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ALGORITHM FOR MODELING COORDINATES OF CORNERS OF BUILDINGS DETERMINED WITH RTN GNSS TECHNOLOGY USING VECTORS TRANSLATION METHOD

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ABSTRACT. The paper presents an innovative solution which increases the reliability of determining the coordinates of corners of building structures in the RTN GNSS mode. Having performed the surveys of the base points in real time, it is proposed to use the method of *line-line intersection*, which results in capturing the Cartesian coordinates X, Y of the corners of buildings. The coordinates which were obtained in this way, are subjected to an innovative solution called the method of ***vectors translation***. This method involves modeling the coordinates obtained by the algorithm developed by the author. As a result, we obtain the Cartesian coordinates X and Y of the corners of building structures, the accuracy and reliability of determining which is on a very high level.

Keywords: RTN-RTK-RTX GNSS, measurement control, corner of a building, vectors translation, line-line intersection.

1. INTRODUCTION

Field details subject to measurement, which can not be implemented directly, require the use of a variety of indirect solutions. Any such offset affects the accuracy and reliability of determining the coordinates of a given point. Corners of buildings belong to such terrain details which require auxiliary algorithms to determine the X and Y coordinates while using classical surveying methods (tacheometry) and RTN GNSS technology. The importance of this problem is even greater because buildings belong to the details of the first-order accuracy. Not only the mean position error of that point should be relatively small (up to the maximum of ± 0.10m), but also the reliability of the X and Y coordinates obtained from the measurement should be unquestionable. Determining a large-size structure (a building) in the field in the RTN GNSS mode is a task which requires the knowledge of not only the surveying technique itself but, most importantly, the predicted values of the final measurement results, depending on the use of indirect methods. The analysis of the results of the surveys performed on the corners of buildings, depending on the use of indirect methods, has been presented in Krzyżek (2014 and 2015). In these articles the author has presented both the theoretical aspects regarding the accuracy of the surveys in real-time, as well as he proposed innovative algorithms to improve the reliability of determining horizontal coordinates of the structures subject to surveying. Studies on the surveys performed on structures in the urban environment (corners of buildings) have also been presented in (Klukas et al., 2010). In their deliberations, the authors present the results of a close integration of the GPS survey and the ultrawideband (UWB) observations to determine the accurate position of several points of the corners of a building structure. The solutions at the sub-meter level are maintained by a close conjugation of the observations.

The few studies aimed at increasing the accuracy and reliability of determining the position of the corners of buildings in the RTN GNSS mode are now supported by modern surveying systems. One of the support systems is a network of ASG EUPOS reference stations or other commercial stations. Regardless of the choice of the reference station network, an important factor in their selection is a possibility to use multi-constellation during the surveys, and so the GPS and GLONASS systems. The advantages of such a solution have been presented, inter alia, in (Angrisano et al., 2013) and (Pirti et al., 2013), emphasizing the major benefits of the positioning integrity.

Using a network of reference stations requires to ensure a continuous communication between the user's receiver and the stations. Sometimes, however, for various reasons, there are temporary interruptions in obtaining adjustments between a reference station and a rover. A modern solution supporting real-time surveying, RTX (*Real Time Extended*) is prepared to handle such problems, as it ensures a continuation of the measurement in real time up to a few minutes after the user's receiver has lost communication with the network of reference stations. The principle of operation of the RTX technology using the xFill function has been presented in (Leonardo et al., 2011), and Krzyżek (2013) as well as Krzyżek (2014) present the research results in the context of using RTX to establish measurement control networks. Using the RTX technology with the xFill function for surveying corners of buildings can be performed at the stage of determining the position of the so-called base points. In fact, this procedure aims to establish measurement control points for the local needs to meet the accuracy requirements of the regulation MIA (2011).

Taking into account the author's previous study results as well as other related publications on the subject, and the possibilities provided by modern information technology today, in this paper the author has presented an innovative solution (the algorithm), which models via iteration the coordinates of the corners of a building obtained from real time surveys. This algorithm is to identify possibilities of using the innovative solution which significantly increases the reliability of determining the horizontal coordinates of the corners of buildings in the RTN GNSS mode and can be used in the topographic surveys of buildings, just like tacheometry, which is the classical surveying method.

