

THICKNESS OPTIMISATION OF TEXTILES SUBJECTED TO HEAT AND MASS TRANSPORT DURING IRONING

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Abstract:

Let us next analyse the coupled problem during ironing of textiles, that is, the heat is transported with mass whereas the mass transport with heat is negligible. It is necessary to define both physical and mathematical models. Introducing two-phase system of mass sorption by fibres, the transport equations are introduced and accompanied by the set of boundary and initial conditions. Optimisation of material thickness during ironing is gradient oriented. The first-order sensitivity of an arbitrary objective functional is analysed and included in optimisation procedure. Numerical example is the thickness optimisation of different textile materials in ironing device.

Keywords:

heat and mass transport, thickness optimisation, ironing

Nomenclature

A	matrix of thermal conduction coefficients, $W/(m K)$,	$R_1; R_2$	sorption rates during first- and second-stage of sorption, $kg/(m^3 s)$, $kg/(m^3 s)$,
b	vector of design parameters, m ,	R_f	mean radius of fibres, m ,
$C; C_0$	constraint functional and imposed constant value of constraints,	R_v	specific gas constant for water vapour, that is, in relation to molar mass, $J/(kg K)$,
c	volumetric heat capacity, $J/(kg K)$,	$s_1; s_2$	adjustable parameters of sorption process,
D	matrix of mass transport coefficients, m^2/s ,	T	temperature, $K/^\circ C$,
$div_{\Gamma} \mathbf{v}^p$	tangent divergence of vector \mathbf{v}^p on external boundary Γ ,	$T_{\infty}(x,t)$	surrounding temperature, $K/^\circ C$,
$E_a; E_f$	pressures of saturated water vapour in air and within fibres, Pa , Pa ,	t	real time in primary and additional structures, s ,
$e_a; e_f$	water vapour pressure in air and within fibres, Pa , Pa ,	t_{eq}	time to reach quasi-equilibrium during sorption process, s ,
$f; f_w$	heat and mass source capacity; J/s ; kg/s ,	$\mathbf{v}^p(\mathbf{x}, \mathbf{b}, t)$	transformation velocity field associated with design parameter b_p ,
F, G	objective functional, objective functional of particular physical interpretation,	$\mathbf{v}_n^p = \mathbf{n} \cdot \mathbf{v}^p$	transformation velocity normal to external boundary Γ ,
$g_p = Dg/Db_p$	global (material) derivative of g in respect of design parameter b_p ,	w_a	water vapour concentration in air filling interfibre void space, kg/m^3 ,
$g^p = \partial g/\partial b_p$	partial (local) derivative of g in respect of design parameter b_p ,	w_f	water vapour concentration within fibres, kg/m^3 ,
H	mean curvature of external boundary Γ , $1/m$,	W_c	fractional water content on fibre surface,
$H_a; H_f$	relative humidity of air and fibres,	\mathbf{x}	coordinate vector of crucial geometrical points, m ,
$h; h_w$	heat and mass convection coefficient, $W/(m^2 K)$, m/s ,	β	approximation coefficient,
N	number of functionals during sensitivity analysis,	Γ	external boundary of structure,
n	unit vector normal to external boundary Γ , directed outwards to domain Ω bounded by this boundary,	ε	effective porosity of textile material,
P	number of design parameters,	σ	Stefan-Boltzmann constant, $W/(m^2 K^4)$,
ρ	proportion between sorption rates during first stage of sorption process,	η	absorption coefficient,
q	vector of heat flux density, W/m^2 ,	λ_w	cross transport coefficient, that is, heat of sorption of water vapour by fibres, J/kg ,
\mathbf{q}_w	vector of mass flux density, $kg/(m s)$,	Σ	discontinuity line between adjacent parts of piecewise smooth boundary Γ ,
$\mathbf{q}_n = \mathbf{n} \cdot \mathbf{q}$	heat flux density normal to external boundary, W/m^2 ,	ρ	density of fibres, kg/m^3 ,
$\mathbf{q}_{nw} = \mathbf{n} \cdot \mathbf{q}_w$	mass flux density normal to external boundary, $kg/(m s)$,	τ	transformed time in adjoint structure, s ,
		χ	Lagrange multiplier,
		Ψ, γ	domain and boundary integrands of objective functional,
		Ω	structural domain, m^2 ,
		∇	gradient operator.

Introduction

Heat and mass transport are the fundamental phenomena within textile structures subjected to ironing process. The reason is the difference between temperatures as well as water vapour concentrations within the structure or between the structure and surrounding. The special protective clothing of advanced technology can be also provided with the internal heat and mass sources. Heat and mass transfer can be different described in respect of textile material and different conditions. Let us analyse the coupled problem, that is, the heat is transported with mass whereas the mass transport with heat is negligible.

Physical model is described by the following state variables: the temperature T and the water vapour concentrations w_p , w_a . Mathematical model introduces the transport equations as well as the set of boundary and initial conditions which can be solved numerically. The variational approach of finite element method is applied. Thus, the applied functional of unambiguous interpretation should satisfy the assumed goal of optimisation procedure, that is, minimisation or maximisation of objective functional. Design variables are the material thicknesses during ironing process determined as the vector of shape parameters.

Some aspects of heat and mass transport are described by different Authors. The combined moisture sorption, condensation and liquid diffusion introduce Li, Zhu [15]; Li, Zhu, Yeung [16]. The discussed model is complicated and its solution approximated. It follows that the variational approach is difficult to apply. The fundamentals of coupled heat and water vapour transport are also described by Li [13]; Li, Luo [14]. The essential assumptions are introduced and the third equation formulated according to David, Nordon [3]. Technologically speaking, the description of coupled heat and water vapour transport according to [13,14] is sufficient. Combination of humidification time, temperature and moisture flux density causes that the void spaces within textiles are filled by water vapour. Mathematical model is easy enough and can be solved by means of typical methods.

The main goal of the presented paper is to determine both physical and mathematical models of the coupled heat and mass transport during ironing as well as optimise the material thicknesses. Design variables are now the thicknesses of material layers. Optimisation procedure is gradient oriented and needs the first-order sensitivities of an arbitrary objective functional. Material derivative concept as well as the direct and adjoint approaches to sensitivity analysis were discussed by Dems, Mróz [4]; Dems, Korycki, Rousselet [5]; Korycki [9-11]. Numerical optimisation of material thickness is always cheaper and more universal than the practical one and can give measurable benefits concerning computation time and total costs.

