SOME NUMERICAL RESULTS OF MULTIPOINTS BOMNDARY VALUE PROBLEMS ARISE IN ENVIRONMENTAL PROTECTION

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Abstract: In this paper, we investigate two problems arise in pollutant transport in rivers, and we give some numerical results to approximate this solutions. We determined the approximate solutions using two numerical methods: 1. B-splines combined with Runge-Kutta methods, 2. BVP4C solver of MATLAB and then we compare the run-times.

Key words: B-spline, Nonlocal and multipoint boundary value problem, Multistep, Runge-Kutta and extrapolation method. AMS Classifications (2010): 65D07,34B10,65L06

The statement of the problems

The aim of this work is to compare the run-times of two numerical methods used to determine the approximate solutions of multipoints value problems with boundary conditions at infinity, they appears in pollutants transport in rivers.

Ames and Lohner (1981), [1], study models for a transport, reaction and dissipation of pollutants in rivers. One model gives rise to a system of three first-order PDE in one space variable x and time t . By looking for traveling wave solutions, that depend only the variable: $z = x - t$ they reduce PDE, to ODE:

$z=x-t$ they reduce PDE, to ODE like:

$$\begin{aligned}f'' &= \beta gf \\ g'' &= -\beta gh \\ h'' &= \lambda \beta gh\end{aligned}$$

Here f represents a pollutant, g bacteria, and h carbon; the physical parameters λ and β are constants. After showing that the equations for g and h imply that:

$$g(z) = E - (h(z)/\lambda) \quad (2)$$

where E is a given value for $g(\infty)$. They reduce the system of ODE so:

$$h'' = \lambda \beta (E - (h/\lambda))h, \quad (3)$$

$$f'' = \beta (E - (h/\lambda))f. \quad (4)$$

This equations are be solved subject to boundary conditions:

$$h(0)=1, h(\infty)=0, f(0)=1, f(\infty)=0 \quad (5)$$

Since $h(\infty)=0$ the equation (3) can be approximated for large z by:

$$h'' = E\lambda\beta h$$

For $\beta=\lambda=10$ and $E=1$, this equation becomes:

$$h'' = 100h \quad (6)$$

with solution:

$$h(z) = Ae^{10z} + Be^{-10z}$$

Because $h(0)=1, h(\infty)=0$ implies $A=0$ and $B=1$, then the approximate solution $h(z)$ of the problem (6) with conditions (5) is asymptotically multiple of e^{-10z}

With $h(z) = e^{-10z}$, $\beta = \lambda = 10$, $E = 1$ the equation (4) becomes:

$$f'' = 10(1 - e^{-10z}/10)f, \quad (7)$$

subject to boundary conditions:

$$f(0) = 1, f(\infty) = 0 \quad (8)$$

Also for large z the equation (7) can be approximate by:

$$f'' = 10f$$

Solving this approximating ODE we find that:

$$f(z) = Ae^{\sqrt{10}z} + Be^{-\sqrt{10}z}$$

For constants A, B. The solution is to have $f(\infty) = 0$, so $A = 0$.

From this we see that the solution of the given equation is approximately a multiply $e^{-\sqrt{10}z}$. For practical purposes it is interesting solution behavior in transient area (an interval bounded by that starts from 0) and not the behavior of the asymptotically area where $h(z) \approx 0$, $f(z) \approx 0$.

2. Numerical results

We present two approximation methods for exact solutions of the problems (6) and (7) with conditions (5), and (8). We divide the interval $[0, \infty)$ in two subintervals like:

1. a transient area $[0, 10]$ and
 2. the asymptotically area $(10, \infty)$,
- where $z = 10$ is the positive solution of equation:
- $$r^2 - 100 = 0.$$

2.1. B-splines and Runge-Kutta methods

A natural choice of the start solution for this problem is:

$$h(t) = 1/(t+1)^3, f(t) = 1/(t+1)^3$$

since:

$$\begin{aligned} h(0) &= 1, h(\infty) = 0, \\ f(0) &= 1, f(\infty) = 0. \end{aligned}$$

We obtained the approximation solutions of the problem (6) with conditions (5), like:

1. On the interval $[0, 10]$ with B-splines functions of order $(k+1)$,
2. And on $[10, \delta)$ with Runge-Kutta methods (k -stages), solver ode113 or ode45 ([4]).

The convergence given in ([3], [5]).

After 5 iterations obtained the following results fig :1(a), fig:1(b):

(a) Approximate solution for the problem: (6)+(5)

(b) Approximate solution for the problem: (7)+(8)

Figure 1: B-spline + Runge-Kutta

2.2. The solver BVP4C

Following the idea giving in [2, pp:146-152], we solve the problems (6) + (5) and (7)+(8) using the solver BVP4c of MATLAB suggest by [6, pp: 153] and obtained the following results:

(a) Approximate solution for the problem: (6)+ (5)

(b) Approximate solution for the problem: (7)+ (8)

Figure 2: BVP4C

Conclusions

Using the functions tic-toc of Malab we obtained the following results:

Problems	Method	Tolerance	Running Times
(6)+(5)	B-Splines+Runge-Kutta	10^{-15}	1.645805 seconds
(6)+(5)	BVP4C	10^{-15}	2.1725 seconds
(7)+(8)	B-Splines+Runge-Kutta	10^{-15}	0.931149 seconds
(7)+(8)	BVP4C	10^{-15}	2.0004 seconds

For more details concerning the run-times you can be used the function profile viewer of Matlab. This problem is set on an infinite interval, so some experimentation is necessary to verify that a sufficiently large δ has been specified.

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