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ROSBOROUGH FORMULATION IN SATELLITE GRAVITY GRADIOMETRY

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ABSTRACT. Following the launch of CHAMP, a new era was born in the gravity field determination from satellite observations. Many methods have been proposed and applied for the recovery of the Earth's gravity field from the observations of the satellite missions CHAMP, GRACE and GOCE. This paper deals with the Rosborough formulation in gravity field modelling. This formulation is derived from the transformation of time-wise representation from the orbital into the spherical coordinate systems. Base functions of the Rosborough formulation depend on the type of the functional of the gravity field and the inclination of the orbit. Unlike the space-wise approach, the Rosborough approach can easily deal with both isotropic and non-isotropic functionals. The proposed formulation is implemented on the GOCE data in order to show its efficiency. Numerical results show that the Rosborough formulation is a powerful and efficient tool in the case of GOCE gradiometry data processing.

Keywords: Rosborough formulation, gravity field recovery, GOCE, gravitational gradient tensor.

1. INTRODUCTION

The satellite missions CHAMP [CHAllenging Minisatellite Payload; (Reigber et al., 1999)], GRACE [Gravity Recovery And Climate Experiment; (Tapley et al., 2004)] and GOCE [Gravity field and steady-state Ocean Circulation Explorer; (ESA, 1999)] have opened a new era in the global gravity field study of the Earth. These dedicated gravity field missions have made significant improvements in our knowledge of the geopotential field of the Earth. Besides, benefits from these gravity missions will develop many fields of studies such as geodesy, oceanography, geophysics and hydrology. As an example, the GOCE mission has a great impact on studies of the interior structure of the Earth, ocean circulation and unification of the height systems (Rummel et al., 2002). Huge number of unknowns and observations, especially for the latter two missions, makes the estimation of the potential coefficients a difficult task. Many authors have developed different representations of the gravity field, and consequently different data processing strategies, to solve such a huge system of equations (see e.g. Reguzzoni and Tselfes, 2009; Pail et al., 2010; Xu et al., 2008; Sneeuw, 2000).

Basically, there has been a distinction between two different approaches, namely the time-wise and the space-wise. In the former approach, observations are considered as a time-series along the orbit of the satellite, while the latter regards the observations as a function of the spatial coordinates (Rummel et al., 1993). The Rosborough approach is a combination of both approaches.

In this study, the Rosborough formulation and the gravity field recovery based on this formulation is discussed, especially for GOCE data processing. Rosborough (1986) expressed the orbital perturbations of a satellite in the spherical coordinates. The transformation that was carried out by Rosborough was the reverse transformation of that of Kaula (1966), which means, he again expressed the along-orbit observables in the spherical coordinates. The representation derived from this transformation is different from that of the spherical harmonic series. Contrary to the base functions of the spherical harmonic series, i.e., spherical harmonic functions, the new base functions depend on the functional of the geopotential field and also on the characteristics of the orbit of the satellite. The point about this transformation is that, due to the expansion of sine and cosine functions in the binomial series, cumbersome expressions are derived and their implementation leads to a very slow algorithm. Sneeuw (2003) proposed an improved and fast algorithm for the Rosborough approach based on the complex notation and without any need for the expansion in the binomial series.

In this paper, the Rosborough formulation is first introduced. Then, the gravity field recovery from this formulation is discussed in section 3. In section 4, the application of this approach for GOCE data processing is considered. In section 5, the results of the numerical experiments and accuracy evaluation of the Rosborough formulation are presented and the achievements are discussed. The summary and conclusions are expressed in the last section.

2. ROSBOROUGH FORMULATION

A functional of the gravity field (l) can be expressed in complex notation as (Sneeuw, 2000):

$$l(r,u,I,\Lambda)=\sum_{n=0}^{N}\sum_{m=-n}^{n}\sum_{k=-n}^{n}\bar{K}_{nm}\,H_{nmk}^{l}e^{i\psi_{km}}, \qquad (1)$$

Where $\bar{K}_{nm}$ are complex normalized spherical harmonic coefficients , $\psi_{km}=ku+m\Lambda$ and transfer functions H_{nmk}^{l} are stated in terms of the normalized inclination function ($\bar{F}_{nmk}(I)$):

$$H_{nmk}^{l}=\frac{GM}{R}\left(\frac{R}{r}\right)^{n+1}h_{nmk}^{l}\,\bar{F}_{nmk}(I). \qquad (2)$$

and h_{nmk}^{l} is the transfer coefficients of the functional l.