The gradient oriented optimisation of textile structures subjected to coupled heat and mass transfer is the extension of the previous works [9-11]. The novelty elements are now the compact modelling of coupled heat and mass transport, the formulation of optimisation problem and the practical

application during ironing. Additionally, the material parameters are described as moisture-dependent. The obtained results will be verified in the consecutive paper using the ironing device equipped with thermoelements as well as the "sweating guarded hotplate" which simulate the coupled emission of heat and moisture from the skin.

Physical and mathematical model of coupled heat and mass transport

Let us introduce the complex textile structure made of different materials subjected to coupled heat and mass transport during heating, cooling or conditioning. The repeatable structure consists of fibres and void spaces between fibres. Technologically speaking, the non-homogeneous fibres are manufactured by spinning from the yarn made of short natural or synthetic filaments. The internal structure of fibres is also porous and the void spaces between filaments are filled by water vapour during ironing. Effective porosity depends on the type of textile, that is, woven fabric, knitted fabric, nonwovens. The complex structure is characterised by the "external" porosity caused by the void interfibre spaces and the "internal" porosity because of the inhomogeneity of fibres. Thus, two-stage homogenisation procedure is necessary to create the homogenised textile material of the specified heat and mass transfer coefficients. Heat and mass are transferred in the homogenised structure of the effective material porosity ϵ . Let us apply the most popular homogenisation method, that is, the "rule of mixture" [6].

Physical model is characterised by the following assumptions.

- Heat is transported by conduction through the material and convection on the external surfaces of fibres to interfibre spaces. Mass is transported within fibres and interfibre spaces by diffusion [13].
- Volume changes of fibres caused by mass diffusion can be neglected even if material is sterile, dry, subjected to transient heat and mass transport, cf. [13,14]. The typical ironing process is relatively short whereas the considerable changes of volume are slower.
- Orientation of fibres can influence mass transport although its diameters are small and water vapour is transported much more rapidly through interfibre spaces than in fibres. The reason is different structure and layout of fibres within woven fabrics, knitted fabrics and nonwovens. Textile structure and materials applied should be always deeply analysed.
- The instantaneous thermodynamical equilibrium exists between the textile material and fluid in free spaces irrespective of time characteristics concerning coupled problems. Additionally, textiles have small diameters and large surface/volume ratio.

Mathematical model of coupled transfer contains: (i) the equations of heat and mass balances; (ii) the constitutive equations characterising the material in respect of heat and mass transfer; (iii) the state equations, that is, relations between the state variables; (iv) the physical and chemical correlations which define each phase of material.

Let us first introduce the heat balance. Heat is supplied by: (i) the coupled heat and water vapour transport which is defined mathematically by means of the effective porosity ε and the heat sorption of water vapour by fibres λ_w ; (ii) internal heat source of the capacity f . Heat is lost by: (i) transport to surrounding which is described by the vector of heat flux density \mathbf{q} on the external surface Γ ; (ii) accumulation within material of the heat capacity c .

Next we have to define the mass balance. Mass can be emitted by internal source of the capacity f_w . Mass is lost by: (i) transport to surrounding which is described by the vector of mass flux density \mathbf{q}_w on the external surface Γ ; (ii) accumulation within fibres; (iii) accumulation in the void spaces between fibres.

Introducing both balance equations and Ostrogradski-Gauss theorem, we determine the heat and mass transport equations after some transformations [9].

$$\begin{cases} (1-\varepsilon)\frac{dw_f}{dt} + \varepsilon\frac{dw_a}{dt} = -\text{div}\mathbf{q}_w + f_w; & \mathbf{q}_w = \mathbf{D} \cdot \nabla w_f; \\ \rho c \frac{dT}{dt} - \lambda_w(1-\varepsilon)\frac{dw_f}{dt} = -\text{div}\mathbf{q} + f; & \mathbf{q} = \mathbf{A} \cdot \nabla T; \end{cases} \quad (1)$$

State variables are the temperature T and the water vapour concentrations w_a ; w_f . Therefore, third equation is necessary to solve the problem and describe the sorption/desorption of water vapour on the contact surface of fibres and interfibre spaces. Let us introduce the correlation according to David, Nordon [3]; Li [13]; Li, Luo [14]. The sorption is described as two-stage process. The first stage is described by the Fick's law with constant diffusion coefficient. The second stage is defined by empirical correlation because the fibres are subjected to viscoelastic relaxation of more random distribution. Li, Holcombe introduce this problem for strongly hygroscopic wool fibres, according to Watt. The ironing process is no longer than 30s, that is, determined by the Fick's diffusion during the first stage of sorption. Fractional water content on fibre surface is under the prescribed boundary value. Material of lower hygroscopicity (e.g. polyester) is also described by the Fick's diffusion.

$$\frac{dw_f}{dt} = (1-p)R_1 + pR_2 \quad \begin{cases} p = 0 & \text{for } W_C < 0,185 \text{ and } t < t_{eq}; \\ p = 0,5 & \text{for } W_C \geq 0,185 \text{ and } t < t_{eq}; \\ p = 1 & \text{for } t > t_{eq}. \end{cases} \quad (2)$$

Equilibrium time t_{eq} is determined experimentally for different materials, cf. [7,14]. The model is characterised by the instantaneous thermodynamic equilibrium between fibres and the void spaces, cf. Li [13] which allows to describe the state variable w_f in the form.

$$w_f(\mathbf{x}, R_f, t) = \rho W_C(H_f) \quad (3)$$

The fractional water content W_C is determined using the sorption isotherms of fibres. Assuming cylindrical fibres, sorption during the first phase R_1 is described by differential equation according to Crank [2] in respect of Eq.(2) and proportion factor $p=0$.

$$\frac{dw_f}{dt} = R_1(\mathbf{x}, t) = \frac{1}{r} \frac{d(r\mathbf{D} dw_f)}{dr^2}. \quad (4)$$

Let us analyse the physical phenomena between fibres and void interfibre spaces, that is, in the nonhomogeneous two-phase environment. The sorption/desorption of water vapour on the common surface of fibres/void spaces can be approximated by the coefficient β , cf. Crank [2], Li [13].

