ARTIFICIAL SATELLITES, Vol. 49, No. 4 – 2014
DOI: 10.2478/arsa-2014-0017

POSITIONAL ACCURACY OF GPS SATELLITE ALMANAC

Lihua Ma, Shangli Zhou
National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China
e-mail: mlh@nao.cas.cn

ABSTRACT. How to accelerate signal acquisition and shorten starting time are key problems in the Global Positioning System (GPS). GPS satellite almanac plays an important role in signal reception period. Almanac accuracy directly affects the speed of GPS signal acquisition, the start time of the receiver, and even the system performance to some extent. Combined with precise ephemeris products released by the International GNSS Service (IGS), the authors analyse GPS satellite almanac from the first day to the third day in the 1805th GPS week (from August 11 to 13, 2014 in the Gregorian calendar). The results show that mean of position errors in three-dimensional coordinate system varies from about 1 kilometer to 3 kilometers, which can satisfy the needs of common users.

Keywords: Global Positioning System (GPS), satellite almanac, precise ephemeris

1. INTRODUCTION

The Global Positioning System (GPS) is a space-based satellite navigation system that provides location and time information in all weather conditions, wherever on or near the Earth where there is an unobstructed line of sight to four or more GPS satellites. GPS signals contain ranging signals, which are used to measure the distance between the receiver and the satellite, and navigation message. The navigation message include ephemeris data, which is used to calculate the position of each satellite in orbit, and information about time and status of the entire space constellation, and are referred to as satellite almanac (Warren & Raquet, 2003; Kaplan & Hegarty, 2006; Hofmann-Wellenhof et al., 2008; Ma & Wang, 2013). In satellite navigation, how to accelerate signal acquisition and shorten starting time of the receiver have always been two of the most significant problems. Actually GPS satellite almanac plays an important role in the signal reception period (Urschl et al., 2007; Chen et al., 2008). The almanac ,which is mainly used to solve the approximate position of every satellite, to aid the receiver to capture satellite signals quickly, is contained in the subframes of the GPS navigation message. The positional accuracy has direct impacts on the speed of signal acquisition, the starting time of the receiver, and even the system performance to some extent. In this work, combined with precise ephemeris products published by the International GNSS Service (IGS), the authors analyse positional accuracy of the GPS satellite almanac from the first day to third day in GPS week 1805 (from August 11 to 13, 2014 in Gregorian calendar).

2. DESCRIPTION OF SATELLITE ALMANAC

The fourth and fifth subframes of navigation message contain components of the satellite almanac. Each frame contains only 1/25th of the total almanac. A receiver must process data of 25 whole frames to retrieve the entire almanac message which is 15000 bit in size. At this rate, 12.5 minutes are required to receive the entire almanac from a single satellite. In case there is a disruption in the ability to make updates regularly, satellite data is updated every 24 hours in general with up to 60 days' data loaded. Typically the updates contain ephemerides, with new almanacs uploaded less frequently. The control segment guarantees that during normal operations new almanacs are uploaded at least every 6 days. Actually, with the SEM and Yuma formats, the CelesTrak website (http://www. celestrak.com) publishes daily GPS satellite almanac series released by the 2nd Space Operations Squadron and US Coast Guard Navigation Center (USCG NAVCEN). Table. 1 gives the interpretation of every parameter in Yuma almanac[1].

Table1. GPS Yuma almanac parameters

Parameter	Interpretation
ID	Pseudo-Random Number (PRN) of the Space Vehicle Number (SVN)
Health	Indication of the health of satellite
Eccentricity	Eccentricity of orbit (e)
Time of Applicability (s)	The number of seconds in the orbit when the almanac was generated (t_{oa})
Orbital Inclination (rad)	Orbital inclination (i)
Rate of Right Ascen (r/s)	Rate of change in the measurement of the angle of right ascension as defined in the Right Ascension mnemonic ($\dot{\Omega}$)
SQRT(A) (m 1/2)	This is defined as the measurement from the center of the orbit to either the point of apogee or the point of perigee ($\sqrt{a}$)
Right Ascen at Week (rad)	Right Ascension is an angular measurement from the vernal equinox (Ω_0)
Argument of Perigee (rad)	An angular measurement along the orbital path measured from the ascending node to the point of perigee, measured in the direction of the SV's motion (ω)
Mean Anom (rad)	Angle traveled past the longitude of ascending node (M_0)
Af0(s)	SV clock bias in seconds (a_{f0})
Af1(s/s):	SV clock drift in seconds per seconds (a_{f1})
week	GPS week (0000-1023), every 7 days since 1999 August 22.

3. ACCURACY ANALYSIS OF GPS ALMANAC

Table. 2 provides the algorithm by which the receiver computes the position vector of the satellite $(x_k,\ y_k,\ z_k)$ in the Earth-Centered Earth-Fixed (ECEF) coordinate system from the satellite almanac.

