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AN OPTIMIZED TRIAD ALGORITHM FOR ATTITUDE DETERMINATION

Xiaoning Zhu, Ming Ma, Defu Cheng, Zhijian Zhou
(College of Instrumentation & Electrical Engineering, Jilin University, China, 130000)

ABSTRACT: The classic TRIAD was used to obtain the attitude of air vehicles. However, the accuracy was dominated by the sensor noise and the calculation order. To improve that in this paper, a new method based on weighting the vectors summation and difference was proposed. Then both simulation and experiment verified the advantages of the optimized algorithm.

1. INTRODUCTION

The classic TRIAD (tri-axial attitude determination) algorithm for attitude determination of satellites was proposed in 1981. It used two vectors to solve the Wahba's problem [1]. The algorithm is easy for computation. In satellite, star sensors obtained those vectors [2-5]. In 2012, the TRIAD was used to calculate the nine terms of the direct cosine matrix (DCM) in an attitude and heading reference system (AHRS), the two vectors used for TRIAD were geomagnetic field and gravitational field [6]. In [7] a TRIAD based on GPS signal was proposed to estimate attitude of the unmanned airborne vehicles (UAVs). According to the error matrix of TRIAD derived in [8], the accuracy of attitude determination was dominated by the wideband noise of vector observations. Later in [9] it has been illustrated that different order of reference vectors affected the results, because of weighting too much on the first vector. The two vectors were distinguished by the order. Accordingly, they were named as major vector and secondary vector, respectively. Therefore, two problems need to be considered here: a) the calculation order, b) the wideband noises. Many improvement works have been tried in past research [10-16]. In [10] an improved TRIAD was proposed by calculating the summation and difference of two vectors, then two new vectors were used in classic TRIAD solving the order problem. Reference [11] pointed out that this method was only feasible when two vectors have the same accuracy. Therefore, it proposed a new TRIAD algorithm by creating an optimal major vector to solve the different noise problem. In [12-16], TRIAD was performed with Kalman Filter, which increased the computation time. However, neither of them considered both problems that deviated the TRIAD accuracy.

In this paper, after mathematical calculations both TRIAD improvements mentioned above were proofed as non-optimal solutions. Moreover, an optimized TRIAD algorithm was proposed by weighting the vectors summation and difference to solve the order and noise problems simultaneously. The first section illustrated the theory of the classic TRIAD algorithm and two improved TRIAD algorithms including their limitations. The optimized TRIAD was proposed in the fourth section. At the end of this paper, simulation and experiment based on an AHRS were used to verify the performance of the optimized TRIAD algorithm.

2. THEORY

2.1 Classic TRIAD

TRIAD uses two unparalleled unit vectors to construct a new coordinate [1]. The attitude matrix was calculated by the algebraic method. The equation is given as,

$$\begin{cases} \hat{r}_1 = \hat{V}_1 \\ \hat{r}_2 = \left(\hat{V}_1 \times \hat{V}_2\right) / \left|\hat{V}_1 \times \hat{V}_2\right| \\ \hat{r}_3 = \left(\hat{V}_1 \times \hat{r}_2\right) / \left|\hat{V}_1 \times \hat{r}_2\right| \end{cases} \qquad \begin{cases} \hat{s}_1 = \hat{W}_1 \\ \hat{s}_2 = \left(\hat{W}_1 \times \hat{W}_2\right) / \left|\hat{W}_1 \times \hat{W}_2\right| \\ \hat{s}_3 = \left(\hat{W}_1 \times \hat{s}_2\right) / \left|\hat{W}_1 \times \hat{s}_2\right| \end{cases} \tag{1}$$

$$M_{ref} = \left[\hat{r}_1 \vdots \hat{r}_2 \vdots \hat{r}_3\right] \quad M_{obs} = \left[\hat{s}_1 \vdots \hat{s}_2 \vdots \hat{s}_3\right] \tag{2}$$

where $\hat{V}_1$ and $\hat{V}_2$ are the initial observations of two vectors. $\hat{W}_1$ and $\hat{W}_2$ are the current observations. $\hat{r}_1$, $\hat{r}_2$, and $\hat{r}_3$ indicate the base vectors of reference matrix, M_{ref}, after orthogonalizing the reference vectors. $\hat{s}_1$, $\hat{s}_2$, and $\hat{s}_3$ indicate the base vectors of observation matrix, M_{obs}. Then the attitude matrix is derived as,