Rosborough formulation is obtained from expressing ψ_{km} in terms of spherical coordinates (ϕ,λ) (Sneeuw, 2003). According to the definition of ψ_{km} (Kaula, 1966) :

$$e^{i(ku+m\Lambda)}=e^{i\left(ku+m(\lambda-(\alpha-\Omega))\right)}=e^{i(ku-m(\alpha-\Omega))}\;e^{im\lambda}\,. \qquad (3)$$

where u is the argument of the latitude, Λ is the longitude of the ascending node and α is the right ascension of the satellite. Now we transform the variables u and $(\alpha-\Omega)$ into expressions in latitude (ϕ) and inclination(I). From spherical trigonometry, it is:

$$\sin u=\frac{\sin\phi}{\sin I}\,. \qquad (4)$$

$$\cos u=\cos\phi\cos(\alpha-\Omega), \qquad (5)$$

$$\sin u=\frac{\cos\phi\sin(\alpha-\Omega)}{\cos I}, \qquad (6)$$

From the former, the following equation is obtained (Sneeuw, 2003):

$$\cos u = \pm \frac{1}{\sin I} \sqrt{\sin^2 I - \sin^2 \phi}. \tag{7}$$

where the signs + and − are used for the ascending and descending tracks respectively.

Substituting last three equations in Eq. (3) gives:

$$e^{i\psi_{km}} = e^{i(ku - m(\alpha - \Omega))} . e^{im\lambda} = \Psi^{\pm}_{mk}(\phi, I)\, e^{im\lambda}, \tag{8}$$

where

$$\Psi^{\pm}_{mk}(\phi, I) = \frac{1}{\sin^{m+k} I} \left(\pm\sqrt{\sin^2 I - \sin^2 \phi} + i \sin\phi\right)^k . \left(\frac{\pm\sqrt{\sin^2 I - \sin^2 \phi} - i \sin\phi \cos I}{\cos\phi}\right)^m . \tag{9}$$

Eq. (9) is a stable expression which leads to a fast algorithm. Also, the risk of overflow for polar regions and for large m has been taken into account. The difference between this expression and the one that was given by Rosborough (1986) shows the advantage of using complex notation instead of real one. Substitution of Eq. (8) in Eq. (1) results in Rosborough formulation:

$$l^{\pm}(\phi, \lambda, I) = \sum_{n=0}^{N} \sum_{m=-n}^{n} \sum_{k=-n}^{n} \bar{K}_{nm}\, H^{l}_{nmk} \Psi^{\pm}_{mk}(\phi, I)\, e^{im\lambda}. \tag{10}$$

new base functions $Q^{\pm}_{nm}(\phi, I)$ of this representation are defined using $\Psi^{\pm}_{mk}(\phi, I)$. These new base functions depend on the functional of the gravity field, inclination of the orbit and are also different for ascending and descending tracks:

$$l^{\pm}(\phi, \lambda) = \sum_{n=0}^{N} \sum_{m=-n}^{n} \bar{K}_{nm}\, Q^{\pm}_{mk}(\phi, I)\, e^{im\lambda}, \tag{11}$$

$$Q^{\pm}_{mk}(\phi, I) = \sum_{k=-n}^{n} H^{l}_{nmk}\, \Psi^{\pm}_{mk}(\phi, I) . \tag{12}$$

It should be noted that by applying constant radous approximation, the dependence of $Q^{\pm}_{mk}$ on variable r vanishes. Eq. (11) is similar to the spherical harmonic series but with the new base functions $Q^{\pm}_{mk}(\phi, I)$ that are not orthogonal in general. Therefore the data processing strategy in Rosborough approach is very similar to that of the space-wise approach (Sneeuw, 2003).

3. GEOPOTENTIAL RECOVERY IN ROSBOROUGH APPROACH

3.1. DATA PROCESSING STRATEGY

Data processing strategy in the Rosborough approach is similar to that of the space-wise approach. In comparison with the space-wise approach, in the Rosborough method the data are reduced on two spheres; one for the data on ascending tracks (l^+) and one for the data on descending tracks (l^-). One can use either ascending or descending observations to recover the spherical harmonic coefficients. Alternatively, both ascending and descending observations are usually combined into spatially mean and variable parts:

$$\begin{cases} l^M = \dfrac{l^+ + l^-}{2}; & \text{spatially mean} \\ l^V = \dfrac{l^+ - l^-}{2}; & \text{spatially variable} \end{cases} . \tag{13}$$

l^M is used in the recovery since it contains the dominant part of the signal while l^V is the nuisance part and is almost zero except for the polar regions.

The Rosborough approach has at least two advantages in comparison with the space-wise approach, although data processing strategy of these two approaches is almost similar. Firstly, the Rosborough formulation is derived from the time-wise representation; therefore, any isotropic or non-isotropic functional can be dealt with in this formulation, just as the time-wise approach. Secondly, distinguishing between the ascending and descending observations can be very useful and beneficial in some cases, for instance in the case of the gradiometry observation equation of the GRACE satellites (Sharifi, 2006). In Sharifi (2006), it was shown that the along-track gradiometry observation equation of the GRACE satellites is different for the ascending and descending tracks. He solved the non-isotropic gradiometry equation in an iterative strategy based on the Banach's fixed point theorem using the space-wise approach. Also, it was assumed there that the resulting errors due to the differences in the number of the ascending and descending observations are hopefully negligible. It is obvious that this non-isotropic observation equation can be treated easily and efficiently by the Rosborough approach.