$$w_f = \rho\beta w_a. \quad (5)$$

Heat and mass transfer equations during the first phase of water sorption within fibres have consequently the form in respect of Eq.(1)-Eq.(5).

$$\begin{cases} \left(1-\varepsilon + \frac{\varepsilon}{\rho\beta}\right)\frac{dw_f}{dt} = -\text{div}\mathbf{q}_w + f_w; & \mathbf{q}_w = \mathbf{D} \cdot \nabla w_f; \\ \rho c \frac{dT}{dt} + \lambda_w(1-\varepsilon)\frac{dw_f}{dt} = -\text{div}\mathbf{q} + f; & \mathbf{q} = \mathbf{A} \cdot \nabla T. \end{cases} \quad (6)$$

Sorption rate during the second phase R_2 can be determined experimentally according to Li [13] in the form.

$$R_2(\mathbf{x}, t) = s_1 \text{sign}(H_a - H_f) \exp\left(s_2 |H_a - H_f|^{-1}\right) \quad (7)$$

The equation is difficult to solve and has unique solution for the specific humidity functions H_a , H_f . Let us analyse the absolute and relative humidity and the general law of perfect gases [8] on the contact surfaces between fibres and interfibre spaces. Assuming the thermodynamic equilibrium, the same parameters exists on the common surfaces of fibres and interfibre spaces $E_a=E_f$; $T_a=T_f$.

$$\frac{H_f}{H_a} = \frac{R_v^{-1} \frac{e_f}{T_f}}{R_v^{-1} \frac{e_a}{T_a}} = \frac{e_f}{e_a} = \eta; \quad \frac{w_f}{w_a} = \frac{\frac{e_f}{E_f} \cdot 100\%}{\frac{e_a}{E_a} \cdot 100\%} = \frac{e_f}{e_a} = \eta. \tag{8}$$

Physically speaking, the obtained factor of proportionality η is the absorption coefficient of water vapour on contact surfaces of fibres and interfibre spaces. Therefore, the state variables are connected according to Eq.(8). Finally, the transport equations during the second phase of sorption have the form in respect of Eq.(1), Eq.(7), Eq.(8), [9].

$$\begin{cases} \left(1 - \varepsilon + \frac{\varepsilon}{\eta}\right) \frac{dw_f}{dt} = -\text{div} \mathbf{q}_w + f_w; & \mathbf{q}_w = \mathbf{D} \cdot \nabla w_f; \\ \rho c \frac{dT}{dt} + \lambda_w (1 - \varepsilon) \frac{dw_f}{dt} = -\text{div} \mathbf{q} + f; & \mathbf{q} = \mathbf{A} \cdot \nabla T. \end{cases} \tag{9}$$

The transport equations have the similar form during both phases of sorption, cf. Eq.(6), Eq.(9). Let us next introduce the set of boundary and initial conditions [9]. The first-kind conditions specify the state variables T and w_f on the external boundary portions Γ_T, Γ_f . The second-kind conditions describe the heat and mass flux densities normal to the part of external boundaries Γ_q, Γ_2 . The third-kind conditions determine the convective heat and mass flux densities normal to part of the external boundaries Γ_c, Γ_3 . The fourth-kind condition define the same state variables on the common internal boundaries Γ_4 . The radiation condition is formulated using the fourth power of temperature or as integral according to Bialecki [1]. The initial conditions determine the state variables at the beginning of the coupled transport. All the conditions have the form.

$$\begin{aligned} T(\mathbf{x}, t) &= T^0(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_T; & w_f(\mathbf{x}, t) &= w_f^0(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_f; \\ q_n(\mathbf{x}, t) &= \mathbf{n} \cdot \mathbf{q} = q_n^0(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_q; & q_{nw}(\mathbf{x}, t) &= \mathbf{n} \cdot \mathbf{q} = q_{nw}^0(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_2; \\ q_n(\mathbf{x}, t) &= h[T(\mathbf{x}, t) - T_\infty(\mathbf{x}, t)] \quad \mathbf{x} \in \Gamma_c; & q_{nw}(\mathbf{x}, t) &= h_w[w_f(\mathbf{x}, t) - w_{f\infty}(\mathbf{x}, t)] \quad \mathbf{x} \in \Gamma_3; \\ T^{(i)}(\mathbf{x}, t) &= T^{(i+1)}(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_4; & w_f^{(i)}(\mathbf{x}, t) &= w_f^{(i+1)}(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_4; & q_n^r(\mathbf{x}, t) &= \sigma T^4 \quad \mathbf{x} \in \Gamma_r; \\ T(\mathbf{x}, 0) &= T_0(\mathbf{x}, 0) \quad \mathbf{x} \in (\Omega \cup \Gamma); & w_f(\mathbf{x}, 0) &= w_{f0}(\mathbf{x}, 0) \quad \mathbf{x} \in (\Omega \cup \Gamma). \end{aligned} \tag{10}$$

Application of sensitivity approach

Let us apply the sensitivity approach to determine the optimal thicknesses. The first-order sensitivity is defined as material derivative of an arbitrary functional F in respect of design parameter $F_p = DF/Db_p$. The objective functional determine now the optimum in view of moisture transport. We can denote according to [10].

$$F = \int_0^{t_f} \left[\int_{\Omega(\mathbf{b})} \Psi(f_w) d\Omega + \int_{\Gamma(\mathbf{b})} \gamma(w_f, q_w, w_{f\infty}) d\Gamma \right] dt; \tag{11}$$

where Ψ, γ are continuous and differentiable functions of defined arguments. The sensitivity is analysed by means of both direct and adjoint approaches [4,9-11].

The direct approach introduces the set of additional problems concerning the heat and mass transfer associated with each design parameter, that is, each material thickness. Introducing P design parameters, we have to solve $(P+1)$ problems of heat and mass transfer: the primary and P additional problems. This approach is convenient to optimise the thicknesses described by the low number of design variables. Primary and additional problems have the same shape as well as heat and mass transport conditions but the different values of some fields within structural domain and on its external boundary. State variables are the temperature T_p and the water vapour concentrations w_f^p, w_a^p within each additional structure. The correlations are formulated by differentiation of the appropriate equations for primary problem [10]. The transport equations have the unified form in respect of Eq.(1), Eq.(6).

$$\begin{cases} \left(1 - \varepsilon + \frac{\varepsilon}{Z}\right) \frac{dw_f^p}{dt} = -\text{div} \mathbf{q}_w^p + f_w^p; & \mathbf{q}_w^p = \mathbf{D} \cdot \nabla w_f^p; \\ c \frac{dT^p}{dt} + \lambda_w \frac{dw_f^p}{dt} = -\text{div} \mathbf{q}^p + f^p; & \mathbf{q}^p = \mathbf{A} \cdot \nabla T^p. \end{cases} \tag{12}$$

$Z = \beta\rho$ for $p = 0$; $Z = \beta\eta$ for $p > 0$.