2. ALGORITHM FOR MODELING THE POSITION OF THE CORNERS OF BUILDING STRUCTURES USING VECTORS TRANSLATION METHOD

The algorithm developed by the author has been presented hereunder. It results in the significant improvement in the reliability of determining the X and Y coordinates of the corners of building structures using RTN GNSS technology and the indirect method of *line-line intersection*. This algorithm, called the method of ***vectors translation***, performs the modeling of the coordinates of the same corner of a building obtained in several variants. This modeling partially uses the *half-angle* method (Krzyżek, 2015), and translation of the vectors resulting from the application of the *half-angle* method relative to the points (corners of buildings) calculated from the classical method of *line-line intersection*. The whole algorithm also takes into account the two-times theoretical assumption of right angles at the corners of a building, the method of *vector addition* (the method which was examined in detail by the author), and a few additional details presented below. Figure 1 (a general scheme) and the detailed drawings 1.1-1.9 depict a graphical presentation of the algorithm implementation. These drawings illustrate sample images of the structure (a building) whose geometry and the mutual relationship of individual elements (wall faces, right angles, etc.) have been strongly deformed in order to visualize the described algorithm better.

Vectors translation algorithm:

1. Having measured the base points *A, B, C, D,* etc., and having performed the calculations from the line-line intersection, we obtain the X, Y coordinates of the corners of buildings *1-6* (Fig. 1).

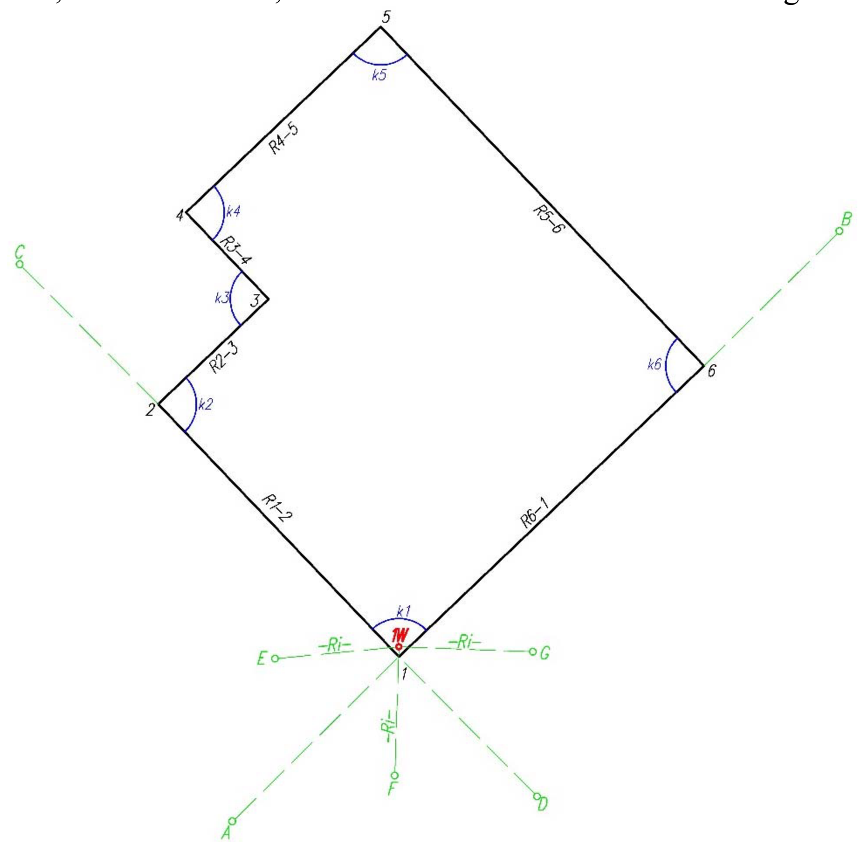

Fig. 1. Measurement of the base points and determining the X, Y coordinates of the corners of building structures

2. For one of the points (*corner No. 1*) we perform measurement using distance-distance intersection (base points E, F, G) with the supernumerary observations and we calculate the X, Y coordinates of this point by the method of least squares to obtain the new position of the point number *1W*. We adopt the position of this point as a constant in relation to other determined corners of the building. When selecting the proper point (corner), it is advised to consider the best possible measurement conditions for the base points determined in the RTN GNSS mode, from which distance-distance intersection is implemented (Fig. 1).

