DYNAMICAL MODELING AND EXCITATION RECONSTRUCTION AS FUNDAMENTAL OF EARTH ROTATION PREDICTION

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ABSTRACT. Though pure mathematical approximations such as regression models and neural networks show good results in Earth rotation forecasting, dynamical modeling remains the only base for the physically meaningful prediction. That assumes the knowledge of cause-effect relationships and physical model of the rotating Earth. Excitation reconstruction from the observed Earth orientation parameters (EOP) is a crucial stage, needed for comparison with known causes, such as tidal forcing, atmospheric (AAM), oceanic (OAM) angular momentum changes, and uncovering unknown ones. We demonstrate different approaches, which can be used to avoid ill-conditionality and amplification of noises during the inversion. We present amplitude and phase studies of the model and reconstructed excitations of Chandler wobble. We found out, that modulation of Chandler excitation is synchronous with 18-yr tidal effects in the Earth's rotation rate changes. The results of the study can be used for excitation and EOP forecast. The key issues of the EOP prediction are discussed.

Keywords: Earth's rotation, Chandler wobble, excitation functions, inverse problems.

1. INTRODUCTION

The results of the EOP PCC have shown, that various mathematical methods of EOP prediction compete with each other at approximately equal level, but the best results are given by the methods based on the physical modeling of the Earth's rotation. They use dynamical model and involve a number of additional data about the processes that cause variations in the rotation velocity and orientation of the poles of the planet. Mathematical approximations are based on the selection of dependencies, which approximates observations well, perhaps with a large number of parameters. For long-term forecast such models can be unsuitable. Physical models are based on the identified cause-effect relationships, they also have mathematical representation, possibly with fewer parameters. They should explain observations and have a predictive power. The following key issues related to the EOP prediction by means of physical modeling can be raised

- Do we have at our disposal the good enough dynamical model of the rotating Earth?
- Can we reconstruct the excitation with sufficient accuracy?
- Do we fully understand the causes of variations in the Earth's rotation?
- Can we predict the excitation and derive from it the resultant EOP variations?

In this paper we're trying to consider these questions, however, not pretending to give complete answers to them. We offer a variety of approaches for the excitation functions reconstruction, in particular, looking for the Chandler oscillation physical causes. More than a century has passed since the discovery in 1891 by S.C. Chandler the periodicity, named in his honour, Chandler (1891), Carter (2000). Despite this, its final explanation still has not been found. It's not clear what causes the modulation of the Chandler oscillation, what process makes a major contribution to this resonance phenomenon, Yatskiv (2000). Though, as it is shown in Gross (2000), Salstein (2000), Brzezinski and Nastula (2002) Liao et al. (2003), Bizouard and Seoane (2010) most of the energy can be explained by oceanic and atmospheric excitation, as well as changes in the bottom pressure, the correlation in time with these processes on the long periods remains questionable. Our attention will be focused on the polar motion (PM). First, we shall analyse it, then consider the dynamical model of the Earth's rotation, reconstruct excitation by different methods and analyse the results. In conclusion, we shall return to the above questions.

Fig. 1. (a) Coordinates of the pole since 1846 yr and (b) their accuracy as given in the IERS EOP $\pi_{\pi,01}$ bulletin.

2. SINGULAR SPECTRUM ANALYSIS OF POLAR MOTION

The pole motion since 1846 yr is represented on Figure 1a. The data are taken from the IERS EOP C01 bulletin. It gives the coordinates of the pole since 1846 up to 1899 with a 0.1 yr step, and since 1900 up to now (2010) with a step of 0.05 yr. The data prior to 1899 are derived from astronomical observations, obtained before the creation of the International Latitude Service (ILS). The early data were linearly interpolated to bring all the series to 0.05 yr sampling. The Figure 1b represents the growth of observational accuracy. Observations before 1900 are the least accurate. Optical observations of ILS were replaced in the 1970-th by the space geodesy techniques (VLBI, LLR, SLR, GPS). This led to a significant increase of accuracy. Therefore, the trajectory on Figure 1a is crooked and unsmooth at first, and smooth and even in recent decades. Now the accuracy of the observations greatly exceeds the accuracy of EOP forecasts, so considerable efforts are aimed at improving the accuracy of the latter.