$$A = M_{obs} \big/ M_{ref} \tag{3}$$

where A is a DCM represents the attitude matrix. From the equation (1) it can be seen, that both reference and observation matrices were weighted twice by the first vector. It means the first vector dominated the accuracy of attitude determination, which was called as major vector. The second vector was named as secondary vector. When the noise of the major vector is bigger than the secondary vector, then the classic TRIAD is not the optimized solution. In [9] the error model of TRIAD was given as:

$$P_{TRIAD} = \sigma_1^2 I + \frac{1}{\left|\hat{w}_1 + \hat{w}_2\right|^2}\left[\sigma_1^2\left(\hat{w}_1 \bullet \hat{w}_2\right)\left(\hat{w}_1 \hat{w}_2{}^T + \hat{w}_2 \hat{w}_1{}^T\right) + \left(\sigma_2^2 - \sigma_1^2\right)\hat{w}_1 \hat{w}_1{}^T\right] \tag{4}$$

where P_{TRIAD} is the attitude covariance matrix. I is an identity matrix. σ_1^2 and σ_2^2 are the wideband noise variances of two vector sensors, respectively. $\hat{w}_1$ and $\hat{w}_2$ are the current observations of two vectors. $\hat{w}_1 \bullet \hat{w}_2$ indicates that the included angle of two vectors is another parameter dominating the attitude accuracy. From equation (4), it can be seen that when $\sigma_1^2 > \sigma_2^2$, the error matrix is bigger than the case of $\sigma_1^2 < \sigma_2^2$. This is the drawback of TRIAD algorithm [8]. The proposed method in this paper tried to solve this problem.

Here are two traditional solutions to optimize TRIAD. The first method constructed two new vectors, which have the same noises, leads to the calculation order is no more important. The second method constructed an optimal vector with lower noise as the major vector to reduce the error. However, it ignored the secondary vector. Neither of them is the optimized solution.

2.2 Improved TRIAD based on summation and difference vectors

The included angle was assumed as constant. The wideband noises of two sensors were normal distribution. In [7] a method was proposed to create two vectors, which have the same noises. The algorithm was given as,

$$\hat{\chi}_1 = \hat{V}_1 + \hat{V}_2 \tag{5}$$

$$\hat{\chi}_2 = \hat{V}_1 - \hat{V}_2 \tag{6}$$

where $\hat{\chi}_1$ and $\hat{\chi}_2$ are the summation and difference vectors, respectively. Use them to construct a new reference matrix as follows.

$$\begin{cases} \hat{r}_1 = \hat{\chi}_1 \\ \hat{r}_2 = (\hat{\chi}_1 \times \hat{\chi}_2)/|\hat{\chi}_1 \times \hat{\chi}_2| \\ \hat{r}_3 = (\hat{\chi}_1 \times \hat{r}_2)/|\hat{\chi}_1 \times \hat{r}_2| \end{cases} \tag{7}$$

The observation matrix was accordingly changed. Then attitude matrix was calculated with these new reference and observation matrix. Due to the equations (5) and (6), the noise variances of $\hat{\chi}_1$ and $\hat{\chi}_2$ are the same, i.e. $\sigma_1^2 = \sigma_2^2$. So the calculation order is no longer important. The covariance matrix of this method was derived as:

$$P_{TRIAD-1} = \sigma^2 I + \frac{1}{|\hat{w}_1 + \hat{w}_2|^2}\left[\sigma^2(\hat{w}_1 \bullet \hat{w}_2)(\hat{w}_1 \hat{w}_2^T + \hat{w}_2 \hat{w}_1^T)\right] \tag{8}$$

where $TRIAD-1$ represents the TRIAD based on summation and difference vectors. $\sigma^2 = \max(\sigma_1^2, \sigma_2^2)$. Therefore, $P_{TRIAD\text{-}1} > P_{TRIAD}$. Though this improvement balanced the sensors weight, the attitude algorithm was not an optimal solution.