3. Basing on the obtained coordinates of the points *1-6* from the line-line intersection, we calculate the horizontal angles of the corners of the building k_1-k_6 (Fig. 1).

4. We calculate the angular deviation at each corner of the building $f_k = 100^g - k_i$

5. We calculate the angular adjustment for each corner of the building $v_k = \frac{f_k}{2}$

6. From the corner of the building No. *1,* resulting from intersection of the lines *A, B, C, D*, we calculate the X and Y coordinates of the next point – No. *2'*- using the X and Y coordinates of the positions (the corner *1*) and the polar coordinates, i.e. the angle equal to $v_k = \frac{f_k}{2}$ together with the length equal to the tie distance R_{1-2} (Fig. 1.1).

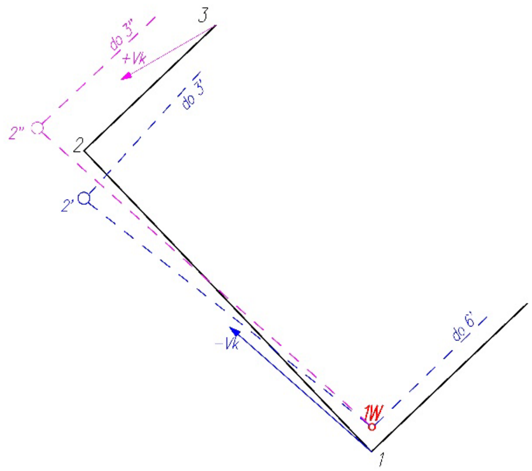

Fig. 1.1. Determining the X, Y coordinates of the corner of the building 2' in the main direction, and of the 2" in the counter direction

Going in the clockwise direction, we perform the same operations on each successive point of the positions (*2, 3, 4, 5*) to obtain a new position (the X, Y coordinates) of the corners of the building - *3', 4', 5', 6'* (Fig. 1.2-1.5) (Krzyżek, 2015). Here, attention should be paid to the sign of the angular adjustment v_{ki} - when laying off the angle from the reference line to a neighboring point (corner) in the clockwise direction - the "+" sign is assumed, and in the opposite direction - the "-". According to the assumption contained in the section *2)*, when we reach the starting point *1W*, we do not calculate its new coordinates.

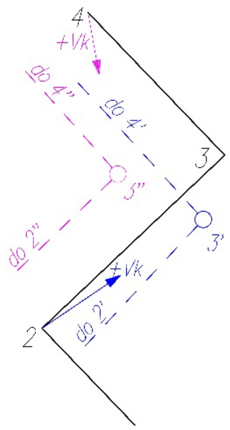

Fig. 1.2.

Determining the X, Y coordinates of the corner of the building 3' in the main direction, and of the 3" in the counter direction

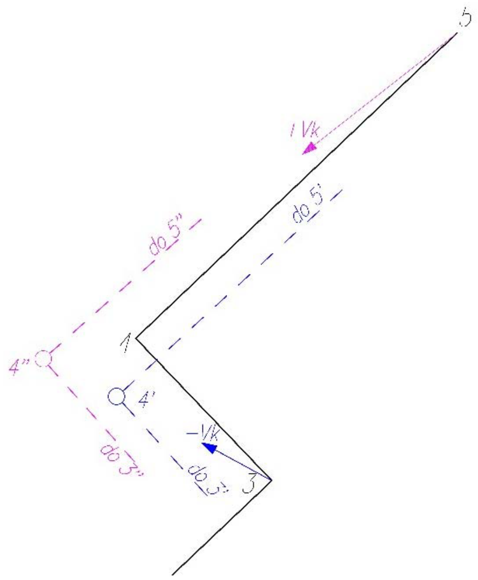

Fig. 1.3.

