Prime filters of hyperlattices

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Abstract

The purpose of this paper is the study of prime ideals and prime filters in hyperlattices. I -filter and the filter generated by $a \in L$ are introduced. Moreover, we introduce dual distributive hyperlattices, and I -filter in dual distributive hyperlattices. Some properties of hyperlattices are studied and the relationship between prime ideals and prime filters in hyperlattices is discussed.

1 Introduction

Hyperstructures theory was firstly introduced by F. Marty in the eighth congress of Scandinavians in 1934 [16]. This theory has been developed in various fields. The theory of hyperfields and hyperrings was initiated by Krasner and the results were published in 1983 in [15]. In the hyperring $(H, +, \cdot)$, $(+)$ is a hyperoperation and $(\cdot)$ is a binary operation [15]. Schweigert studied congruence of multialgebra [22]. Ameri and Nozari studied relationship between the categories of multialgebra and algebra [2]. Ameri and Rosenberg also studied congruences and strong congruences of multialgebras [3]. Hyperstructures were studied in many papers, e.g. [1, 7, 8, 9, 10, 12, 18, 17] and books e.g. [7, 24]. Some applications can be found e.g. in [5, 6, 11, 23].

The theory of hyperlattices was introduced by Konstantinidou in 1977 [14]. Barghi considered the prime ideal theorem for distributive hyperlattices [19]. Koguep, Nkuimi, and Lele studied ideals and filters in hyperlattices [13]. Rasouli and Davvaz defined fundamental relation on a hyperlattice and obtained

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a lattice from a hyperlattice. Moreover, they defined a topology on the set of prime ideals of a distributive hyperlattice [20, 21].

In this paper, we introduce dual distributive hyperlattices, and I -filter in dual distributive hyperlattices and study their properties. We consider relationship between prime ideals and prime filters in hyperlattices. Moreover, I -filter and the filter generated by $a \in L$ are introduced.

2 Preliminaries

In this section we give some results of hyperstructures and mainly hyperlattices that we need to develop our paper.

Definition 2.1. [16] Let H be a nonempty set and $P^*(H)$ denotes the set of all nonempty subsets of H . Maps $f: H \times H \longrightarrow P^*(H)$, are called *hyperoperations*.

Definition 2.2. [14] Let L be a nonempty set, $\wedge -$ be a binary operation, and $\vee -$ be a hyperoperation on L . L is called a *hyperlattice* if for all $a, b, c \in L$ the following conditions hold:

- (i) $a \in a \vee a$, and $a \wedge a = a$;
- (ii) $a \vee b = b \vee a$, and $a \wedge b = b \wedge a$;
- (iii) $a \in [a \wedge (a \vee b)] \cap [a \vee (a \wedge b)]$;
- (iv) $a \vee (b \vee c) = (a \vee b) \vee c$, and $a \wedge (b \wedge c) = (a \wedge b) \wedge c$;
- (v) $a \in a \vee b \implies a \wedge b = b$.

Let $A, B \subseteq L$. Then define:

$$A \vee B = \bigcup \{a \vee b \mid a \in A, b \in B\};$$

$$A \wedge B = \{a \wedge b \mid a \in A, b \in B\}.$$

Definition 2.3. [13] Let L be a hyperlattice. L is called *bounded* if there exist $0, 1 \in L$ such that for all $a \in L, 0 \leq a \leq 1$. We say that 0 is the *least element* and 1 is the *greatest element* of L .

Definition 2.4. [19] Let L be a hyperlattice and I be a nonempty subset of L . I is called an *ideal* if the following conditions hold:

- (i) If $a, b \in I$, then $a \vee b \subseteq I$;

(ii) If $a \in I, b \leq a$, and $b \in L$, then $b \in I$.

Ideal I is called a *prime ideal* if $a \wedge b \in I$ then $a \in I$ or $b \in I$ for all $a, b \in L$.

Definition 2.5. [19] Let L be a hyperlattice and F be a nonempty subset of L . F is called a *filter* if the following conditions hold:

- (i) If $a, b \in F$, then $a \wedge b \in F$;
- (ii) If $a \in F, a \leq b$, and $b \in L$, then $b \in F$.

Filter F is called *prime filter* if $(a \vee b) \cap F \neq \emptyset$, then $a \in F$ or $b \in F$ for all $a, b \in L$.