The data processing scheme in the Rosborough approach is represented in Fig 1.

3.2. BLOCK-DIAGONAL LEAST SQUARES SOLUTION

Linear system of equations constructed by Eq. (10) cannot be inverted using ordinary PCs due to the huge number of observations and unknowns. After rearranging the parameters according to spherical harmonic order (m), the system turns out to become block-diagonal:

$$\sum_{n=0}^{N}\sum_{m=-n}^{n}\sum_{k=-n}^{n} = \sum_{m=-N}^{N}\sum_{n=|m|}^{m}\sum_{k=-n}^{n} . \tag{14}$$

swapping summations in Rosborough formulation according to Eq. (14) and carrying out inner summations over k and n, results in a single summation formula with the new coefficients which are called *lumped coefficients:*

$$l^{\mathrm{M}}(\phi,\lambda) = \sum_{\mathrm{m}=-\mathrm{N}}^{\mathrm{N}} \mathrm{D}_{\mathrm{m}}^{l}(\phi) e^{im\lambda}, \tag{15}$$

with:

$$\mathrm{D}_{\mathrm{m}}^{l}(\phi,\lambda) = \sum_{n=|m|}^{\mathrm{N}} \mathrm{Q}_{\mathrm{nm}}^{\mathrm{M}}(\phi,\lambda)\bar{K}_{nm}, \tag{16}$$

$$\mathrm{Q}_{\mathrm{nm}}^{\mathrm{M}}(\phi,\lambda) = \sum_{k=-n}^{\mathrm{n}} H_{nmk}^{l}\, \Psi_{mk}^{M}(\phi, I). \tag{17}$$

where l^{M} is the spatially mean contribution of the signal and $\Psi_{mk}^{M}(\phi, I)$ is the average of the $\Psi_{mk}^{+}(\phi, I)$ and $\Psi_{mk}^{-}(\phi, I)$. Eq. (15) can be interpreted as the *finite Fourier series*. The *lumped coefficients* $\mathrm{D}_{\mathrm{m}}^{l}(\phi)$ are computed using a *Fast Fourier Transform* (FFT) along each parallel. These lumped coefficients are the quasi-observations of the linear system of equations represented by Eq. (16).

To estimate the geopotential coefficients, the corresponding linear system of equations is inverted by block-wise least squares adjustment. This inversion can be written in matrix notation as:

$$\boldsymbol{D} = \boldsymbol{Q}\,\boldsymbol{K}\,, \tag{18}$$

where $\boldsymbol{K}$ is the vector of geopotential coefficients according to each spherical harmonic order and $\boldsymbol{D}$ is the vector of lumped coefficients:

$$\boldsymbol{D} = \begin{pmatrix} D_m^l(\phi_1) \\ D_m^l(\phi_2) \\ \vdots \\ D_m^l(\phi_{N_\theta}) \end{pmatrix}, \tag{19}$$

$$\boldsymbol{K} = \begin{pmatrix} \bar{K}_{|m|\,m} \\ \bar{K}_{|m+1|\,m} \\ \vdots \\ \bar{K}_{N\,m} \end{pmatrix}, \tag{20}$$

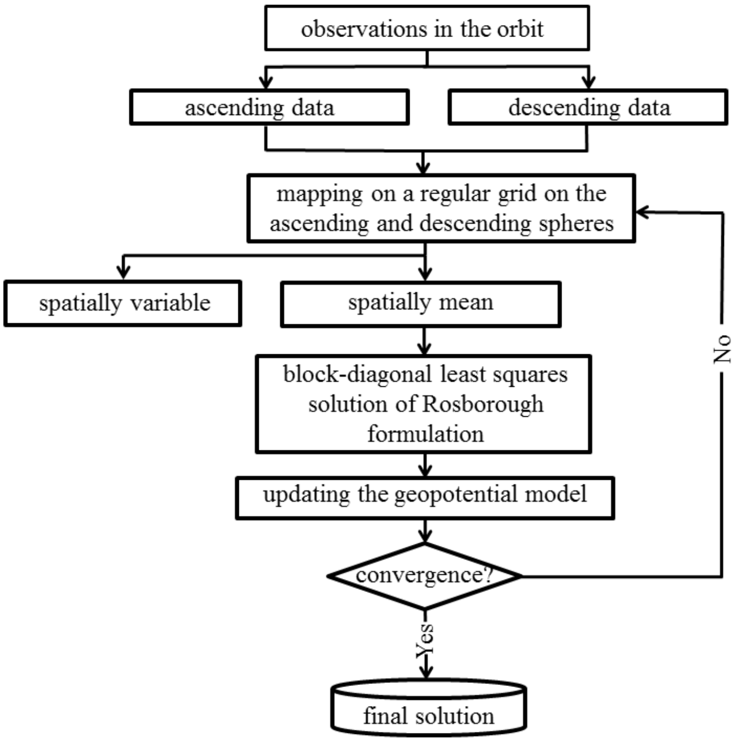

Fig. 1. Flowchart of the data processing strategy in the Rosborough approach.