Determining the X, Y coordinates of the corner of the building 4' in the main direction, and of the 4" in the counter direction

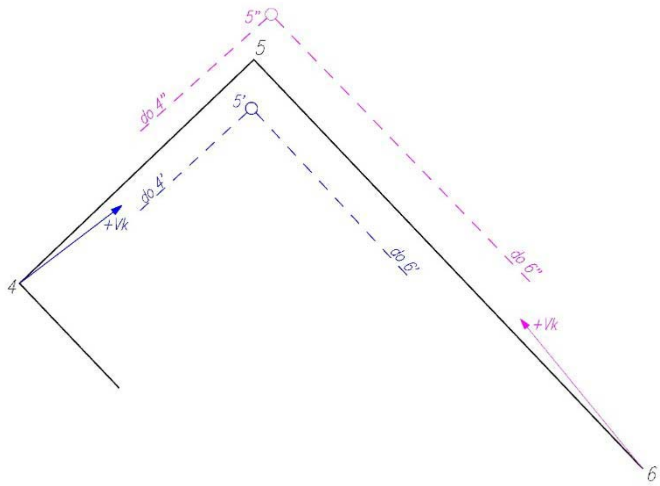

Fig. 1.4. Determining the X, Y coordinates of the corner of the building 5' in the main direction, and of the 5" in the counter direction

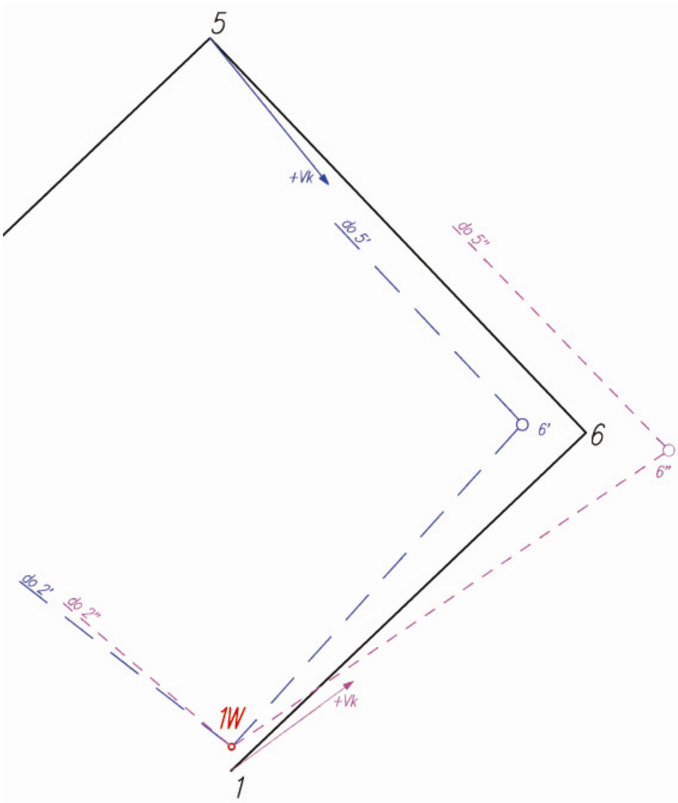

Fig. 1.5. Determining the X, Y coordinates of the corner of the building 6' in the main direction, and of the 6" in the counter direction

7. The operations listed in the section *6)* we repeat in the counterclockwise direction to obtain the new coordinates at every corner of the building. The newly established points we denote as 6", 5", 4", 3", 2" (Fig. 1.1-1.5).

8. In this way, we obtained a double location of the building - from the main direction (*2′-6′*) and from the counter direction (*6″-2″*). For each of these positions, the common link is the point (corner) No. *1W* adopted as a constant (the assumption contained in the section *2*).

9. In this way, we also obtained the dL_i vectors (from $dL_{2'}$ to $dL_{6'}$ – Fig.1.6 and from $dL_{6''}$ to $dL_{2''}$ – Fig.1.7) from the newly established points from the main direction (*2′-6′*) – Fig. 1.6, and from the counter direction (*6″- 2″)* – Fig.1.7, relative to the points *2-6* (corners) obtained from intersection of the lines *A, B, C, D*, etc. For each of the newly established corners of the building (independently

for the main direction (no.') and for the counter direction (no."), we perform a translation by the dL_i vector obtained from the previous point, in order to obtain a new position of the corner of the building once again: for the main direction (the points *3'T-6'T*) and for the counter direction (the points *5''T-2''T*). The operations described in this section do not apply to the first point in the main direction (*2'*) or in the counter direction (*6''*), because the vector of the point which was previous to them, or the fixed point *1W,* has been adopted at zero.