2.3 Improved TRIAD based on the optimal vector

To overcome the noise affection, in [11] an optimal vector was constructed based on different sensor accuracies. The equation was given as,

$$\hat{\chi} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\hat{V}_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\hat{V}_2 \tag{9}$$

where $\hat{\chi}$ represents the optimal vector. The variance $\hat{\chi}$ was also given in the paper [17].

$$\sigma_{opt}^2 = Var(\hat{\chi}) = \left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^{-1} \tag{10}$$

The equation (10) indicates that the optimal vector, $\hat{\chi}$, is more accurate than both original vectors. The proof procedure was illustrated in appendix in [17]. Then the optimal vector was used as the major vector in proposed TRIAD [11]. Therefore, according to Equation (1), the reference matrix of this method is derived as:

$$\begin{cases} \hat{r}_1 = \hat{\chi} \\ \hat{r}_2 = (\hat{\chi} \times \hat{V}_2)/|\hat{\chi} \times \hat{V}_2| \\ \hat{r}_3 = (\hat{\chi} \times \hat{r}_2)/|\hat{\chi} \times \hat{r}_2| \end{cases} \tag{11}$$

The observation matrix is similarly. It is noteworthy that the secondary vector is still the second vector $\hat{V}_2$, which deviated the accuracy of reference matrix again. This attitude algorithm is not the optimal solution either.

$$P_{TRIAD-2} = \sigma_{opt}^2 I + \frac{1}{|\hat{w}_1 + \hat{w}_2|^2}\left[\sigma_{opt}^2(\hat{w}_1 \bullet \hat{w}_2)(\hat{w}_1 \hat{w}_2^T + \hat{w}_2 \hat{w}_1^T) + (\sigma_2^2 - \sigma_{opt}^2)\hat{w}_1 \hat{w}_1^T\right] \tag{12}$$

where $TRIAD-2$ represents the TRIAD based on the optimal vector. Though the noise variance of the major vector is σ_{opt}^2 instead of σ_1^2. Then $P_{TRIAD-2} < P_{TRIAD}$. However, the noise variance of the secondary vector is still σ_2^2. Therefore, this TRIAD improvement is also not the optimal solution.

2.4 Optimized TRIAD based on weighting the vectors summation and difference

To optimize the algorithm, it only needs to find another unparalleled vector with less noise than two original vectors. An optimal differential vector was proposed according to equation (9).

$$\hat{\varsigma} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\hat{V}_1 - \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\hat{V}_2 \tag{13}$$

where $\hat{\varsigma}$ represents the optimal differential vectors which variance is $Var(\hat{\varsigma}) = \left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^{-1}$.

The proof is as the same as the optimal vector. At last, both two optimal vectors were used to construct the reference and observation matrix.

$$\begin{cases} \hat{r}_1 = \hat{\chi} \\ \hat{r}_2 = (\hat{\chi} \times \hat{\varsigma}) / |\hat{\chi} \times \hat{\varsigma}| \\ \hat{r}_3 = (\hat{\chi} \times \hat{r}_2) / |\hat{\chi} \times \hat{r}_2| \end{cases} \qquad \begin{cases} \hat{s}_1 = \hat{\kappa} \\ \hat{s}_2 = (\hat{\kappa} \times \hat{\xi}) / |\hat{\kappa} \times \hat{\xi}| \\ \hat{s}_3 = (\hat{\kappa} \times \hat{s}_2) / |\hat{\kappa} \times \hat{s}_2| \end{cases} \tag{14}$$

where $\hat{\kappa}$ and $\hat{\xi}$ represent the observation vector after optimizing. From the equations above, the optimal attitude matrix could be finally derived. The error model of the optimized TRIAD was given as,

$$P_{Opt-TRIAD} = \sigma_{opt}^2 I + \frac{1}{|\hat{w}_1 + \hat{w}_2|^2}\left[\sigma_{opt}^2 (\hat{w}_1 \bullet \hat{w}_2)(\hat{w}_1 \hat{w}_2{}^T + \hat{w}_2 \hat{w}_1{}^T)\right] \tag{15}$$

where σ_{opt}^2 represents the noise variance of both two optimal vectors. And $\sigma_{opt}^2 = Var(\hat{\chi}) = Var(\hat{\varsigma})$. Then, $P_{Opt-TRIAD} < P_{TRIAD-2}$. Therefore, the optimized TRIAD was mathematical proofed more accurate than the three TRIAD algorithms.

4. SIMULATION

To verify the advantage of the proposed algorithm, Monte Carlo simulations were performed. First of all the relative position of the reference vectors was assumed as invariant. Then the wideband noise of first vector set as 1% of measurement range, the other was set as the same as the first one. After 1000 runs of Monte Carlo, the simulation results were shown in Fig. 1.