It is known that the main PM components, besides the long-periodic trend and decadal variations, are the annual and Chandler periodic components, Sidorenkov (2009). The annual component is quite stable in frequency and amplitude, what is not so for the Chandler oscillation. To separate these components and release them from noise we used singular spectrum analysis (SSA), Golyadina (2004). This technique is based on the separation of principal components of the time series correlation in a multidimensional embedding space, Jolliffe (2002), and goes back to Takens (1981) works on a dynamical chaos. SSA allows to separate components of different frequencies, present in the time series, to separate the trend and release signal from noise. SSA was used previously for the EOP analysis by Gorshkov (2003), Bougeard et al. (2002), Zotov (2005). Here we applied SSA not to the x and y pole offset components independently, but to a complex time series¹ $m = m_x + im_y$, what gave a better separation of the Chandler and annual components. Figure 2a represents an x-coordinate of the pole time series. Figure 2b shows a plot of SSA-separated components. We illustrate the results on the example of x-coordinates of the pole, the picture for the y-component is similar. The main parameter of SSA is lag L , it was chosen to be 240 points, i.e. 12 years, almost equal to the double beat period of the annual and Chandler oscillations (6.4 yr).

Fig. 2. (a) x-coordinate of the pole and (b) result of singular spectrum analysis for it.

¹Here m is a small correction to the Earth's rotation angular velocity vector, whose components can be obtained from the x and y offsets of the pole, expressed in radians, according to the formulas $m_x \approx x$ $m_y \approx -y$.

Figure 3a shows spectrum of the annual and Chandler components, separated by means of SSA. The spectrum of the original PM series is represented as a background. The annual component is almost completely separated from Chandler by the SSA. As the separated components have finite dimensionality and are free from noise, they are easier to predict than the original time series.

Fig. 3. (a) Spectrum of the SSA-separated component and the original PM spectrum. (b) Frequency response of the inverse, regularized and corrective smoothing operators.

3. DYNAMICAL MODELLING AND EXCITATION RECONSTRUCTION

The problem of the Earth's rotation dynamical modeling and excitation functions reconstruction was raised by Jeffreys (1940) among the first. The dynamical system of the rotating Earth can be written as

$$\frac{i}{\sigma_c} \frac{dm(t)}{dt} + m(t) = \chi(t), \quad (1)$$

with a complex parameter $\sigma_c = 2\pi f_c(1+i/2Q)$. Here Q is a quality-factor, f_c is a Chandler frequency. This first order equation is based on the linearisation of the original Euler-Liouville equation and is a simple but an adequate approximation of the real system, Munk and MacDonald (1960), Wilson and Chen (1996), Vicente and Wilson (2002), Bizouard and Seoane (2010). In the works of Vicente and Wilson (1997), Spiridonov and Tsurkis (2008), Furuya and Chao (1996) the question about Q and Chandler frequency values is investigated. Their values are still under clarification. For example, Spiridonov and Tsurkis (2008) list studies, where the band for Q is 20-180, and for the Chandler period is 420-450 days.

The transfer function of equation (1) has the form

$$L(f) = \frac{\sigma_c}{\sigma_c - 2\pi f}. \quad (2)$$

Figure 4 represents the amplitude-frequency (AFR) $|L|$ and phase-frequency (PFR) $\arg(L)$ response of the dynamical system of rotating Earth for two different values of σ_c . In this research we use values $Q = 175$, $f_c = 0.843$, what corresponds to the Chandler period $T_c \approx 433.3$, obtained by Vicente and Wilson (1997).

Fig. 4. Amplitude (a) and phase (b) response of the system (1) for different parameters.

Developed in the works of Wilson (1985), Wilson and Chen (1996), Vicente and Wilson (2002) and improved in Zhou et al. (2005) Jeffreys-Wilson filter is a convenient way to reconstruct the excitation from discrete equally spaced observations

$$\chi(t) = \frac{ie^{-i\pi f_c \Delta t}}{\sigma_c \Delta t} \left[m_{t+\frac{\Delta t}{2}} - e^{i\sigma_c \Delta t} m_{t-\frac{\Delta t}{2}} \right], \quad (3)$$

where $\Delta t/2$ is a time interval between the equidistant observation's read outs. However, the Jeffreys-Wilson filter does not contain any corrective procedure, required for inverse problems solution, in particular, the differentiation of observations. The filter can be improved by introduction of the appropriate corrective procedure (stabilizer), Tikhonov and Arsenin (1977), Panteleev (2001a), (2001b). The necessity to stabilize the solution is caused by observational errors, especially large at the early epochs. The uncertainty of the dynamical system parameters is an additional source of errors in the reconstructed excitation functions.