$$\boldsymbol{Q} = \begin{pmatrix} Q^M_{|m|m}(\phi_1, I) & Q^M_{|m+1|m}(\phi_1, I) & \dots & Q^M_{N\,m}(\phi_1, I) \\ Q^M_{|m|m}(\phi_2, I) & Q^M_{|m+1|m}(\phi_2, I) & \dots & Q^M_{N\,m}(\phi_2, I) \\ \vdots & \vdots & \ddots & \vdots \\ Q^M_{|m|m}(\phi_{N_\theta}, I) & Q^M_{|m+1|m}(\phi_{N_\theta}, I) & \dots & Q^M_{N\,m}(\phi_{N_\theta}, I) \end{pmatrix}, \tag{21}$$

where N_θ is the number of latitudinal grid points. Each array of the matrix $\boldsymbol{Q}$ is computed by Eq. (17) as follows:

$$Q^M_{nm}(\phi, I) = \sum_{k=-n}^{n} \mathrm{H}^l_{\mathrm{nmk}} \Psi^{\mathrm{M}}_{\mathrm{mk}}(\phi, I) \;\Rightarrow\; Q^M_{n\,m} = \langle \boldsymbol{H}^l, \boldsymbol{\Psi}^{\mathrm{M}} \rangle, \tag{22}$$

where $\langle\, ,\, \rangle$ indicates the inner product of two vectors. Vectors $\boldsymbol{H}$ and $\boldsymbol{\Psi}^{\mathrm{M}}$ are given as:

$$\boldsymbol{H}^l = \left(\mathrm{H}^l_{\mathrm{nm},-\mathrm{n}} \quad \mathrm{H}^l_{\mathrm{nm},-\mathrm{n}+1} \quad \cdots \quad \mathrm{H}^l_{\mathrm{nm}0} \quad \cdots \quad \mathrm{H}^l_{\mathrm{nm,n}-1} \quad \mathrm{H}^l_{\mathrm{nm,n}}\right), \tag{23}$$

$$\boldsymbol{\Psi}^{\mathrm{M}} = \left(\Psi^{\mathrm{M}}_{\mathrm{m},-\mathrm{n}} \quad \Psi^{\mathrm{M}}_{\mathrm{m},-\mathrm{n}+1} \quad \cdots \quad \Psi^{\mathrm{M}}_{\mathrm{m}0} \quad \cdots \quad \Psi^{\mathrm{M}}_{\mathrm{m,n}-1} \quad \Psi^{\mathrm{M}}_{\mathrm{m,n}}\right), \tag{24}$$

Finally, the geopotential coefficients of the spherical harmonic of order m are obtained by the method of least squares:

$$\widehat{\boldsymbol{K}} = (\boldsymbol{Q}^T\boldsymbol{Q})^{-1}\boldsymbol{Q}^T\boldsymbol{D}\,. \tag{25}$$

4. APPLICATION OF THE ROSBOROUGH FORMULATION IN GOCE GRADIOMETRY DATA PROCESSING

GOCE is the first mission that benefits from gradiometry technique. Gravitational gradiometry is the measurement of the second derivatives of the gravitational potential. A second-order tensor field with 3×3 components, known as gravitational gradient tensor (GGT), is formed from these gravitational gradients. The three-axis GOCE gradiometer measures the diagonal components V_{xx}, V_{yy}, V_{zz} and one of the off-diagonal components V_{xz} with high precision, while the other off-diagonal components, V_{xy} and V_{yz}, are of low precision (Rummel et al., 2011). In the framework of High Level Processing Facility (HPF) for GOCE mission, three different approaches, namely time-wise, space-wise and direct approaches, are implemented to determine the gravity field from GOCE orbit and gradiometry data (Pail et al., 2011).

In order to apply the Rosborough method, to recover the geopotential coefficients form the four high-precision GGT components, the transfer functions, H^l_{nmk}, of these functionals should be obtained. Based on the data processing strategy in the Rosborough approach (see Fig. 1), the procedure to recover the potential coefficients from gravitational gradients is as follows: first, the ascending and descending observations are separated. Then, they are mapped onto two spheres separately. After mapping along-track data, the spatially mean contribution of each gravitational gradient component is computed. Finally, by substitution of the equivalent transfer functions of these GGT components from Table 1 in Eq. (25), the spherical harmonic coefficients are retrieved.

Note that the only isotropic component is the second radial derivative, V_{zz}. In the next section, some numerical experiments are presented in order to show the performance of the Rosborough approach for GOCE gradiometry data analysis.