Definition 2.6. [19] Let L be a hyperlattice. L is *distributive* if for all $a, b, c \in L$:

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c). \quad (2.1)$$

Example 2.7. [13] Let $L = \{0, a, b, 1\}$ and $\wedge$ and $\vee$ are given with Table 1. Then $(L, \vee, \wedge, 0, 1)$ is a distributive hyperlattice.

$\wedge$	0	a	b	1	$\vee$	0	a	b	1
0	0	0	0	0	0	{0}	{a}	{b}	{1}
a	0	a	0	a	a	{a}	{0, a}	{1}	{b, 1}
b	0	0	b	b	b	{b}	{1}	{0, b}	{a, 1}
1	0	a	b	1	1	{1}	{b, 1}	{a, 1}	L

(a)
(b)

Table 1

Theorem 2.8. [13] Let L be a distributive hyperlattice. Then $0 \vee 0 = 0$.

Definition 2.9. [4] Let L be a nonempty set. Then L is called $\wedge$ -*semilattice* if for all $a, b, c \in L$ the following conditions hold:

- (i) $a \wedge a = a$;
- (ii) $a \wedge b = b \wedge a$;
- (iii) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$.

3 Ideals and filters in hyperlattices

In this section, some properties of hyperlattices are studied and the relationship between prime ideals and prime filters in hyperlattices is discussed. Finally, I -filter and the filter generated by $a \in L$ are introduced.

In the sequel, L denotes a bounded hyperlattice.

Remark 3.1. The converse of condition (v) in Definition 2.2 holds. By (iii) in Definition 2.2 and $a \wedge b = b$, we have $a \in a \vee (a \wedge b) = a \vee b$. Thus $a \in a \vee b$.

Definition 3.2. We define the order $\leq$ on L by:

$$a \leq b \iff b \in a \vee b \iff a \wedge b = a.$$

Remark 3.3. The binary relation $\leq$ is reflexive, antisymmetric, and transitive. Thus $(L, \leq)$ is a poset.

Proposition 3.4. For all $a, b \in L$ the following conditions hold:

- (i) $a \in a \vee 0$;
- (ii) $1 \in a \vee 1$;
- (iii) If $a, b \neq 0$ and $a \wedge b = 0$, then $a, b \notin a \vee b$.

Proof. (i), (ii): L is bounded, so $0 \leq a \leq 1$, for all $a \in L$. Therefore, by Definition 2.2(v), $a \in a \vee 0$, and $1 \in a \vee 1$.

(iii): Suppose $a \wedge b = 0$. If $a \in a \vee b$, by Definition 2.2(v), $b = a \wedge b = 0$. Thus $b = 0$ which is a contradiction. If $b \in a \vee b$, we obtain that $a = 0$, which is a contradiction. $\square$

Theorem 3.5. If P is a prime ideal of L , then $L \setminus P$ is a prime filter of L .

Proof. Let $a, b \in L \setminus P$. We show that $a \wedge b \in L \setminus P$. It is clear that $a, b \notin P$. If $a \wedge b \in P$, then $a \in P$ or $b \in P$ because P is prime filter, which is a contradiction. So $a \wedge b \in L \setminus P$. Assume $x \in L, a \in L \setminus P$ such that $a \leq x$, we show that $x \in L \setminus P$. It is clear that $a \notin P$. Suppose $x \notin L \setminus P$, so $x \in P$. We have $a \leq x$ and P is an ideal, therefore $a \in P$ that is a contradiction. So, $L \setminus P$ is a filter. It is enough to show that $L \setminus P$ is a prime filter. Suppose $a, b \in L$ and $(a \vee b) \cap (L \setminus P) \neq \emptyset$. So there exists $x \in L$ such that $x \in a \vee b$ and $x \in L \setminus P$. If $a, b \notin L \setminus P$, then $a, b \in P$, therefore $a \vee b \subseteq P$. Hence $(a \vee b) \cap (L \setminus P) = \emptyset$, which is a contradiction. $\square$

Corollary 3.6. If P is a prime ideal of L , then $L \setminus P$ is a $\wedge$ -semilattice.