In this work we are trying to reconstruct Chandler excitation, using different techniques with a corrective procedure. The essence of this kind of procedure is in rejecting the improbable solutions, i.e. in additional filtering.

On figure 3b the solid line shows the amplitude-frequency response of the operator, inverse to (2). The Jeffreys-Wilson filter can be used as an approximation of inverse operator $L^{-1}(f)$, Wilson et al. (1996), Zhou et al. (2005), Brzezinski (2007), at least near the Chandler frequency the difference is not not significant. Reconstruction of $\chi(t)$ from $m(t)$ can be represented in the frequency domain by multiplication of the PM spectrum by this inverse operator.

$$\hat{\chi}(f) = L^{-1}(f)\hat{m}(f),$$

here hat represents Fourier transform.

Evidently from Fig. 3b that the inverse operator weakens the component at the Chandler frequency much more, then the components on the left and right-hand sides from it. This is due to the fact that Chandler frequency is resonance, just a small excitation can be enough to cause oscillation. Thus, unless special attempts are undertaken during the reconstruction, the Chandler excitation can get lost in the strong excitations at neighbour frequencies, for example in the annual one. To prevent this, it is necessary to cut off the frequency components not related to the Chandler oscillation.

Since SSA release the signal from noise and separates the annual component from the Chandler, we can apply Wilson filter to the letter and get the Chandler excitation. There

is no need in additional corrective procedure so that it is already implicitly implemented by means of SSA. The result of the Chandler excitation reconstruction, separated by means of SSA through Jeffreys-Wilson filtering, is shown on Figure 5a by a solid line. For greater confidence, we shall try to prove the result by other SSA-independent methods.

To solve the ill-posed problems regularization can be used, Tikhonov and Arsenin (1977). Reconstruction of $\chi(t)$ can be regularized, so that the inverse operator will be written as

$$L_{reg}^{-1}(f) = \frac{L^*(f)}{L^*(f)L(f) + \alpha}, \quad (4)$$

where α is a regularization parameter. When $\alpha \rightarrow 0$, $L_{reg}^{-1}(f)$ tends to the inverse operator $L^{-1}(f)$. On the figure 3b, together with AFR of $L^{-1}(f)$ operator, the AFR of $L_{reg}^{-1}(f)$ operator (4) with parameter $\alpha = 500$ is represented. This parameter was chosen to make the results more or less consistent with SSA results. Regularized operator is a filter, which suppresses those frequencies, where $|L(f)|$ is small. However, its AFR very slowly decreases at the edges. It is hard to make annual component not to pass. Instead, we estimated the parameters of annual component by means of least squares method (LSM) and subtracted it from the original EOP C01 series. The result of regularization in frequency domain is represented on Figure 5a for comparison.

Table 1. Annual harmonic parameters, adjusted by LSM.

	amplitude	phase for 1846.0
x-coordinate	$0.088'' \pm 0.005''$	$231^\circ \pm 3^\circ$
y-coordinate	$0.078'' \pm 0.006''$	$148^\circ \pm 4^\circ$

Pantelev corrective smoothing was developed for the marine gravity observations processing, Pantelev (2001a), and can be used instead of regularization. The essence of the procedure is in the additional filtering, applied along with the inverse operator. Such reconstruction is based on the assumption that the noise spectrum, amplified by the inverse operator, has the frequency range different from the range of the useful signal, we are interested in. The bandwidth of the additional filter is chosen to suppress this noise. The operator of the corrective smoothing is built in the frequency domain according to the expression

$$L_{corr}^{-1}(f) = \frac{L_{filter}(f)}{L(f)}, \quad (5)$$

here $L_{filter}(f)$ is a transfer function of the additional smoothing filter. We used Pantelev filter, see eq. (7), Pantelev and Chesnokova (2004), centred at the Chandler frequency

$$L_{filter}(f) = \frac{f_0^4}{(f - f_c)^4 + f_0^4}. \quad (6)$$

The parameter $f_0 = 0.04$ was chosen to suppress low and high frequencies, including the annual. Then, there is no need to subtract the annual component from the PM signal before $\chi(t)$ reconstruction. In addition, this f_0 makes the filter (5) characteristics near the Chandler frequency similar to the characteristics of the regularized inverse operator (4) with parameter $\alpha = 500$. The AFR of the Pantelev corrective filter (5) is represented on Figure 3b together with those for the inverse and regularized operators. The advantage of the filter (6) is that it has sufficiently quickly decreasing AFR and everywhere zero PFR.