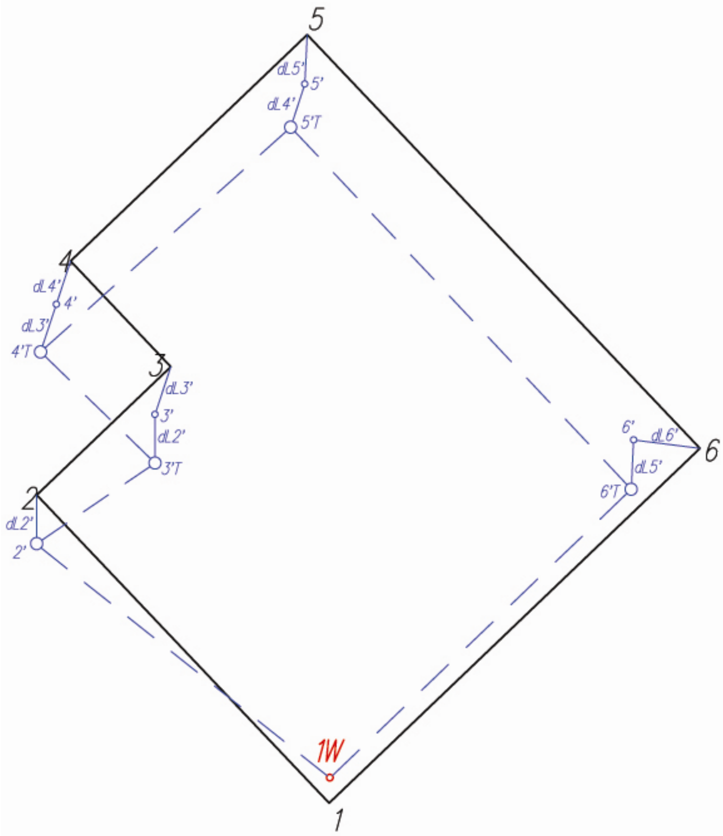

Fig. 1.6. Translation of the newly obtained points in the main directions, of the length of the vector *dLi*

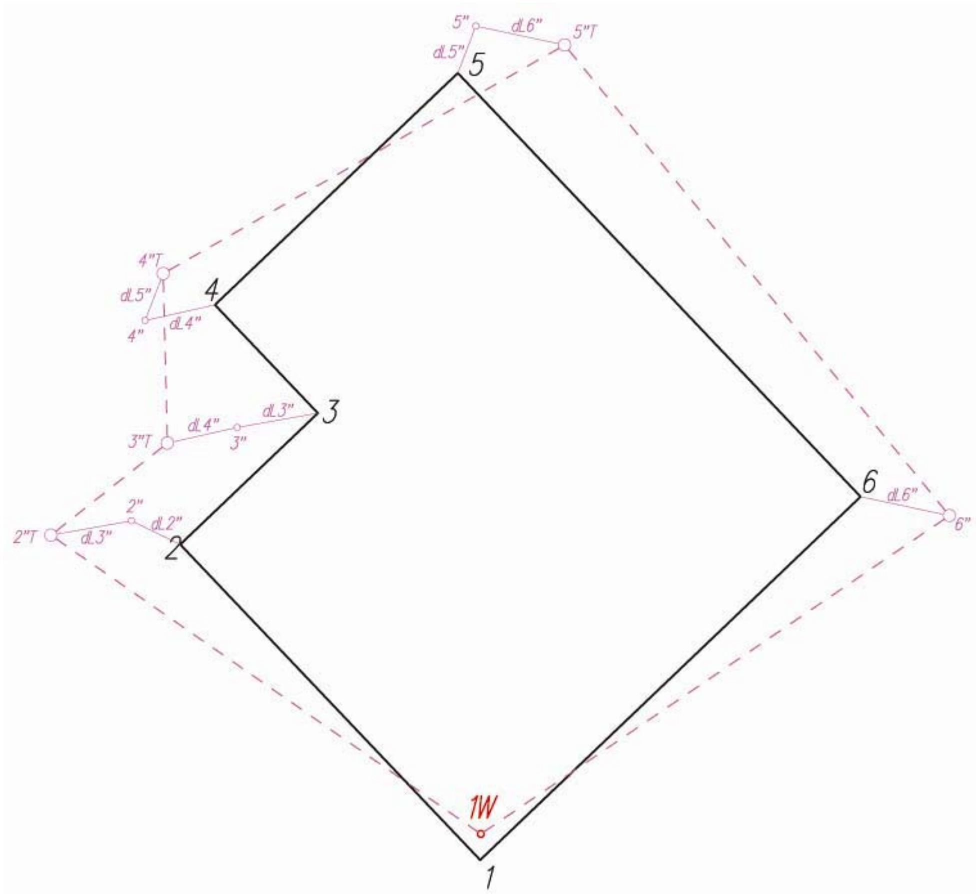

Fig. 1.7. Translation of the newly obtained points in the counter directions, of the length of the vector *dLi*