The converse of Theorem 3.5 holds. Now, we state it by the following theorem:

Theorem 3.7. *Let F be a prime filter of L . Then $L \setminus F$ is a prime ideal of L .*

Proof. Let $a, b \in L \setminus F$. If $a \vee b \notin L \setminus F$, then there exists $x \in a \vee b$ such that $x \notin L \setminus F$. So $x \in a \vee b$ and $x \in F$. Hence $x \in (a \vee b) \cap F$ and it implies that $(a \vee b) \cap F \neq \emptyset$. Thus $a \in F$ or $b \in F$, which is a contradiction. Let $a \in L \setminus F, x \leq a$. Assume $x \notin L \setminus F$. Thus we have: $x \in F$ and $x \leq a$. Since F is a filter, $a \in F$, which is a contradiction. Therefore, $L \setminus F$ is an ideal. We must show that $L \setminus F$ is prime. Assume $a, b \in L$ such that $a \wedge b \in L \setminus F$. If $a \notin L \setminus F$ and $b \notin L \setminus F$, then $a \in F$ and $b \in F$. Since F is a filter, $a \wedge b \in F$. So $a \wedge b \notin L \setminus F$, which is a contradiction. Thus $L \setminus F$ is a prime ideal. $\square$

In the following example, we show that F must be prime in Theorem 3.7.

Example 3.8. Let $L = \{0, a, 1\}$ and $\wedge$ and $\vee$ are given with Table 2. Consider $F = \{1\}$, F is a filter of L , but it is not prime. We have $L \setminus F = \{0, a\}$, $0 \vee 0 = L$, and $L \not\subseteq L \setminus F$, so $L \setminus F$ is not a ideal of L .

$\wedge$	0	a	1	$\vee$	0	a	1
0	0	0	0	0	$\{0, a, 1\}$	$\{a, 1\}$	$\{1\}$
a	0	a	a	a	$\{a, 1\}$	$\{a\}$	$\{1\}$
1	0	a	1	1	$\{1\}$	$\{1\}$	$\{1\}$

(a)
(b)

Table 2

Theorem 3.9. *Let F be a non-empty subset of L such that it is a $\wedge$ -semilattice. Then F is a prime filter if and only if for all $a, b \in L$, the following conditions hold:*

- (i) *If $a \notin F$ and $b \notin F$, then $a \vee b \subseteq L \setminus F$;*
- (ii) *If $a \in F$ and $x \in x \vee a$, then $x \in F$.*

Proof. Suppose that F is a prime filter. (i) If $a \notin F$ and $b \notin F$, then we have $a, b \in (L \setminus F)$. Since by Theorem 3.7, $(L \setminus F)$ is an ideal, $a \vee b \subseteq (L \setminus F)$.

(ii) Let $x \in x \vee a$. By Definition 3.2, $a \leq x$. Since F is a filter, $a \leq x$, and $a \in F$, so $x \in F$.

Conversely, suppose $(a \vee b) \cap F \neq \emptyset$. If $a \notin F$ and $b \notin F$, by (i) we conclude that $a \vee b \subseteq (L \setminus F)$. Therefore $(a \vee b) \cap F = \emptyset$, which is a contradiction. $\square$

Definition 3.10. Let L be a non-empty set and $\vee-$ be a hyperoperation on L . Then L is called a $\vee-$ semi-hyperlattice if for all $a, b, c \in L$ the following conditions hold:

- (i) $a \in a \vee a$;
- (ii) $a \vee b = b \vee a$;
- (iii) $a \vee (b \vee c) = (a \vee b) \vee c$.

Example 3.11. Let $L = \{0, a, 1\}$. Hyperoperation $\vee-$ on L is given by Table 3. Then $(L, \vee)$ is a $\vee-$ semi-hyperlattice.

$\vee$	0	a	1
0	$\{0\}$	$\{a, 1\}$	$\{1\}$
a	$\{a, 1\}$	$\{a\}$	$\{1\}$
1	$\{1\}$	$\{1\}$	$\{1\}$

Table 3

Proposition 3.12. Let L be a $\wedge-$ semilattice and a $\vee-$ semihyperlattice. Then L is a hyperlattice if and only if for all $a, b \in L$ the following conditions hold:

- (i) $a \in [a \vee (a \wedge b)] \cap [a \wedge (a \vee b)]$;
- (ii) $a \in a \vee b \implies b \leq a$.

Definition 3.13. Let $(L, \vee, \wedge)$ be a hyperlattice and S be a non-empty subset of L . Then S is called a *sub-hyperlattice* of L if $(S, \vee, \wedge)$ is a hyperlattice.