Such a filter does not introduce phase distortion. The result of the corrective smoothing, made in spectral domain, is also represented on Figure 5a.

As it is seen from Figure 5a, the results of the Chandler excitation reconstruction obtained by three different methods are similar, what gives hope that they are reliable. Since regularization and corrective smoothing were performed in the spectral domain with the back transform to time domain, their results suffer from boundary effects. They are seen at the edges and last not more than the half-width of the corresponding time-window of the filter. In case of Panteleev corrective smoothing corresponding window $h_{filter}(t)$ is represented on Figure 6a and for $f_0 = 0.04$ it's half-width is less than 20 yrs. Regularization window is less extent in time, but for the selected α is wider in the frequency domain, what allow more side-frequencies to pass. This results in larger amplitude variability of the excitation, obtained by regularization. SSA-reconstruction also has boundary-effects which cover about 12 years (L points) at both edges.

Fig. 5. Comparison of (a) Chandler excitation amplitude, obtained by three different methods with (b) zonal tides model for LOD.

4. CHANDLER EXCITATION STUDY

To study the reconstructed Chandler excitation we used Gabor transform that is spectrum analysis in a sliding window, Vityazev (2001).

$$G_h f(\omega, t) = \int_{-\infty}^{\infty} f(\tau) h_{\omega, t}^*(\tau) d\tau = \int_{-\infty}^{\infty} f(\tau) h(\tau - t) e^{-i\omega\tau} d\tau,$$

here $h_{\omega, t}(\tau) = h(\tau - t)e^{i\omega\tau}$, $h(t)$ is a symmetric time window, centred in the origin of coordinates, f is an analysing signal. If to fix ω at the Chandler frequency $2\pi f_c$, the

absolute value and phase of $e^{i2\pi f_c t} G_h f(\omega, t)$ would give us an envelope and phase evolution of the signal, passing through the filter $\hat{h}(f)$, centred at f_c (by hat we denoted Fourier-transform). We modified Gabor transform and instead of the usual Gauss window we used Pantelev window, Pantelev (2004),

$$h(t) = \frac{\omega_0}{2\sqrt{2}} e^{-\frac{\omega_0 |t|}{\sqrt{2}}} \left(\cos \frac{\omega_0 t}{\sqrt{2}} + \sin \frac{\omega_0 |t|}{\sqrt{2}} \right), \quad (7)$$

here $\omega_0 = 2\pi f_0$. Frequency response $\hat{h}(f - f_c)$ is given by (6). When the time-window (7) is filled with oscillation $h_{2\pi f_c, 0}(\tau) = h(\tau) e^{i2\pi f_c \tau}$, AFR becomes centered at the Chandler frequency. To let a large part of the reconstructed excitation to pass through this kind of Gabor transform with Pantelev window and to obtain the envelope we have chosen a rather broad spectral window $\hat{h}(f)$, by selection of parameter $f_0 = 0.15$. For the phase evaluation, to make it less curved and divergent, we used $f_0 = 0.04$ as in the corrective smoothing. Figure 6 represents Pantelev filter for two values of parameter f_0 in time and frequency domains. It helps to judge about the temporal and frequency resolution of corrective smoothing and of Gabor transform used in our work.

Fig. 6. (a) Pantelev window in time domain, and (b) corresponding amplitude response, centred at Chandler frequency.

The envelopes of Chandler excitations, obtained from the Gabor transform module, are represented at Figure 5a. The phase change, obtained from its argument, is shown on Figure 7 together with the phase evolution of the SSA-separated PM Chandler component, obtained in the same way. The well-known $\sim 3\pi/2$ phase jump in the Chandler wobble around 1930-th, Vondrak (1999), Malkin and Miller (2009), Gibert and Le Mouel (2008), almost disappears in excitation. This can be explained by the influence of the dynamical system (1), which introduces π phase shift (see Figure 4b). It could be possibly explained by the excitation transition in the frequency domain from the left-hand side from the Chandler frequency to the right-hand side (private discussions with Gorshkov). At the same time Figure 7 shows that other interesting phase transitions appear in excitation. This phase changes causes the fact, that we do not see in the Chandler PM component (Fig. 2b) such amplitude changes, as in it's excitation (Fig. 5a). The interesting fact to note is that in the 1930-th, when the observed amplitude of the Chandler oscillation decreases, the excitation on the contrary is quite large. It could be that two large excitation waves with small phase differences neutralize each other in the 1930-th (private discussions with Bizouard).