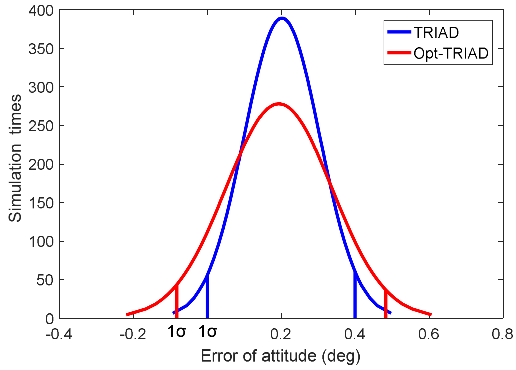

Fig. 1. Simulation results of attitude determination in the same accuracy case

Opt-TRIAD represents the optimized TRIAD. Figure 1 indicates the variance of proposed algorithm is smaller the classic TRIAD. 1σis the standard deviation which covered 94.7773% proportion under the curve. In other words, the optimized TRIAD is more accurate than TRIAD. On the other hand, the different accuracy case was shown in Fig. 2, in which the wideband noise of the first vector was set as 1% of measurement range and the other was set as 2%.

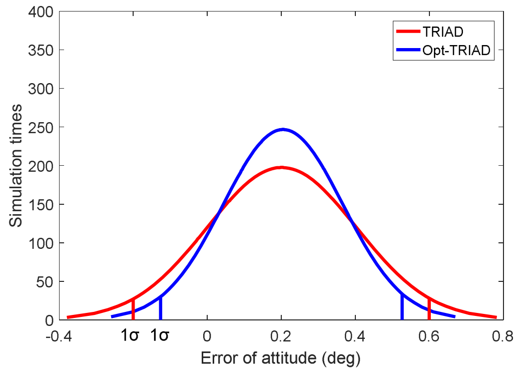

Fig. 2 Simulation results of attitude determination in different accuracy case

5. EXPERIMENT

In this section, an inertial navigation system (INS) unit produced by Chinese company RION was used to verify the proposed algorithm. INS contains a tri-axial magnetometer with resolution of 500 *nT* and a tri-axial accelerometer with resolution of 10 *mg*. They were used to measure the geomagnetic field and the gravitational field, respectively. The microcontroller unit (MCU) used for data sampling and computation is STM32F303. Furthermore, a nonmagnetic turntable with a resolution of 0.01° was used to provide the actual attitude reference. Fig. 3 shows the experiment devices.

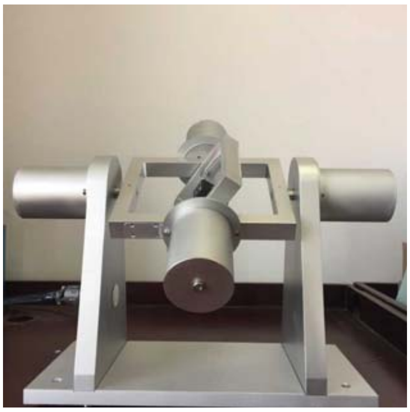

Fig. 3. Experiment device including an INS and a nonmagnetic turntable

The experimental results of four TRIAD algorithms were shown in Table. 1. The mean value is the average of the error of attitude determination. The standard deviation is the square of the variance of the error. It can be seen that the optimized TRIAD proposed in this paper was statistically more accurate than the other three algorithms. Though the computation time of proposed algorithm is larger, it is enough for attitude updating frequency (50Hz).

Table 1. The experimental results of four TRIAD algorithms

	Mean value	Standard deviation	Computation Time (μs)
TRIAD	0.388°	0.6138°	158
TRIAD-1	0.392°	0.6452°	177
TRIAD-2	0.379°	0.4864°	173
Opt-TRIAD	0.382°	0.3245°	197

6. CONCLUSION

This paper presented two shortcomings of TRIAD algorithm: the measurement noises of vectors and the calculation order. Two past research improved only one aspect. Therefore, an optimized TRIAD algorithm that could suppress two shortcomings simultaneously was proposed. The improvement was mathematical proofed through the covariance matrix. The experiment shows that the proposed method was statistically more accurate than the other three TRIAD algorithms. Moreover, for low-cost MCU it is still easy computation.

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