Example 3.14. Let L be the hyperlattice in Example 3.8 and $S = \{a, 1\}$. Then S is a sub-hyperlattice of L .

Proposition 3.15. Let L be a hyperlattice and let S be a non-empty subset of L . Then S is a sub-hyperlattice of L if and only if $a \wedge b \in S$ and $a \vee b \subseteq S$ for all $a, b \in S$.

Remark 3.16. Let L be a distributive hyperlattice. $\{0\}$, and L are sub-hyperlattice of L which are called the *trivial sub-hyperlattices* of L . It is necessary that L be a distributive hyperlattice. In Example 3.8, since $0 \vee 0 = \{0, a, 1\}$ and $0 \vee 0 \not\subseteq \{0\}$, $\{0\}$ is not a sub-hyperlattice of L .

In this part we introduce I -filters and filters generated by an element a in a type of hyperlattices that are called *dual distributive*.

Definition 3.17. A hyperlattice L is called *quasi dual distributive* if for all $a, b, c \in L$.

$$a \vee (b \wedge c) \subseteq (a \vee b) \wedge (a \vee c);$$

L is called *weak dual distributive* if:

$$a \vee (b \wedge c) \cap [(a \vee b) \wedge (a \vee c)] \neq \emptyset;$$

L is called *dual distributive* if:

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c).$$

Clearly a quasi dual distributive hyperlattice is weak dual distributive and a dual distributive hyperlattice is quasi distributive and weak dual distributive.

Example 3.18. (i) Let L be the hyperlattice L in Example 3.8. Then $(L, \vee, \wedge)$ is a dual distributive hyperlattice but it is not distributive. Since we have $0 \vee 0 = L$, by Theorem 2.8, L isn't distributive.

(ii) Let $L = \{0, a, b, 1\}$. $\wedge$ operation and $\vee$ hyperoperation on L are given by Table 4. Then hyperlattice L is not distributive, also L is not dual distributive.

$\wedge$	0	a	b	1	$\vee$	0	a	b	1
0	0	0	0	0	0	{0}	{a, 1}	{b, 1}	{1}
a	0	a	0	a	a	{a, 1}	{a, 1}	{1}	{1}
b	0	0	b	b	b	{b, 1}	{1}	{b, 1}	{1}
1	0	a	b	1	1	{1}	{1}	{1}	{1}

(a)
(b)

Table 4

Since $a \wedge (b \vee a) = a \wedge 1 = a$ and $(a \wedge b) \vee (a \wedge a) = 0 \vee a = \{a, 1\}$, therefore L is not distributive and since $0 \vee (a \wedge b) = 0 \vee 0 = 0$, and $(0 \vee a) \wedge (0 \vee b) = \{a, 1\} \wedge \{b, 1\} = \{0, a, b, 1\}$, L is quasi dual distributive, but it is not dual distributive.

The hyperlattices such that are distributive and dual distributive, are called *strongly distributive hyperlattices*.

Example 3.19. Let $L = \{0, a, 1\}$. $\vee$ and $\wedge$ are given by Table 5. Then L is a strongly distributive hyperlattice.

Lemma 3.20. Let L be a dual distributive hyperlattice and $a \in L$. We define $F_a = \{x \in L : x \neq 0, 1 \in x \vee a\}$, then F_a is a filter of L .

Proof. Suppose x and $y \in F_a$. So $1 \in x \vee a$ and $1 \in y \vee a$. Hence $1 \in (x \vee a) \wedge (y \vee a)$. Since L is dual distributive, we have $a \vee (x \wedge y) = (a \vee x) \wedge (a \vee y)$.

$\wedge$	0	a	1	$\vee$	0	a	1
0	0	0	0	0	$\{0\}$	$\{a, 1\}$	$\{1\}$
a	0	a	a	a	$\{a, 1\}$	$\{a\}$	$\{1\}$
1	0	a	1	1	$\{1\}$	$\{1\}$	$\{1\}$

(a)
(b)

Table 5

So $1 \in (x \wedge y) \vee a$. Therefore, $(x \wedge y) \in F_a$. Let $x \in F_a$ and $y \in L$, such that $x \leq y$. Hence $1 \in x \vee a$ and $x \wedge y = x$. So we have $1 \in x \vee a = (x \wedge y) \vee a$. Since L is dual distributive, $1 \in (a \vee x) \wedge (a \vee y)$. So $1 \in a \vee y$ and it implies that $y \in F_a$. Thus F_a is a filter. $\square$

In Lemma 3.20, L must be dual distributive. In the following example we show that the converse of Lemma 3.20 does not hold.