[1] https://celestrak.com/GPS/almanac/Yuma/definition.asp

Table 2. Computation of a GPS satellite's ECEF position vector

$a = \left(\sqrt{a}\right)^2$	Orbital semi-major axis
$n = \sqrt{\frac{\mu}{a^3}}$	Mean motion, $\mu = 398600.5 \times 10^8\ m^3/s^2$
$t_k = t - t_{oa} + (CurWeek - Week) \cdot 604800.0$	Time from the almanac epoch
$M_k = M_0 + n \cdot t_k$	Mean anomaly
$M_k = E_k - e \cdot \sin E_k$	Eccentric anomaly (solved iteratively for E_k)
$\upsilon_k = \tan^{-1} \frac{\sqrt{1-e^2} \sin E_k / (1 - e\cos E_k)}{(\cos E_k - e)/(1 - e\cos E_k)}$	True anomaly
$\phi_k = \upsilon_k + \omega$	Argument of latitude
$r_k = a(1 - e \cdot \cos E_k)$	Orbital radius
$\begin{cases} x_p = r_k \cdot \cos\phi_k \\ y_p = r_k \cdot \sin\phi_k \end{cases}$	The x- and y-coordinate in the orbital plane
$\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e) \cdot t_k - \dot{\Omega}_e \cdot t_{oa}$	Corrected longitude of node
$\begin{cases} x_k = x_p \cdot \cos\Omega_k - y_p \cdot \cos i \cdot \sin\Omega_k \\ y_k = x_p \cdot \sin\Omega_k + y_p \cdot \cos i \cdot \cos\Omega_k \\ z_k = y_p \cdot \sin i \end{cases}$	The x-, y- and z-coordinate in the ECEF frame

Where, $CurWeek$ and t is the current time in the form of weeks and seconds; $\dot{\Omega}_e$ is the Earth rotation angular velocity. The meaning of some parameters is given in Table. 1.

According to Table. 2, the spatial position of every GPS satellite can be computed. In this work the precise ephemeris products is released by the IGS for comparison. The IGS provides the IGS Rapid (IGR), Ultra-Rapid (IGU) and Final (IGS) products. Here the IGS Final product with high precision is adopted as the spatial positional reference. For the interval of the IGS ephemeris is 15 minutes, the position of each GPS satellite is calculated every 15 minutes. Meanwhile the process can effectively avoid the error from the interpolation technique (Gao et al., 2012; Wang et al., 2014).

Firstly, the ECEF x-, y-, and z-coordinate are computed from the GPS satellite almanac which is from the first day to the third day in the 1805th GPS week. then subtract the ECEF x-, y-, and z-coordinate of the IGS precise ephemeris from the above results respectively. The mean value (mean) and standard deviation (std) of the difference (Δx, Δy and Δz) are given as follows. Furthermore, the positional error (Δr) with three-dimensional coordinates is analysed in mean and std methods. According to the bulletin of USCG NAVCEN (United States Coast Guard Navigation Center), the satellite with PRN03 is unavailable starting from August 1, 2014. Tables. 3-5 display the results of the first, second and third day in the GPS week 1805, respectively. The average value (MEAN) of all available satellites in one day is given in the last row of every table.

Table 3. Positional error of GPS satellite almanac in the first day of GPS week 1805