5. NUMERICAL EXPERIMENTS

In this section, some numerical examples are constructed in order to evaluate the formulation derived before. It has to be noted that the main emphasis of this paper is to show the performance and efficiency of the Rosborough formulation in satellite gravity gradiometry, and as a proof-of-concept study, this approach is implemented on the GOCE gradiometry data. Therefore, other problems such as mapping along-track data on grid points, polar gap problem, filtering the coloured noise of the gradiometry data, etc. is not investigated here. This study deals with the second part of the Rosborough approach, i.e. the estimation of the harmonic coefficients from gridded data on global grids on two spheres. To this end, gravitational gradients of disturbing potential are simulated in Local Orbital Reference Frame (LORF) using non-singular expression (Petrovskaya and Vershkov, 2006).

Table 1. Transfer functions of the diagonal components of the gravitational gradient tensor (from Sneeuw, 2000).

Gravitational gradient	Transfer function
V_{xx}	$H_{nmk}^{xx} = \frac{\mathrm{GM}}{\mathrm{R}^3}\left(\frac{R}{r}\right)^{n+3}\left[-\left(k^2+n+1\right)\right]\bar{F}_{nmk}(I)$
V_{yy}	$H_{nmk}^{yy} = \frac{\mathrm{GM}}{\mathrm{R}^3}\left(\frac{R}{r}\right)^{n+3}\left[k^2-(n+1)^2\right]\bar{F}_{nmk}(I)$
V_{zz}	$H_{nmk}^{zz} = \frac{\mathrm{GM}}{\mathrm{R}^3}\left(\frac{R}{r}\right)^{n+3}\left[(n+1)(n+2)\right]\bar{F}_{nmk}(I)$

Geographic mapping of the spatially mean and variable contributions is illustrated in Fig. 2. Due to the sun-synchronous orbit of the GOCE satellite with the inclination of about 96.6°, orientation of the x- and y-axis in LORF, which respectively coincides with the along-track and cross-track directions, is different for the ascending and descending tracks. Therefore, T_{xx} and T_{yy} observations are not identical in the two tracks. The maximum variations of the orientation occur at high latitudes. Hence, the spatially variable component has its maximum magnitude in polar regions. The second radial derivative T_{zz} is isotropic. It means that the radial direction is identical for the ascending and descending tracks. This fact can be seen in Fig. 2 where the spatially variable component of T_{zz} is zero.

In order to assess the accuracy of the Rosborough formulation, the point-wise gravity gradients are simulated in LORF with egm96 geopotential model (Lemoine et al., 1998) to degree and order 200 on a 200×400 global grid and on two separate spheres, one for the ascending data and the other for the descending data, both at 250 Km altitude. The data in this *ideal* simulation are noise-free and both the synthesis as the analysis, is truncated at degree 200, so there is no aliasing error. Therefore, an error in the estimation of harmonic coefficients from diagonal components of the GGT is due to the error in the Rosborough formulation and block-wise least squares solution. Since the study of polar gap problem is not the topic of this paper, polar gaps are filled in with computed values from egm96 model. However, In the case of GOCE real data, polar gap problem can be dealt with an iterative solution, where in each iteration step the polar gaps are filled in with pseudo observations generated using the gravity field solution of the previous step. The results in terms of error degree r.m.s (root mean square) show that the coefficient reconstruction is performed perfectly (see Fig. 3). In all the three cases the difference between the reference and estimated coefficients is less than 10^{-5} for all degrees, although the geopotential coefficients determined from T_{xx} and T_{yy} components show a slight increase of error for higher degrees. At this step, the space-wise approach was applied on T_{zz} gridded data of the ideal simulation to estimate the potential coefficients. A comparison between the solutions of the two approaches from the T_{zz} component shows that the accuracy of the potential coefficients estimated by the Rosborough approach is about two orders of magnitude higher than that of the space-wise approach. In order to gain a better view of the accuracy of the coefficients, Fig. 4 shows the error distribution coefficient by coefficient. Similar to the error degree r.m.s, Fig. 4 reveals that all of the potential coefficients are retrieved with high precision, which shows the efficiency of the proposed approach for GOCE gradiometry data analysis.