The boundary and initial conditions are now formulated according to Eq.(10) and [9]. It is evident that during ironing the state variables as well as the heat and mass flux densities are design parameter independent, that is, $(T^0)_p = (w_f^0)_p = (q_n^0)_p = (q_w^0)_p = 0$. All plane surfaces have the curvature $H \rightarrow 0$, which simplify sensitivity correlations.

$$\begin{aligned}
 T^p(\mathbf{x}, t) &= -\nabla T^0 \cdot \mathbf{v}^p \quad \mathbf{x} \in \Gamma_T; & w_f^p(\mathbf{x}, t) &= -\nabla w_f^0 \cdot \mathbf{v}^p \quad \mathbf{x} \in \Gamma_1; \\
 q_n^p(\mathbf{x}, t) &= \mathbf{q}_\Gamma^0 \cdot \nabla_\Gamma v_n^p - \nabla_\Gamma q_n^0 \cdot \mathbf{v}_\Gamma^p - q_{n,n}^0 v_n^p \quad \mathbf{x} \in \Gamma_q; \\
 q_{nw}^p(\mathbf{x}, t) &= \mathbf{q}_{w\Gamma}^0 \cdot \nabla_\Gamma v_n^p - \nabla_\Gamma q_{nw}^0 \cdot \mathbf{v}_\Gamma^p - q_{nw,n}^0 v_n^p \quad \mathbf{x} \in \Gamma_2; \\
 q_n^p(\mathbf{x}, t) &= h(T^p - T_\infty^p) + \mathbf{q}_\Gamma \cdot \nabla_\Gamma v_n^p \quad \mathbf{x} \in \Gamma_c \\
 q_{nw}^p(\mathbf{x}, t) &= h_w(w_f^p - w_{f\infty}^p) + q_{w\Gamma} \cdot \nabla_\Gamma v_n^p \quad \mathbf{x} \in \Gamma_3; & q_n^{rp} &= 4\sigma T^3 T^p \quad \mathbf{x} \in \Gamma_r; \\
 T^{p(i)}(\mathbf{x}, t) &= T^{p(i)}(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_4; & w_f^{p(i)}(\mathbf{x}, t) &= w_f^{p(i)}(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_4; \\
 T_0^p(\mathbf{x}, 0) &= T_{0p} - \nabla T_0 \cdot \mathbf{v}^p \quad \mathbf{x} \in (\Omega \cup \Gamma); & w_{f0}^p(\mathbf{x}, 0) &= w_{f0p} - \nabla w_{f0} \cdot \mathbf{v}^p \quad \mathbf{x} \in (\Omega \cup \Gamma).
 \end{aligned}
 \tag{13}$$

The first-order sensitivity expression can be determined in final form, cf. [10].

$$\begin{aligned}
 F_p &= \int_0^{t_f} \left\{ \int_\Omega \Psi_{,f_w} f_w^p d\Omega + \int_{\Gamma_1} [-\gamma_{w_f} (\nabla_\Gamma w_f^0 \cdot \mathbf{v}_\Gamma^p + w_{f,n}^0 v_n^p) + \gamma_{q_{nw}} (q_{nw}^p - \mathbf{q}_{w\Gamma} \cdot \nabla_\Gamma v_n^p)] d\Gamma_1 + \right. \\
 &\quad \left. \int_{\Gamma_2} [\gamma_{w_f} w_f^p - \gamma_{q_{nw}} (\nabla_\Gamma q_{nw}^0 \cdot \mathbf{v}_\Gamma^p + q_{nw,n}^0 v_n^p)] d\Gamma_2 + \int_{\Gamma_3} [\gamma_{w_f} w_f^p + \gamma_{q_{nw}} h_w (w_f^p - w_{f\infty}^p)] d\Gamma_3 + \right. \\
 &\quad \left. \int_\Gamma (\Psi + \gamma_n) v_n^p d\Gamma + \int_\Gamma \gamma_{w_{f\infty}} w_{f\infty}^p d\Gamma + \int_\Sigma \gamma \mathbf{v}^p \cdot \mathbf{v} \right\} dt, \quad p = 1, 2, \dots, P.
 \end{aligned}
 \tag{14}$$

The adjoint approach determines the set of adjoint problems associated with each objective functional. Applying N objective functionals, $(N+1)$ coupled problems are solved, that is, the primary and N adjoint heat and mass transfer problems. The approach is convenient for the low number of objective functionals. Primary and adjoint problem have the same shape as well as the heat and mass transfer conditions but the fields within the structure and on the surrounding boundary are different. State variables are the temperature T^a and the water vapour concentrations w_f^a , w_a^a within each adjoint structure. The transport equations, the boundary and initial conditions have the similar form as the corresponding correlations for primary structure, cf. Eq.(1), Eq.(6), Eq.(10), [9].

$$\begin{cases}
 \left(1 - \varepsilon + \frac{\varepsilon}{Z}\right) \frac{dw_f^a}{d\tau} = -\text{div} \mathbf{q}_w^a + f_w^a; & \mathbf{q}_w^a = \mathbf{D} \cdot \nabla w_f^a; \\
 c \frac{dT^a}{d\tau} + \lambda_w \frac{dw_f^a}{d\tau} = -\text{div} \mathbf{q}^a + f^a; & \mathbf{q}^a = \mathbf{A} \cdot \nabla T^a.
 \end{cases}
 \tag{15}$$

$$Z = \beta\rho \text{ for } p = 0; \quad Z = \beta\eta \text{ for } p > 0.$$

The detailed analysis of transport equations allows to formulate the set of conditions [9].