ID	Δx(m)		Δy(m)		Δz(m)		Δr(m)	
	mean	std	mean	std	mean	std	mean	std
1	225.63	2452.14	198.58	1476.41	-702.12	1931.86	3340.91	1111.38
2	75.98	1326.15	-177.42	1894.87	723.46	1538.48	2704.45	944.24
4	296.09	2066.36	43.05	1340.54	-749.78	1784.77	2987.48	944.40
5	-115.46	1862.77	291.54	2456.93	-470.02	2575.88	3852.68	1210.30
6	216.27	1623.60	-100.13	1515.05	-140.63	1700.13	2749.04	512.36
7	193.42	1795.63	61.35	892.95	816.77	1809.94	2666.50	910.07
8	207.42	1771.52	24.46	812.85	1177.75	1626.70	2629.75	950.04
9	-34.89	1854.38	259.35	1817.40	161.79	2839.47	3785.68	646.26
10	-203.09	1858.06	80.99	2349.95	-846.32	2544.05	3816.88	1223.51
11	178.95	2082.53	288.68	1963.88	-870.16	1732.17	3233.66	1228.53
12	-62.57	1605.33	126.82	1766.92	-92.73	1882.49	2879.90	943.74
13	-118.86	1648.50	-4.69	1197.92	78.03	2194.78	2870.43	814.85
14	105.01	1528.82	-162.21	2044.86	443.36	2560.13	3536.51	819.07
15	89.67	1972.55	224.92	1933.45	16.24	3189.97	4144.93	709.44
16	78.37	1881.46	-184.37	2092.87	-106.19	2262.98	3474.24	946.38
17	70.32	965.34	33.60	742.81	68.76	1596.51	1895.48	642.45
18	176.30	1994.44	141.44	1709.45	308.49	2532.68	3528.70	937.81
19	-14.85	1057.65	99.37	635.57	-636.50	1404.42	1820.56	751.82
20	-38.05	1649.46	-213.40	1559.30	-482.82	1898.64	2830.48	971.64
21	-319.95	2291.79	-51.40	1884.79	549.83	1866.78	3425.97	916.74
22	178.09	1783.37	-29.45	1896.18	95.01	2295.57	3370.03	781.63
23	-115.55	1425.10	-93.19	1555.79	347.24	2501.30	3186.60	768.55
24	-29.98	1498.56	-105.08	1117.21	146.05	1772.36	2443.60	796.67
25	-23.34	1534.16	136.88	1689.73	33.23	1790.06	2761.46	853.12
26	-8.74	1975.28	181.75	1182.62	-128.46	2418.40	3222.70	839.05
27	-85.58	893.93	42.62	1054.57	-388.18	1652.45	2038.34	777.66
28	-98.75	1659.69	-60.67	1854.36	304.39	1993.79	3085.21	810.51
29	-150.21	863.32	-117.26	829.61	-336.36	1035.23	1499.84	618.94
30	201.53	1609.94	1.16	896.78	399.42	1769.08	2485.45	696.82
31	-71.54	1099.44	-3.55	1094.53	180.63	1624.87	2127.86	713.89
32	50.21	2124.43	-268.98	1842.18	-955.55	2455.72	3643.96	1231.88
MEAN	27.48	1669.54	21.44	1519.43	-34.04	2025.21	2969.01	871.73

Table 4. Positional error of GPS satellite almanac in the second day of GPS week 1805

ID	Δx(m)		Δy(m)		Δz(m)		Δr(m)	
	mean	std	mean	std	mean	std	mean	std
1	169.63	1250.38	198.69	557.73	-363.29	1162.80	1730.58	634.46
2	70.40	808.54	-145.45	978.54	598.89	1184.64	1773.18	475.64
4	248.97	1040.65	85.51	607.84	-367.07	1274.64	1730.77	506.16
5	-66.88	820.33	141.13	1278.90	-425.15	944.86	1698.83	701.56
6	192.41	826.52	-56.14	733.05	48.70	1175.64	1557.99	439.03
7	126.59	1083.70	80.63	657.19	449.67	1298.66	1789.44	533.00
8	152.02	1046.45	63.03	641.22	692.44	1141.30	1756.65	449.85
9	-4.97	853.77	213.21	938.76	276.70	1550.14	1914.64	657.31
10	-75.25	1009.52	18.74	1209.42	-706.21	973.68	1836.81	728.40
11	117.94	1103.72	230.03	796.59	-497.70	1375.05	1925.70	560.35
12	-70.48	952.35	95.88	1153.38	-17.11	861.37	1615.50	597.09
13	-130.89	843.51	-43.38	849.47	108.06	1323.56	1665.04	643.74
14	113.61	817.78	-150.83	1040.23	425.63	1380.12	1836.01	684.29
15	54.47	915.02	186.51	983.91	125.01	1694.64	2065.72	647.28
16	87.80	1083.43	-128.91	1188.21	-6.57	951.14	1751.76	643.43
17	-24.50	779.85	33.01	710.71	-23.32	1139.58	1491.78	406.60
18	60.65	969.69	104.80	940.01	109.42	958.12	1566.15	538.68
19	-11.40	732.96	0.95	788.28	-492.78	1135.33	1525.24	585.04
20	53.11	1033.00	-103.60	867.25	-449.35	697.80	1492.92	521.15
21	-266.83	1109.18	-88.59	852.24	427.26	1302.41	1902.51	509.48
22	84.15	880.67	22.70	1004.45	-64.93	843.94	1513.17	441.46
23	-98.66	715.79	-116.96	986.54	276.78	1429.46	1791.46	622.03
24	2.63	890.83	-75.01	783.95	54.06	1377.39	1693.73	644.26
25	-37.87	914.49	114.14	1096.17	82.10	789.29	1538.02	540.25
26	-52.55	1135.97	164.16	529.01	71.48	1428.97	1794.25	627.84
27	-35.43	705.58	-42.63	1039.17	-350.14	1230.92	1686.58	587.69
28	-47.70	987.95	-69.84	1109.76	352.75	813.68	1641.05	530.34
29	-79.80	707.15	-73.49	585.01	-384.52	940.27	1287.47	461.19
30	142.96	960.61	36.20	519.87	174.86	1251.04	1591.96	499.97
31	-60.36	780.94	-38.82	775.04	105.13	1265.66	1578.28	558.27
32	90.06	1236.17	-134.04	1053.65	-849.77	991.25	1966.52	684.85
MEAN	22.70	935.37	16.83	879.21	-19.97	1157.66	1700.31	569.70

