [16] **T. Noiri**, On δ -continuous functions, *J. Korean Math. Soc.*, **16**, (1980), 161–166
- [17] **T. Noiri**, Strongly θ -precontinuous functions, *Acta Math. Hungar.*, **90**, (2001), 307–316
- [18] **T. Noiri and V. Popa**, Strongly θ - β -continuous functions, *J. Pure Math.*, **19**, (2002), 31–39
- [19] **T. Noiri and B. Roy**, Unification of generalized open sets on topological spaces, *Acta Math. Hungar.*, **130**, (2011), 349–357
- [20] **M. Özkoç and G. Aslim**, On strongly θ - e -continuous functions, *Bull. Korean Math. Soc.*, **47(5)**, (2010), 1025–1036
- [21] **J. H. Park**, Strongly θ - b -continuous functions, *Acta Math. Hungar.*, **110**, (2006), 347–359
- [22] **B. Roy**, On a type of generalized open sets, *Applied Gen. Topol.*, **12(2)**, (2011), 163–173
- [23] **G. Xun and G. Ying**, μ -separations in generalized topological spaces, *Appl. Math. Jour. Chinese Univ.*, **25(2)**, (2010), 243–252
- [24] **N. V. Veličko**, H -closed topological spaces, *Mat. Sb.*, **70**, (1966), 98–112

Bishwambhar Roy

Department of Mathematics
 Women's Christian College
 6, Greek Church Row
 Kolkata-700026
 India

E-mail: bishwambhar_roy@yahoo.co.in

T. Noiri

2949-1 Shiokita-cho, Hinagu
Yatsushiro-shi, Kumamoto-ken
869-5142 Japan

E-mail: t.noiri@nifty.com

Received: 28.10.2013

Accepted: 15.12.2013