$$\begin{aligned}
 w_f^a(\mathbf{x}, \tau = 0) &= 0 \quad \mathbf{x} \in (\Omega \cup \Gamma); & f_w^a(\mathbf{x}, \tau) &= 0 \quad \mathbf{x} \in \Omega & w_f^{0a}(\mathbf{x}, \tau) &= \gamma_{q_{nw}}(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_1; \\
 q_{nw}^{0a}(\mathbf{x}, \tau) &= -\gamma_{w_f}(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_2; & w_{f\infty}^a(\mathbf{x}, \tau) &= \frac{1}{h_w} \gamma_{w_f}(\mathbf{x}, t) + \gamma_{q_{nw}}(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_3; \\
 T^a(\mathbf{x}, \tau = 0) &= 0 \quad \mathbf{x} \in (\Omega \cup \Gamma); & f^a(\mathbf{x}, \tau) &= 0 \quad \mathbf{x} \in \Omega & T^{0a}(\mathbf{x}, \tau) &= 0 \quad \mathbf{x} \in \Gamma_T; \\
 q_n^{0a}(\mathbf{x}, \tau) &= 0 \quad \mathbf{x} \in \Gamma_q; & q_n^{ar}(\mathbf{x}, \tau) &= 0 \quad \mathbf{x} \in \Gamma_r; & T_\infty^a(\mathbf{x}, \tau) &= 0 \quad \mathbf{x} \in \Gamma_c;
 \end{aligned}
 \tag{16}$$

Introducing the adjoint approach, the time τ of adjoint problem is transformed in relation to the current time t of primary and additional problems. Thus, the final time t_f is the starting time τ in respect of the current time t , that is, $\tau = t_f - t$.

The first-order sensitivity can be finally expressed by following correlation [10].

$$\begin{aligned}
 F_p &= \int_0^{t_f} \left\{ \int_\Omega [\nabla w_f^a \cdot \mathbf{q}_w^p + \Psi_{,f_w} f_w^p] d\Omega - \int_{\Gamma_1} [\gamma_{w_f} + q_{nw}^a] (\nabla_\Gamma w_f^0 \cdot \mathbf{v}_\Gamma^p + w_{f,n}^0 v_n^p) + \gamma_{q_{nw}} \mathbf{q}_{w\Gamma} \cdot \nabla_\Gamma v_n^p d\Gamma_1 + \right. \\
 &\quad \left. - \int_{\Gamma_2} [\gamma_{q_{nw}} - w_f^a] (\nabla_\Gamma q_{nw}^0 \cdot \mathbf{v}_\Gamma^p + q_{nw,n}^0 v_n^p) + w_f^a \mathbf{q}_{w\Gamma}^0 \cdot \nabla_\Gamma v_n^p d\Gamma_2 + \right. \\
 &\quad \left. \int_{\Gamma_3} [(w_f^a - \gamma_{q_{nw}}) h_w w_{f\infty}^p - w_f^a \mathbf{q}_{w\Gamma} \cdot \nabla_\Gamma v_n^p] d\Gamma_3 + \int_\Gamma (\Psi + \gamma_n) v_n^p d\Gamma + \int_\Gamma \gamma_{w_{f\infty}} w_{f\infty}^p d\Gamma + \int_\Sigma \gamma \mathbf{v}^p \cdot \mathbf{v} \right\} dt.
 \end{aligned}
 \tag{17}$$

Problem of shape optimisation

Thickness optimisation is a search of such vector of design parameters which minimise the objective functional G with imposed equality constraint $C=C_0$. Let us apply the objective functional determining the water vapour transport during ironing. The constraints concerning heat transport can be continuous or discrete of the time-dependent functions known in each point of bounding surface. Discrete constraint is now the specified temperature measured by means of thermoelements in the particular points on the lower plate, that is, the first-kind condition is introduced. Applying the Lagrangian functional and its stationarity conditions, we determine the optimality conditions in the form [9,10].

$$\begin{cases} \frac{DG}{Db_p} = -\chi \frac{DC}{Db_p}; & \frac{DC}{Db_p} = \frac{DT}{Db_p} = T^{0p} + \nabla T^0 \cdot \mathbf{v}^p; \\ T - T^0 = 0. \end{cases} \tag{18}$$

The sensitivity vector of objective functional DG/Db_p is defined using both direct and adjoint approaches. The variational approach is applied hence the objective functional G has the unambiguous physical interpretation. Minimisation of functional G is the solution of differential description of real problem. The difficulty is sometimes that the mathematical functional does not have the clear physical description. The most popular objective functional describes the mass flux density on external boundary [5,9-11]. The ironing should secure the maximal moisture transport from the external surface, that is, maximisation of the functional.

$$G = \int_0^{t_f} \left[\int_{\Gamma} q_{nw} d\Gamma \right] dt; \quad \Gamma \in \Gamma_{external} \tag{19}$$

The other functional introduces the transient global amount of mass source in domain Ω [9]. Technologically speaking, mass sources are the microcapsules which can be released by moisture and heat during ironing in special conditions [11]. These materials are applied to introduce the specific agent into textile product to optimise the underclothing microclimate of physically active people (skiers, mountain climbers, workers in extreme conditions etc.).

$$G = \int_0^{t_f} \left[\int_{\Omega} f_w d\Omega \right] dt \tag{20}$$

The alternative functional describes the transient global measure of maximal local state variable on the external boundary [9].

$$G = \int_0^{t_f} \left[\int_{\Gamma} \left(\frac{w_f}{w_{f0}} \right)^n d\Gamma \right] dt ; \quad n \rightarrow \infty \quad \Gamma \in \Gamma_{external} \tag{21}$$

where w_{f0} is the prescribed value of state variable, n is the index exponent. Minimisation of the functional for $n \rightarrow \infty$ equalises the distribution of state variables on external boundary and minimises the maximal local values of the variables. The maxima of local quantities according to Eq.(21) can have time-dependent locations because the maximal values of concentrations can vary with time. The typical thickness optimisation of textiles subjected to ironing does not introduce the locations of variable extrema.

General form of optimisation algorithm is shown in Fig.1.