10. The next step is to calculate the X, Y coordinates of the line-line intersection, which are the two adjacent faces of the building walls. In any case, the base line is generated by two points (corners) set in the main direction, e.g. the straight line *2'-3'T*, whereas the cutting line is generated by two corners of the adjacent wall determined in the counter direction, for example *4''T-3'' T* (Fig. 1.8). In this way, we obtain yet another position, the third one of the building corners marked with the symbol *Z* (e.g. *3Z*). At this stage, we have three positions of each of the building corners: from the main direction (*2*' and from *3'T* to *6'T*), from the counter direction (*6''* and from *5''T* to *2''T*), and from the line-line intersection (from *2Z* to *6Z*). These steps do not need to be performed for the first point in the main direction (2') or in the counter direction (*6''*) - the substantiation in section 13 of the algorithm.

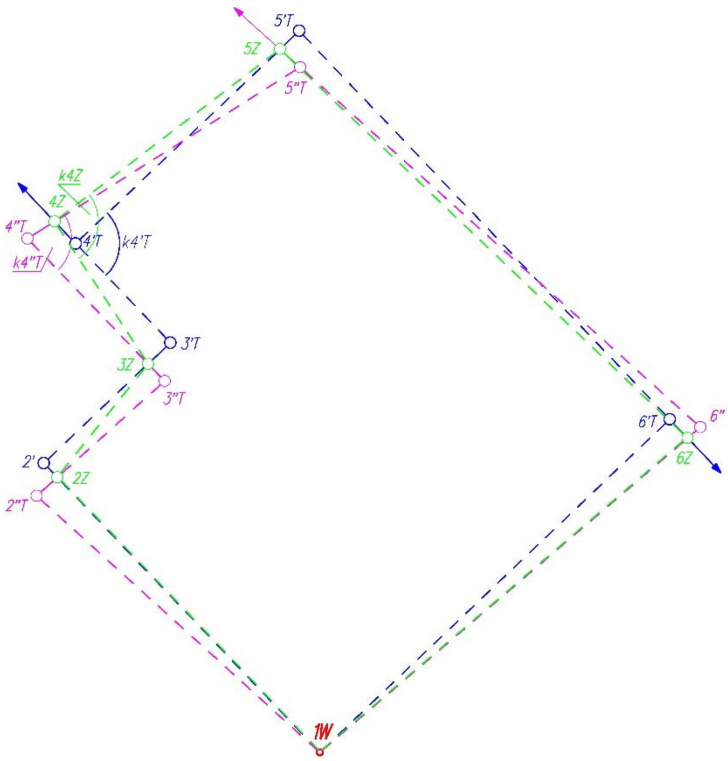

Fig. 1.8. Determining the position of the corners of building structures (*No.Z*) using line-line intersection (wall faces of the building), generated in the main and counter directions

11. In order to obtain an unambiguous position of the point (the most likely X and Y coordinates), we perform calculations to determine the weighted average of the corner position.

12. From the obtained coordinates of the newly established points we again calculate the horizontal angles of the corners *k ', k'', kZ,* independently for each variant *a, b, c* (Fig. 1.8):

a. from the primary direction: *1W-2'-3'T-4'T-5'T-6'T-1W*,
b. from the counter direction: *1W-6''-5''T-4''T-3''T-2''T-1W*,
c. form the line-line intersection of the building wall faces: *1W-2Z-3Z-4Z-5Z-6Z-1W.*