Fig. 7. Phase evolution of the Chandler PM component and its excitation, obtained by the Gabor transform with Panteleev window.

As it was mentioned in introduction, Chandler oscillation supposed to be caused by changes of AAM, OAM, as well as bottom pressure. We will not present the study which proves their influence here. Let us only note one curious detail, which we were able to distinguish. Chandler excitation, as seen on Figure 5a has an amplitude modulation. Figure 5b shows the IERS model of zonal tides for the length of day (LOD). The main long-periodic 18-year tidal harmonic, associated with Saros cycle, is separately highlighted on Figure 5 along the time scale below the reconstructed excitation. This harmonic shows the behaviour, synchronous with modulation of the Chandler excitation. The correlation is not observed in the 1990-th, probably, because of the boundary-effects and for early observations. However, during the XX century, there are 5 synchronous peaks. Thus, when the rotation of the Earth slows down, during the peak tides, an increase of the amplitude of Chandler excitation is observed. This leads us to the hypothesis that the same factor operates both rotational velocity of the Earth and the Chandler motion of the pole. It is possible that the energy from the tidal perturbations is somehow transferred into the Chandler oscillation. The mechanism is not clear, perhaps, the atmosphere and the ocean are the mediators. Zonal tides can not be responsible for PM changes, but plot 5b is a good indicator of the overall activity of the tidal force. One of the difficulty of explanation comes from the fact that the Chandler oscillation with period ~ 433 days in the terrestrial coordinate frame has almost daily period in the inertial reference system.

5. CONCLUSIONS

Let us return to the questions posed in the introduction. Though the dynamical model of the Earth's rotation (1) is a linearised simplification, it is quite adequate. There is some uncertainty about the parameters of Chandler quality Q and frequency f_c . Assuming that the range of their changes is within the reasonable limits, we can use this model.

The physical base for understanding of the Earth's rotation variations is more or less settled. In general, their causes are known, but we would not say, that understanding is complete. There is still room for alternative points of view. For example, the double peak of Chandler wobble spectrum has been analysed whether it is one or two close by frequency oscillations. In the work of Guo et al. (2005) this is discussed in detail and it's

concluded, that we should wait until 2030-th, when, in case of dual components, Chandler oscillations should decrease, as in 1930-th. Though, as it can be seen from Figure 2b there is a tendency of Chandler component to decrease in the last decade, we still supposed in our study, that there is one resonance frequency, what can be physically derived from the theory of the Earth's rotation, as it was developed in works of Euler, Liouville, Poincare and others. But to improve our understanding of causes, excitation studies are required.

Reconstruction of excitation, which is an ill-posed problem of observations differentiation, requires special care, especially for data, aggravated by the errors of optical observations. After the 1970's accuracy has increased and real high-frequency effects became visible. Errors of the differentiation results are difficult to deduce from the errors of initial data. The corrective procedure is highly desirable, such as regularization or Panteleev corrective smoothing. We gave examples of three methods for excitation reconstruction.

Reconstructed excitation, illustrated for Chandler component on Figure 5a, still requires an exhaustive physical explanation. We were able to observe some interesting details in the behaviour of the amplitude and phase of the excitation. It is possible that Luni-Solar tidal influence makes a significant contribution, Avsyuk (1996). Despite the difficulties of interpretation, the excitation prediction is possible. For example, from Figure 5a it can be assumed that Chandler excitation would present a maximum around 2016 yr.

Whereas the Chandler component prediction is one of the most difficult task, the case of the annual and high-frequency components is much easier. The influence of AAM and OAM here is out of doubt. In the public domain there are forecasts of states of the atmosphere and ocean, issued by prediction centres, such as NOAA, NCEP/NCAR. This forecasts can be easily incorporated into the excitation functions prediction and, involving the Kalman filter, Gubanov (1997), Dill et al (2010), immediately transferred to the EOP forecasts.

From our point of view, EOP prediction is on way of continuous improvement with involvement of all sorts of data upon the processes in the Earth system, intensively obtained by modern monitoring systems. That is very useful for prediction of both the excitation and resultant EOP. Though the accuracy of EOP observations today is still higher then even of the short-term predictions, these observations help to deeper understand physics of this amazing overall index of activity of different processes on Earth and in space, namely the Earth's rotation velocity variations and planet's axis orientation changes.

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