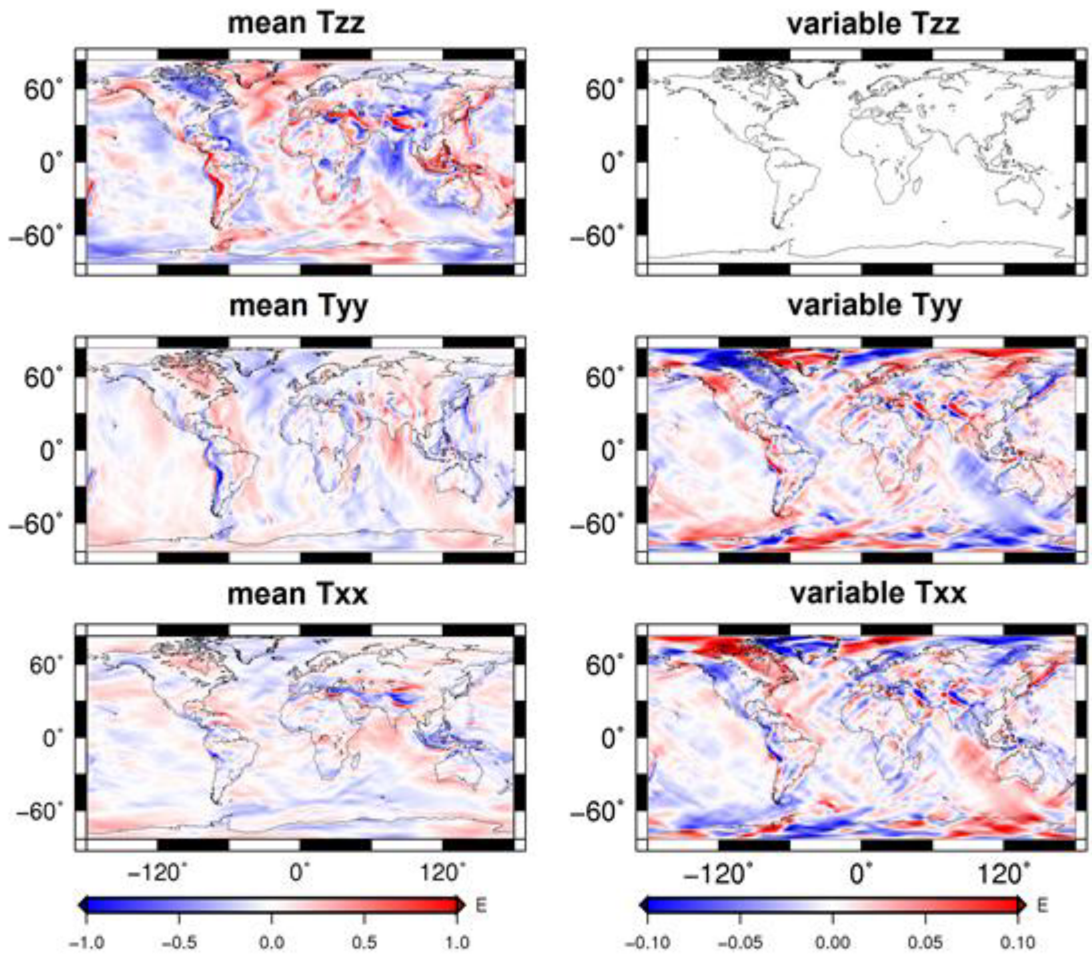

Fig. 2. Spatially mean (left) and variable (right) contributions of the GOCE gravitational gradients in the local orbital reference frame. Note the different scales.

From the numerical experiment presented above, one can easily conclude one of the advantages of the Rosborough approach compared to the space-wise approach, i.e. dealing with all the functionals of gravity field, no matter they are isotropic or non-isotropic. Contrary to the space-wise approach, which cannot deal with non-isotropic functionals in a direct and simple way, Rosborough formulation seems to be a more appropriate strategy for GOCE data processing.

Consequently, another problem that arises from this disadvantage of the space-wise approach is in the gridding of along-track data. Generally, two techniques are available for the gridding: a local least squares gridding, e.g. using radial base functions or rational functions (see e.g. Sharifi, 2006) and a collocation solution. In the former technique all the observations to be interpolated must carry the same spatial information while in the latter technique, different functionals may be used as observations and any other functional can be predicted; for example, estimating the second radial derivative on grid points from a set of observed data $\{T_{xx}, T_{yy}, T_{zz}\}$ (Reguzzoni and Tselfes, 2009). It is clear that in the case of the space-wise approach for GOCE gradiometry data processing, collocation has to be used with its numerical heaviness, while the Rosborough method can benefit from both techniques. In addition, one has to notice that in the space-wise approach, where all gravitational gradients are combined from the second radial derivative on grid points using collocation, the available gradiometry data are significantly reduced. These drawbacks of the space-wise approach