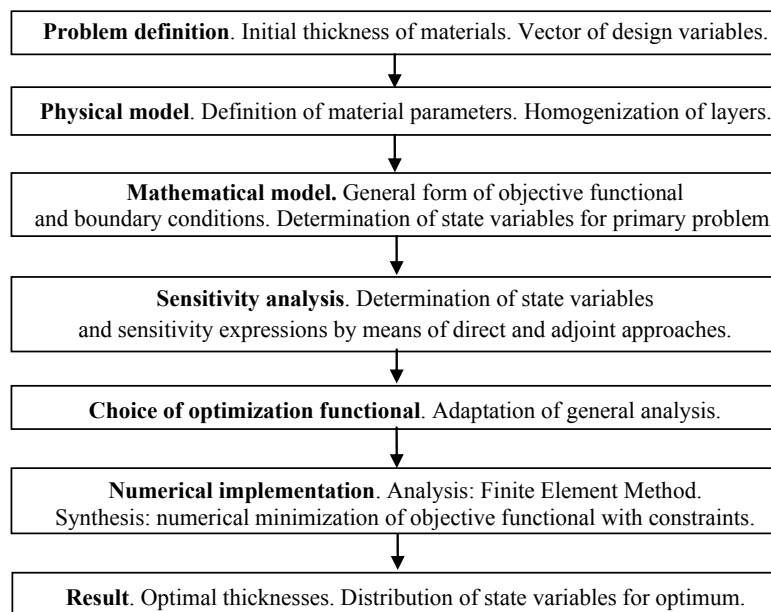


Figure 1. General algorithm of shape optimization

Problem of shape optimisation in ironing device

Let us optimise the material thicknesses in ironing device of the pressure dimensions 1,25m and 0,5m. The heated upper plate is mobile and the lower one stationary. Moisture is transported from the wet woven fabric made of cotton situated on the upper plate of the water content of (200±5)g/m². Let us introduce two different materials subjected to ironing in various configurations: (i) material A: wool (100%), surface mass 228·10⁻³kg/m²; (ii) material B: wool (80%), polyester (20%), surface mass 168·10⁻³kg/m². The prescribed time 30s is sufficient enough to reach the appropriate ironing temperature. The heat and mass transport conditions are the same along the plate. The space 3D problem can be also reduced to an optional cross-section of heating plates and textile material, that is, the plane 2D analysis.

Physical model of ironing introduces the microclimate and operating conditions in the form of material characteristics and boundary conditions. The duration time is considerably less than $t_{eq}=540s$ and there is the first stage of sorption process described by the Fick's diffusion.

Design variables are the layer thicknesses made of materials A and B. The material parameters depend on moisture concentration and density of fibres. The diffusion coefficients of water vapour in fibres during the first-stage of sorption process are assumed according to [7] where (i) denotes the number of layer; $i=1$ contact the upper plate, $i=3$ contact the lower plate. The orthotropic mass transport is described by the following diffusion coefficients [7].

$$\mathbf{D}^{(i)} = \begin{vmatrix} D_{11}^{(i)} & 0 & 0 \\ 0 & D_{22}^{(i)} & 0 \\ 0 & 0 & D_{33}^{(i)} \end{vmatrix}; \quad \text{wool: } D_{11\text{fiber}}^{(i)} = D_{22\text{fiber}}^{(i)} = \left[1.04 - 68.20 \frac{w_f}{\rho} - 1342.59 \left(\frac{w_f}{\rho} \right)^2 \right] \cdot 10^{-14}, \\
 D_{33\text{fiber}}^{(i)} = \left[1.24 - 70.20 \frac{w_f}{\rho} - 1372.59 \left(\frac{w_f}{\rho} \right)^2 \right] \cdot 10^{-14}, \\
 \text{polyester: } D_{11\text{fiber}}^{(i)} = D_{22\text{fiber}}^{(i)} = \left[1.12 - 410 \frac{w_f}{\rho} - 8200 \left(\frac{w_f}{\rho} \right)^2 \right] \cdot 10^{-13}, \\
 D_{33\text{fiber}}^{(i)} = \left[1.32 - 430 \frac{w_f}{\rho} - 8300 \left(\frac{w_f}{\rho} \right)^2 \right] \cdot 10^{-13}; \quad i = 1,2,3. \tag{22}$$

Diffusion coefficient of water vapour in the air is $Da=2,5e^{-5}$. The structure is homogenised by means of „rule of mixture” [6]. The material porosity as well as the sorption coefficient of water vapour by fibres within each layer are assumed as constant in the form.

$$\begin{aligned}
 \text{wool: } \varepsilon^{(i)} &= 0.950; \quad \eta^{(i)} = 0.800; \\
 \text{polyester: } \varepsilon^{(i)} &= 0.750; \quad \eta^{(i)} = 0.400; \quad i = 1,2,3.
 \end{aligned} \tag{23}$$

Let us define the orthotropic heat transport by means of the thermal convection coefficients [7].

$$\mathbf{A}^{(i)} = \begin{vmatrix} \lambda_{11}^{(i)} & 0 & 0 \\ 0 & \lambda_{22}^{(i)} & 0 \\ 0 & 0 & \lambda_{33}^{(i)} \end{vmatrix}; \quad \text{wool: } \lambda_{11\text{fiber}}^{(i)} = \lambda_{22\text{fiber}}^{(i)} = \left[38.49 - 0.720 \frac{w_f}{\rho} + 0.113 \left(\frac{w_f}{\rho} \right)^2 - 0.002 \left(\frac{w_f}{\rho} \right)^3 \right] \cdot 10^{-3}; \\
 \lambda_{33\text{fiber}}^{(i)} = \left[42.49 - 0.800 \frac{w_f}{\rho} + 0.117 \left(\frac{w_f}{\rho} \right)^2 - 0.002 \left(\frac{w_f}{\rho} \right)^3 \right] \cdot 10^{-3}; \\
 \text{polyester: } \lambda_{11\text{fiber}}^{(i)} = \lambda_{22\text{fiber}}^{(i)} = 28.8 \cdot 10^{-3}; \quad \lambda_{33\text{fiber}}^{(i)} = 30.0 \cdot 10^{-3}; \quad i = 1,2,3. \tag{24}$$

The cross-transport coefficient determines the part of heat transported with water vapour during sorption and desorption on external surface of fibres. The parameters $\lambda_w; c$ are assumed according to [7] in the form.

$$\begin{aligned}
 \text{wool: } \lambda_w^{(i)} &= 1602.5 \exp\left(-11.72 \frac{w_f}{\rho}\right) + 2522.0; \quad \text{polyester: } \lambda_w^{(i)} = 2522.0. \\
 \text{wool: } c^{(i)} &= 373.3 + 4661.0 \frac{w_f}{\rho} + 4.221T; \quad \text{polyester: } c^{(i)} = 1610.9; \quad i = 1,2,3.
 \end{aligned} \tag{25}$$