Example 3.21. Let $L = \{0, a, b, 1\}$ and $\wedge$ be an operation and $\vee$ hyperoperation on L are given with Table 6.

$\wedge$	0	a	b	1	$\vee$	0	a	b	1
0	0	0	0	0	0	$\{0, a, b, 1\}$	$\{a, b, 1\}$	$\{b\}$	$\{1\}$
a	0	a	a	a	a	$\{a, b, 1\}$	$\{a, b, 1\}$	$\{b\}$	$\{1\}$
b	0	a	b	b	b	$\{b\}$	$\{b\}$	$\{1, b\}$	$\{1\}$
1	0	a	b	1	1	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

(a)
(b)

Table 6: join table

Then L is a hyperlattice which is not dual distributive, because we have $0 \vee (a \wedge b) = 0 \vee a = \{a, b, 1\}$, and $(0 \vee a) \wedge (0 \vee b) = \{a, b, 1\} \wedge b = \{a, b\}$. So, $0 \vee (a \wedge b) \neq (0 \vee a) \wedge (0 \vee b)$.

We have $F_a = \{a, 1\}$. Since $a \in F_a$ and $a \leq b$, but $b \notin F_a$, So F_a is not a filter. Also, we have $F_b = \{b, 1\}$ that is a filter, so the converse of Lemma 3.20 does not hold.

Proposition 3.22. $F_1 = L \setminus \{0\}$.

Proof. It is clear that $F_1 \subseteq L \setminus \{0\}$. Let $x \in L, x \neq 0$. Since $x \leq 1$, by Definition 3.2, $1 \in x \vee 1$. So $x \in F_1$ and it means that $(L \setminus \{0\}) \subseteq F_1$. So $F_1 = L \setminus \{0\}$. $\square$

Theorem 3.23. Let L be a dual distributive hyperlattice and I be a nonempty subset of L . Define $F_I = \{x \in L : x \neq 0, 1 \in x \vee a, \forall a \in I\}$. Then

- (i) F_I is a filter of L .
- (ii) $F_I = \bigcap_{a \in I} F_a$.
- (iii) $F_a \cap F_b = F_{a \wedge b}$.
- (iv) If $A \subseteq B$, then $F_B \subseteq F_A$.

Proof. We prove (iii), only. Then $x \in F_a \cap F_b$, if and only if $1 \in x \vee a$ and $1 \in x \vee b$, if and only if $1 \in (x \vee a) \wedge (x \vee b) = x \vee (a \wedge b)$, if and only if $1 \in F_{a \wedge b}$. Therefore, $F_a \cap F_b = F_{a \wedge b}$. $\square$

F_I is called I -filter generated by I .

Corollary 3.24. *Let L be a dual distributive hyperlattice. Then the following conditions hold:*

- (i) If $a, b \in L$ and $a \leq b$, then $F_{a \vee b} \subseteq F_b$.
- (ii) $F_a \supseteq F_{a \wedge b}$ and $F_b \supseteq F_{a \wedge b}$.
- (iii) $(\{F_a : a \in L\}, \bigcap)$ is a $\wedge$ -semilattice.

Proof. (i): Let $a \leq b$. By Definition 3.2, $b \in a \vee b$ and it implies that $\{b\} \subseteq a \vee b$. By Theorem 3.23, (iv) we have: $F_{a \vee b} \subseteq F_b$.

(ii) and (iii) are obvious. $\square$

Example 3.25. (i) Let L be the hyperlattice in Example 3.8 and $I = \{a, 1\}$. We have $F_a = \{1\}$ and $F_1 = \{a, 1\}$, then $F_I = F_a \cap F_1 = \{1\} \cap \{a, 1\} = \{1\}$.

- (ii) Let L be the hyperlattice in Example 3.21. We have $a \leq b$, but $F_a \not\subseteq F_b$ and $F_b \not\subseteq F_a$, since $F_a = \{a, 1\}$, and $F_b = \{b, 1\}$.

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