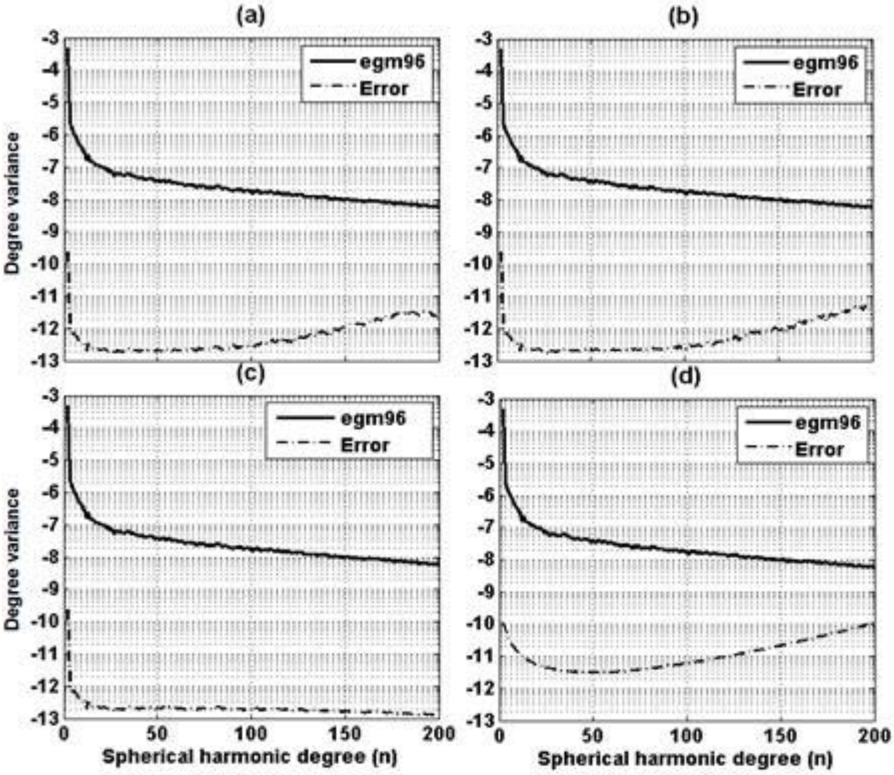

Fig. 3. Error degree r.m.s in log_{10} scale: (a) solution of the Rosborough approach for T_{xx} component; (b) solution of the Rosborough approach for T_{yy} component; (c) solution of the Rosborough approach for T_{zz} component; (d) solution of the space-wise approach for T_{zz} component. The solid and dotted curve represents the egm96 model degree variances and error degree r.m.s respectively.

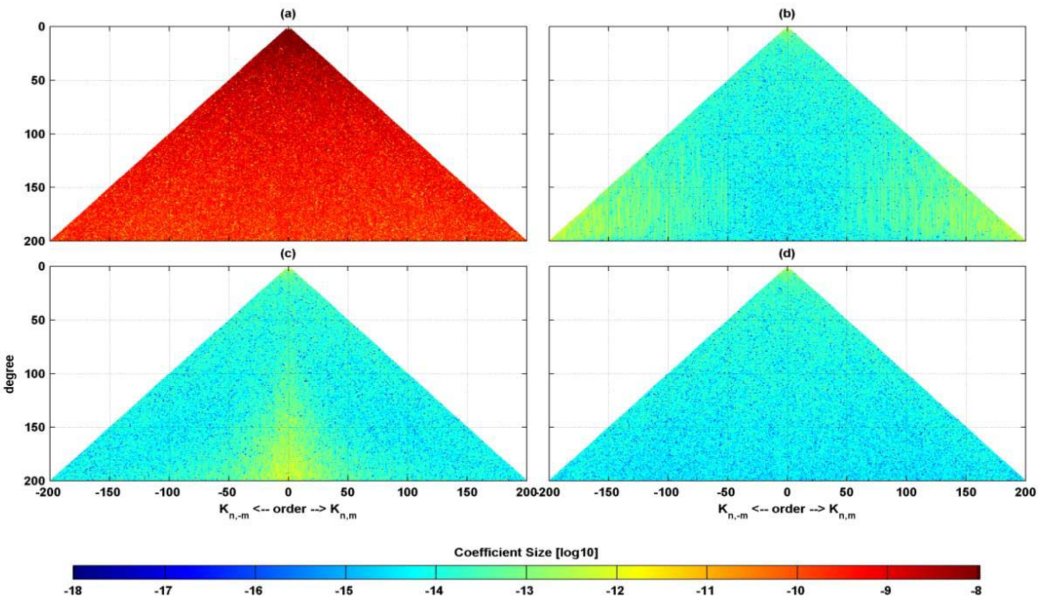

Fig. 4. Harmonic coefficients in log_{10} scale: (a) egm96 coefficients; (b) deviations of the coefficient estimates from egm96, using the T_{xx} component; (c) deviations of the coefficients, using the T_{yy} component; (d) deviations of the coefficients, using the T_{zz} component.

motivated us to use Rosborough formulation instead of the space-wise method for gravity field modeling form GOCE gradiometry observations.