Our next goal is to determine the mathematical model of ironing. Let us assume that the textiles does not contain heat and mass sources, that is, $f=0; f_w=0$. The transport equations for the first phase of sorption in fibres can be adopted from Eq.(6).

$$\begin{cases} \left(1 - \varepsilon^{(i)} + \frac{\varepsilon^{(i)}}{\rho^{(i)}\beta^{(i)}}\right) \frac{dw_f^{(i)}}{dt} = -\text{div}\mathbf{q}_w^{(i)}; & \mathbf{q}_w^{(i)} = \mathbf{D}^{(i)} \cdot \nabla w_w^{(i)}; \\ \rho^{(i)}c^{(i)} \frac{dT^{(i)}}{dt} + \lambda_w^{(i)}(1 - \varepsilon^{(i)}) \frac{dw_w^{(i)}}{dt} = -\text{div}\mathbf{q}^{(i)}; & \mathbf{q}^{(i)} = \mathbf{A}^{(i)} \cdot \nabla T^{(i)}; \quad i=1,2,3; \end{cases} \quad (26)$$

The upper plate is provided with the heating device and additionally subjected to the prescribed moisture flux density normal to its surface from the wet woven fabric. It follows that the upper boundary of the cross-section is subjected to the first-kind condition of heat transport (portion Γ_T) and the second-kind condition of moisture transport (part Γ_2). Both side boundaries are open to surrounding. Thus, heat is lost to surrounding by convection and radiation (boundary portions Γ_C and Γ_r) whereas water vapour by convection (boundary part Γ_3). Therefore, the side boundaries are subjected to the third-kind conditions as well as the radiation condition. Heat and water vapour are transported to the lower plate. Temperature and moisture concentration can be determined by means of exact measurements, that is, we introduce the first-kind conditions on boundary portions Γ_T and Γ_r . Textile materials are pressed and the common boundaries are characterised by the fourth-kind conditions on the parts Γ_4 . Summarising we denote according to Eq.(10), Fig.2.

Upper boundary:

$$t \in \langle 0; 30 \rangle s; \quad T(\mathbf{x}, t) = T^0(\mathbf{x}, t) = \begin{cases} (75t) e^{-x} & t = 1, 2 s; \\ 150 + \sqrt{50(t-2)} e^{-x} & t = 3 \dots 30 s; \end{cases} \quad \mathbf{x} \in \Gamma_T;$$

$$q_{nw}(\mathbf{x}, t) = q_{nw}^0 = 2 \cdot 10^{-3t} e^{-x}; \quad \mathbf{x} \in \Gamma_2;$$

Side boundaries:

$$q_n(\mathbf{x}, t) = h[T(\mathbf{x}, t) - T_\infty] \mathbf{x} \in \Gamma_C; \quad q_{nw}(\mathbf{x}, t) = h_w[w_f(\mathbf{x}, t) - w_{f\infty}] \mathbf{x} \in \Gamma_3; \quad q_n^r(\mathbf{x}, t) = \sigma T^4 \quad \mathbf{x} \in \Gamma_r$$

Lower boundary:

$$t \in \langle 0; 30 \rangle s; \quad T(\mathbf{x}, t) = T^0(\mathbf{x}, t) = \sqrt{426t} e^{-x}; \quad \mathbf{x} \in \Gamma_T; \quad w_f(\mathbf{x}, t) = w_f^0(\mathbf{x}, t); \quad \mathbf{x} \in \Gamma_1;$$

Internal boundaries:

$$T^{(i)}(\mathbf{x}, t) = T^{(i+1)}(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_4; \quad w_f^{(i)}(\mathbf{x}, t) = w_f^{(i+1)}(\mathbf{x}, t) \quad \mathbf{x} \in \Gamma_4; \quad (27)$$

Initial conditions:

$$T(\mathbf{x}, 0) = T_0(\mathbf{x}, 0) = 20^\circ\text{C} \quad \mathbf{x} \in (\Omega \cup \Gamma); \quad w_f(\mathbf{x}, 0) = w_{f0}(\mathbf{x}, 0) = 0,08 \text{ kg/m}^3 \quad \mathbf{x} \in (\Omega \cup \Gamma).$$

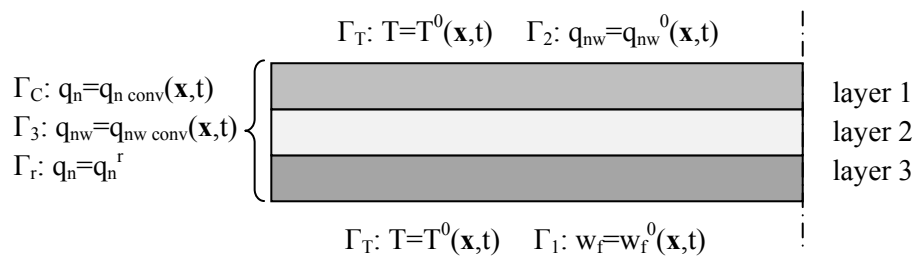


Figure 2. Physical model of boundary and initial conditions in ironing device

The time changes from the initial value $t_0=0$ to the final $t_k=30s$ and the discrete increment of time is assumed equal to $\Delta t=1s$. Convection is characterised by $h=8W/(m^2K)$, surrounding parameters are $T_\infty=20^\circ\text{C}$, $w_{f\infty}=0,06 \text{ kg/m}^3$ [12]. Design variables are three thicknesses of layers made of materials A and B. The optimal thicknesses secure the most effective transport of water vapour during ironing from the textiles into the structure of lower plate and next to surrounding, that is, we determine the moisture radiator. The constraint is the time- and place-dependent temperature. The optimisation problem has the form.

$$\begin{cases} G = - \int_0^{t_f} \left[\int_{\Gamma_l} q_{nw} d\Gamma_l \right] dt \rightarrow \min; \\ T - T^0 = 0 \quad ; \quad \Gamma_l = \Gamma_T \cup \Gamma_1. \end{cases} \quad (28)$$