Table 5. Positional error of GPS satellite almanac in the third day of GPS week 1805

ID	Δx(m)		Δy(m)		Δz(m)		Δr(m)	
	mean	std	mean	std	mean	std	mean	std
1	83.75	533.23	138.02	674.79	-102.53	1118.07	1369.75	361.58
2	29.50	790.68	-83.09	439.57	375.82	1168.53	1464.01	410.55
4	140.48	541.60	89.81	595.41	-86.37	1173.71	1382.37	360.59
5	-14.80	448.61	33.57	567.78	-282.27	362.08	800.87	297.43
6	108.27	626.88	-8.02	483.82	123.15	1099.82	1313.65	346.48
7	54.11	573.35	75.47	555.06	164.07	1014.16	1245.52	365.53
8	78.41	553.19	76.13	552.87	278.38	939.31	1201.67	353.77
9	26.04	378.91	106.92	640.84	261.73	446.49	850.08	323.75
10	24.75	609.19	-7.69	508.14	-456.86	367.76	907.67	378.96
11	58.11	730.57	134.01	515.90	-227.66	1232.91	1491.65	381.41
12	-66.77	465.78	58.66	625.13	27.87	238.33	776.92	251.69
13	-95.47	340.20	-59.63	614.07	94.61	439.87	785.19	291.79
14	84.31	562.18	-112.19	407.84	310.24	379.01	799.50	311.86
15	25.18	481.53	111.24	696.13	113.77	460.08	935.15	265.80
16	78.83	562.92	-67.36	578.98	59.31	275.57	825.00	233.82
17	-77.19	547.76	14.15	644.77	-63.08	466.85	950.73	174.83
18	-16.42	448.81	66.91	541.64	7.53	401.90	781.44	210.03
19	10.76	624.04	-62.30	694.05	-343.02	562.70	1095.39	313.38
20	92.98	589.70	-13.51	428.17	-298.48	405.11	846.09	265.99
21	-158.29	647.56	-108.74	734.13	238.84	1291.48	1575.08	462.77
22	14.69	391.98	54.12	544.65	-111.54	402.91	761.05	207.58
23	-50.98	490.35	-101.28	617.94	168.45	430.89	874.57	276.96
24	29.18	508.47	-42.82	572.20	34.10	1124.86	1292.93	407.31
25	-46.22	452.85	84.65	616.85	88.72	239.17	769.65	248.06
26	-64.52	506.83	103.04	455.05	146.47	458.62	788.06	288.66
27	15.72	651.54	-82.64	741.34	-259.20	548.13	1119.84	286.94
28	4.43	598.21	-54.86	503.54	316.87	220.04	834.66	245.70
29	-24.44	666.58	-36.79	571.46	-344.10	513.15	1039.53	252.67
30	71.23	523.80	50.09	472.85	23.88	1000.62	1172.24	344.71
31	-36.67	469.83	-69.63	502.85	59.38	974.93	1136.40	359.02
32	101.07	656.25	-17.82	571.11	-600.94	297.27	1038.13	362.94
MEAN	15.48	547.53	8.66	569.97	-9.12	646.91	1039.51	311.05

From Tables. 3-5, the std value of the ECEF x-, y- and z-coordinate is remarkably larger than the corresponding mean value in every day. On the average, the amplitude of mean value and dispersion degree of every ECEF coordinate are close to each other. Here, as for the three-dimensional position error, the std value is obviously smaller than the mean value. The MEAN of the three-dimensional position errors' mean value varies from 1 kilometer to 3 kilometers. Specific values relate to GPS satellite and analysis day.

4. SUMMARY

Combined with precise ephemeris product released by the IGS from the first day to the third day in GPS week 1805, the results computed by GPS satellite almanac show that the MEAN of the three-dimensional position errors of all available satellites varies from about 1 kilometer to 3 kilometers, can meet with the need of common user.

Acknowledgments The GPS satellite almanac and precise ephemeris are from the websites: http://www.celestrak.com and ftp://cddis.gsfc.nasa.gov, respectively. The work was carried out under the support of the Key Research Program (Grant No. KJCX2-EW-J01), the Key Research Program of Technology Innovation (KGFZD-125-14-005-2) and the Knowledge Innovation Program (Grant No. KGCX2-EW-407-1) of the Chinese Academy of Sciences.

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Received: 2014-09-28,

Reviewed: 2014-11-25, by Han Yanben,

Accepted: 2014-12-04.