Now let us consider a more realistic simulation with noisy data. A Gaussian white-noise with an error r.m.s of $1\,mE$, which is realistic after gridding along-track data, was added to gridded gradiometry data. Since the gravitational gradients were synthesized up to degree 250 in this simulation, aliasing was also taken into account. In this new simulation, the geopotential recovery is performed using all the diagonal components of the GGT simultaneously. It means that the linear system of equations is formed using three gravitational gradients T_{xx}, T_{yy} and T_{zz}, and a combined solution to degree 200 is obtained. The results in terms of error degree r.m.s show that the accuracy of potential coefficients is about 3 orders of magnitude less when noisy and aliased data are used instead of the ideal data (see Fig. 5). Fig. 6 shows the geoid errors on a regular $1^\circ \times 1^\circ$ grid using both the estimated coefficients and those of the egm96. As can be seen, geoid errors are less than 5 cm for most points on the Earth. The error does not exceed 25 cm.

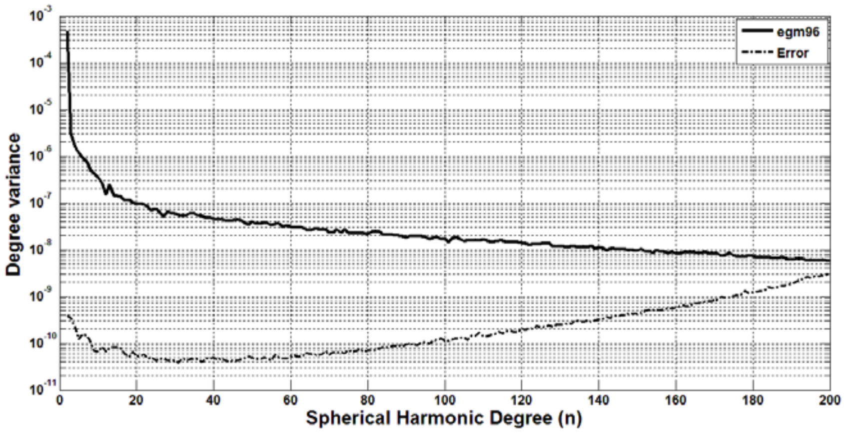

Fig. 5. Error degree r.m.s of the combined solution using noisy and aliased data. The solid and dotted curve represents the egm96 model degree variances and error degree r.m.s respectively.

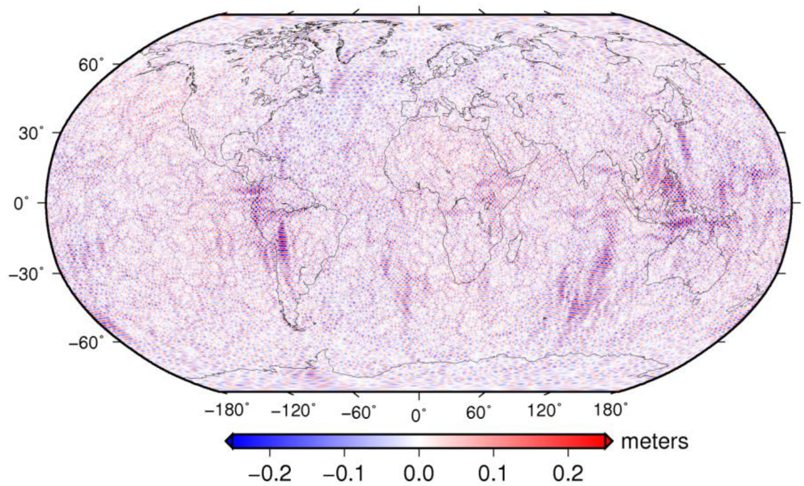

Fig. 6. Geoid undulation differences between the combined solution using noisy and aliased data and egm96 model.

It should be noted that the results of this simulation do not contain the error of projection of the along-orbit observations on a spherical grid. However, this error can be mitigated by an iterative procedure, where in each iteration step, the along-orbit observations are reduced using the data synthesized from the solution of the previous iteration. From the numerical results of the simulation, it can be concluded that the use of Rosborough formulation seems to allow for the reconstruction of the geopotential field up to degree 200 and above with sufficient accuracy.

6. CONCLUSION

Determination of the Earth's gravitational field with unprecedented accuracy is now a reality, thanks to the GRACE and GOCE missions. Many approaches have been proposed by authors for gravity field recovery from dedicated gravity field missions. In this contribution, Rosborough approach was investigated. After derivation of the Rosborough formulation, application of this representation for GOCE data processing was presented in order to show the efficiency of this approach. Numerical results of the simulation show that the potential coefficients can be recovered up to degree 200 and above with sufficient accuracy from this formulation.

Since the Rosborough formulation is the transformation of the time-wise approach from orbital into spherical coordinates, it has characteristics of both time-wise and space-wise methods. For instance, similar to the space-wise approach, it interpolates the along-orbit observations to homogenize the data and makes the numerical solution feasible, or just as the time-wise approach, both the isotropic and non-isotropic observables can be dealt with easily in this approach. In conclusion, it seems that the Rosborough approach has the potential to be used in gravity field recovery, especially for GOCE data processing.

As a future research, Rosborough formulation will be applied to the non-isotropic gradiometry observation equation derived from GRACE satellites.

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