The lower plate is provided with the special sucking-off machine to drain the water vapour from the internal padding. The problem is solved by means of direct and adjoint approaches to sensitivity analysis. The direct approach calls for three additional problems associated with design parameters because three different material thicknesses are optimised. The transport equations and the set of conditions are the simple adaptation of Eqs.(12), Eqs.(13), Eqs.(26), Eqs.(27) for (i) -th layer. The sensitivity expression is determined according to Eq.(14) and Eq.(28) in the final form.

$$G_p = \int_0^{t_f} \left\{ \int_{\Gamma_l} (q_{nw}^p - \mathbf{q}_{w\Gamma} \cdot \nabla_{\Gamma} v_n^p) d\Gamma_l + \int_{\Gamma_l} q_{nw,n} v_n^p d\Gamma_l + \int_{\Sigma} q_{nw} \mathbf{v}^p \cdot \mathbf{v} \right\} dt; \quad p=1, \dots, 3 \quad (29)$$

Let us next introduce the adjoint approach. The problem is described by the only objective functional. Therefore, we have to solve the only adjoint problem. The transport equations are determined by means of Eqs.(15), Eqs.(26) whereas the set of conditions by Eqs.(16), Eqs.(27). The sensitivity correlation is defined in respect of Eq.(17) and Eq.(28) in the form.

$$G_p = \left[\int_{\Omega} \eta \left(1 - \varepsilon + \frac{\varepsilon}{\beta \rho} \right) w_f^a (\nabla w_f \cdot \mathbf{v}^p) d\Omega \right]_{t=0} + \int_0^{t_f} \left\{ - \int_{\Gamma_l} [q_{nw}^a (\nabla_{\Gamma} w_f^0 \cdot \mathbf{v}_{\Gamma}^p + w_{f,n}^0 v_n^p) - \mathbf{q}_{w\Gamma} \cdot \nabla_{\Gamma} v_n^p] d\Gamma_l + \int_{\Gamma_l} q_{nw,n} v_n^p d\Gamma_l + \int_{\Sigma} q_{nw} \mathbf{v}^p \cdot \mathbf{v} \right\} dt. \quad (30)$$

Let us apply three different cases: (i) three layers made of material A, (ii) three layers made of material B, (iii) layers 1, 3 made of material A; layer 2 made of material B, cf. Fig.2. The iterative procedure consists of synthesis and analysis stage. To optimise the thickness we solve the primary, the set of three additional problems and the adjoint problem. Both heat and mass transfer are determined using the same finite element net of the most simple serendipity family according to Zienkiewicz [17]. At the analysis stage, the cross-section of textile structure is approximated by means of 8-nodal rectangular elements, the nodes are located in corners and in the half of boundary. Each layer of textile material is determined by 700 elements of 5600 nodes. At the synthesis stage, the second-order Newton procedure and the first-order method of steepest descent can be alternatively applied to find the directional minimum. The initial and optimal thicknesses are listed in Table 1. The history of optimisation is shown briefly in Figure 3, the changes of objective functional are plotted in terms of iteration number.

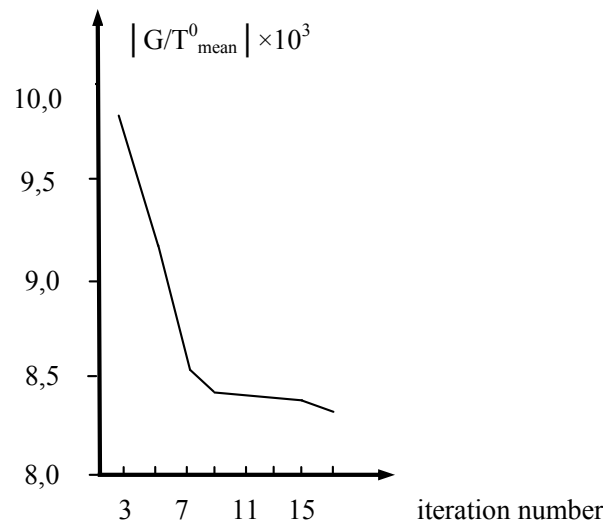


Figure 3. History of thickness optimization T_{mean}^0 – mean value of temperature on lower plate

Table 1. Initial and optimal distribution of material thicknesses

Nr of layer	3 layers of material A		3 layers of material B		2 layers of material A; 1 layer of material B	
	initial	optimal	initial	optimal	initial	optimal
1	1,00	0,77	0,50	0,39	1,00	0,79
2	0,70	0,70	0,40	0,35	0,40	0,32
3	0,60	0,69	0,40	0,35	0,50	0,69

The optimal thicknesses of textile materials depend upon the material distribution. The layers made of the same material A or B determine the comparable optimal thicknesses for each material composition although the lowest layer ($i=3$) is thinner of several percent than the highest ($i=1$). The optimal thicknesses for mixed materials are different. The minimal objective functional is determined in 16 steps and reduced of 12,54% in relation to the initial value.

Conclusions

The presented shape optimisation concerns the real problems of textile engineering, that is, the distribution of material thicknesses subjected to coupled heat and mass transport. Thus, the ironing device must secure the correct sequence of humidification, heating time, temperature and moisture flux density. The main idea is to transport the water vapour from the textiles into the structure of lower plate and next to surrounding, that is, we determine the moisture radiator. The constraint is the temperature distribution on the adequate external boundary. The optimal thicknesses are determined by means of the sensitivity approach and the particular objective functional. It follows that the above description of coupled transport as well as the applied optimisation techniques can be the promising and speed tool for generating the thicknesses of textiles subjected to ironing.

Of course, the objective functional describing the heat transfer can be optimised by means of the same method. The constraint is now the time- and place-dependent moisture flux density. Additional technological constraints can be also applied, for example the minimal or maximal increments in dimensions to secure the correct technological process.

Both physical and mathematical modeling accompanied by the corresponding sensitivity analysis are the universal techniques which can be implemented into coupled heat and mass transport in different engineering fields or other coupled problems in textile engineering. The typical example are the firemen suits subjected to external heat radiation and contact heat from the heat sources, the high-specialised textile composites made of short fibres, pcm-materials, geomaterials etc. The ironing device can be also optimised comprehensively in respect of the material thicknesses and the internal structure of lower plate. Thus, new physical problems and optimisation techniques can be implemented to develop the analysis.

Of course, the obtained theoretical results of optimal thicknesses and distribution of state variables should be compared with the tests. The detailed analysis of such implementations as well as their efficiency and accuracy is beyond the scope of this paper and will be studied in details in consecutive article.

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