13. We calculate a half of the angular deviation at each corner of the building $f_k = \frac{100^g - k_i}{2}$ for each of the three - *a, b, c* - or two - *a, b* – variants, which will be an element of the weight. For the first point, which is after the fixed point in the main and in the counter direction, we calculate the angular deviation f_k taking into account the fixed point *1W* rather than the point *1*, resulting from the intersection of the straight lines *A, B, C, D*. We exclude a total of five corners of the building from these calculations: the fixed point *1W*, and the two points closest to the fixed point in the main direction (*2', 3'T)* and in the counter direction (*6'' and 5''T)*. The substantiation for excluding these points from the calculations of the line-line intersection is associated with a lack of a complete homogeneity of the translations described in section 9: for the points closest to the fixed point in both directions *2'* and *6''* - no vector translation from the previous (fixed) point (*1W*), and for the base or cutting straight lines, drawn from the point which is the first one after the fixed point (*1W*), that is *2'* and *6''*, to the following points, which are respectively *3'T* and *5''T* - no vector translation from the fixed point *1W* for the initial points, i.e. *2'* and *6''* of the base or cutting straight line. For this example (Fig. 1.8), the deviation f_k is calculated only for the point *4Z.*

14. We calculate the weights for each of the three - *a, b, c* - or two - *a, b* – variants: $p_{i_{ta}} = \frac{1}{f^2}$

15. We calculate the X and Y coordinates of the corners of the building as a weighted average. In order to better understand the selection of the appropriate coordinates of the points for weighing, the formulas (1-5) have been presented for a specific example from Fig. 1.8.

$$X_{2W} = \frac{X_{2'} \cdot p_{2'} + X_{2''T} \cdot p_{2''T}}{p_{2'} + p_{2''T}} \qquad Y_{2W} = \frac{Y_{2'} \cdot p_{2'} + Y_{2''T} \cdot p_{2''T}}{p_{2'} + p_{2''T}} \tag{1}$$

$$X_{3W} = \frac{X_{3'T} \cdot p_{3'T} + X_{3''T} \cdot p_{3''T}}{p_{3'T} + p_{3''T}} \qquad Y_{3W} = \frac{Y_{3'T} \cdot p_{3'T} + Y_{3''T} \cdot p_{3''T}}{p_{3'T} + p_{3''T}} \tag{2}$$

$$X_{4W} = \frac{X_{4'T} \cdot p_{4'T} + X_{4''T} \cdot p_{4''T} + X_{4Z} \cdot p_{4Z}}{p_{4'T} + p_{4''T} + p_{4Z}} \qquad Y_{4W} = \frac{Y_{4'T} \cdot p_{4'T} + Y_{4''T} \cdot p_{4''T} + Y_{4Z} \cdot p_{4Z}}{p_{4'T} + p_{4''T} + p_{4Z}} \tag{3}$$

$$X_{5W} = \frac{X_{5'T} \cdot p_{5'T} + X_{5''T} \cdot p_{5''T}}{p_{5'T} + p_{5''T}} \qquad Y_{5W} = \frac{Y_{5'T} \cdot p_{5'T} + Y_{5''T} \cdot p_{5''T}}{p_{5'T} + p_{5''T}} \tag{4}$$

$$X_{6W} = \frac{X_{6'T} \cdot p_{6'T} + X_{6''} \cdot p_{6''}}{p_{6'T} + p_{6''}} \qquad Y_{6W} = \frac{Y_{6'T} \cdot p_{6'T} + Y_{6''} \cdot p_{6''}}{p_{6'T} + p_{6''}} \tag{5}$$

16. We calculate tie distances from the newly determined coordinates of the corners of the building and compare them with the measurement data (tie distances from the survey). In this way we obtain differences in the tie distances – *dR* on individual sides (walls) of the building.

17. In order to eliminate the differences *dR* in the tie distances*,* we implement the method of *vector addition* to obtain the new, and also the final, position (the X and Y coordinates) of the points of the building corners - *1W-6W* (Fig. 1.9). It should be noted, however, that the difference *dR* in the tie distances on the first wall of the building in the main direction and in the counter direction relevant to the fixed point is modified only in the main and counter direction, respectively.

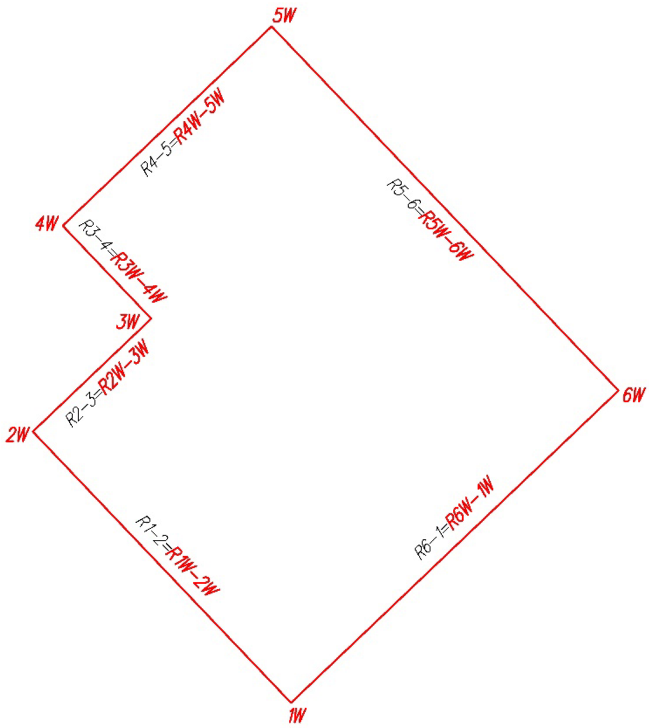

Fig. 1.9. The ultimate position of the corners of the building, taking into account the method of vector translation

18. As a result of the presented algorithm, we determine the final coordinates X, Y, of the corners of the building *1W-6W,* and full compliance of the tie distances calculated from the coordinates relative to the actual dimensions (Fig. 1.9).

In order to verify the presented algorithm in field conditions, a research experiment was performed on a real object. However, due to the large volume of the performed accuracy analysis of the obtained measurement results, the test sample material will be presented in the next article. Also, the mathematical analysis of the formulas of this innovative solution in the context of the obtained final results will be demonstrated in the next series of articles.

3. SUMMARY AND CONCLUSIONS

At the beginning, modern surveying techniques frequently encounter various difficulties associated with the full use of their abilities to carry out surveying tasks. Surveys of buildings in the RTN GNSS mode are examples of such techniques, where due to the obscured horizon, there are significant limitations on the possibility to fully use the fast and modern surveying technology in real-time. In this article, the author suggested that with some modification of the method of *line-line intersection*, used for the direct results of the RTN GNSS surveys, it is possible to determine a reliable position of the corners of buildings. This modification involves the use of the novel algorithm, called ***vectors translation*** to the obtained measurement results. According to the author, the use of the algorithm makes it possible to achieve the level of accuracy and reliability of the determined coordinates of the corners of buildings similar to the classical method of surveying - tacheometry. Some research aspects, such as comparison of tie distances, suggest even greater accuracy of this innovative solution than tacheometry. It should be strongly emphasized though, that the use of ***vectors translation*** method in surveys of buildings in real-time is neither an ideal method nor an option that would replace tacheometry. It cannot be ruled out that, in addition to very good results of its use, there may also occur the values that differ from those adopted as a reference. Although they might only be marginal, one should be aware that such errors are possible. Besides the human factors that affect less accurate measurement results, also field conditions, such as the horizon obscured by trees or buildings, have a direct impact on the reliability of the final results. An additional drawback of this innovative solution is its limited applicability to selected buildings. Due to the assumption of right angles at corner points of buildings, only this type of structures can be determined using ***vectors translation*** algorithm. Where this condition is not met, other surveying methods or integrated surveying should be performed. It is well known that there is no measurement technique which yields the real values of the measured elements. Therefore, the innovative solution of ***vectors translation*** in the RTN GNSS surveys may be considered as yet another factor to improve modern surveying techniques, meeting the accuracy requirements of the regulation MIA (2011). The use of the algorithm developed by the author in measuring the corners of buildings in the RTN GNSS mode leads to the determination of the coordinates of the points close to the true values.

It should also be emphasized that this algorithm does not directly affect the measurement results, but it only models the results already obtained. Thus, an important factor is the method of its use. The idea is to see the advantages of the innovative solution already in the field, and not after analyzing the obtained data in the office. Consequently, an application compatible with the systems of controllers and GNSS receivers should be created. Then, having surveyed the base points in the RTN GNSS mode, and having completed several technical steps, on the controller screen we would obtain coordinates of the corners of buildings, modified by the ***vectors translation*** algorithm and adopted as the most reliable ones.

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