Entropy increase as a measure of energy degradation in heat transfer

ZBIGNIEW DROŻYŃSKI

The Szewalski Institute of Fluid Flow Machinery, Polish Academy of Sciences, Fiszera 14, 80-231 Gdańsk, Poland
EDF Wybrzeże S.A., Swojska 9, 80-867 Gdańsk, Poland

Abstract  Heat transfer is an irreversible process. This article defines the entropy increment as a measure of energy degradation in heat transfer realized in typical surface heat exchangers. As an example of the proposed entropy increase method, presented below are the calculations for heat exchangers working in a typical Clausius-Rankine cycle. The entropy increase in such exchangers inevitably leads to increased fuel consumption and, as a further consequence, to increased carbon dioxide emission.

Keywords: The second law of thermodynamics in heat; Heat and mass transfer; Entropy increment; Surface heat exchangers; Clausius-Rankine cycle

Nomenclature

\[ \begin{align*}
A & \quad \text{heat transfer area, m}^2 \\
F & \quad \text{fluid cross section area, m}^2 \\
l & \quad \text{coordinate, m} \\
L & \quad \text{channel length, m} \\
m & \quad \text{mass flow rate, kg/s} \\
p & \quad \text{static pressure, Pa} \\
Q & \quad \text{heat flux, kW} \\
s & \quad \text{specific entropy, kJ/kgK} \\
S & \quad \text{entropy, kW/K} \\
T & \quad \text{temperature, K} \\
TPC & \quad \text{thermodynamic potential coefficient}
\end{align*} \]

*E-mail: drozd@imp.gda.pl
Greek symbols

- \( \delta \) – thickness
- \( \lambda \) – thermal conductivity
- \( \eta \) – efficiency

Subscripts

- \( c \) – cold
- \( h \) – hot
- \( f \) – friction
- \( w \) – wall
- \( C \) – cycle

1 Introduction

In 1822–23, when working on a steam engine, Nicolas Léonard Sadi Carnot, French physicist, marine engineer and mathematician, published his major work which formulated the mathematical basis for determining entropy. The efficiency of an ideal ‘reversible engine’, taking the name after its author, became the basis for discussion on the laws of thermodynamics. Rudolf Julius Emanuel Clausius, a German physicist, restated Carnot’s principle of the heat engine efficiency. In his paper of 1865 Clausius formulated the first and second laws of thermodynamics in the following way:

1. \textit{Die Energie der Welt ist constant.} The energy of the universe is constant.

2. \textit{Die Entropie der Welt strebt einem Maximum zu.} The entropy of the universe tends to a maximum.

For cycles he set up an integral which was later frequently referred to as the Clausius integral. He also defined the basic equation \( dQ = dU + dW \), where \( dQ \) is the increase of the heat, \( dU \) is the change in internal energy of the body, and \( dW \) is the change in external work done. In 1876 Josiah Willard Gibbs published the work in which entropy and temperature were taken as coordinates.

The mathematical formulation for entropy increase proposed by Clausius is quoted in all basic handbooks of thermodynamics. In the literature on the subject we can also find numerous interpretations of the concept of entropy [1–4]. Methods are known for determining efficiency of thermal cycles from energy and entropy balances [5,6].
2 Definition of entropy increase as a measure of energy degradation in heat flow

The object of analysis is the irreversible phenomenon of steady heat transfer between fluid flows \((m_h, m_c)\) separated by a diaphragm in a surface heat exchanger. Heat transfer takes place only on the surface, \(A\), of the diaphragm separating two flows, while the remaining surfaces are adiabatic.

![Diagram for deriving the relation of entropy increase in heat transfer.](image)

In general examined heat transfer is a function of:

- thermodynamic properties of the heating and cooling fluids which are regarded as known;
- known and constant (for \(0 \leq l \leq L\), parameters along both channels, where \(L\) is channel length:
  - \(m_h\) – mass flow rate of the heating medium,
  - \(F_h\) – heating fluid cross section area,
  - \(m_c\) – mass flow rate of the heated medium,
  - \(F_c\) – heated fluid cross section area,
  - \(\delta\) – diaphragm thickness,
  - \(\lambda\) – thermal conductivity for diaphragm;
- known distributions (for \(0 \leq l \leq L\) of the thermodynamic parameters of the media taking part in the heat transfer:
The searched quantity is the measure of irreversibility of heat transfer in the exchanger. According to the second law of thermodynamics, for each coordinate \( l \) from within the interval \( 0 \leq l \leq L \) of the passage the following relation takes place: \( T_h(l) > T_{wh}(l) > T_{wc}(l) > T_c(l) \). Heat flux transfer from the heating to the heated medium is subject to degradation in the potential field of temperature.

In surface heat exchangers, local entropy changes in the flowing media are caused by the irreversible heat penetration through diaphragm \( dQ(l) \) and friction heat \( dQ_f(l) \) resulting from the turbulent fluid flow in the exchanger passages

\[
dS_h(l) = \frac{dQ_h(l) + dQ_{fh}(l)}{T_h(l)} \tag{1}
\]

and

\[
dS_h(l) = \frac{dQ_c(l) + dQ_{fc}(l)}{T_c(l)}. \tag{2}
\]

The phenomena of heat penetration \( dQ(l) \) through the diaphragm and friction heat \( dQ_f(l) \), observed in the fluid flows in the exchanger passages, are directly dependent on each other. The higher the local Reynolds numbers \((Re_c(l), Re_h(l))\) describing the turbulence of the fluid flows in the passages, the higher the corresponding Nusselt numbers \((Nu_c(l), Nu_h(l))\) and, as a consequence, the higher the local heat transfer coefficient describing the heat penetration between the fluids. Along with this tendency, however, the increasing Reynolds number intensifies the effects of friction heat, which contribute to the additional increase of the entropy. A problem to be solved by the designer is to arrange the flow of fluids in the passages in such a way that the required heat transfer between these fluids is ensured with the lowest possible total entropy increase. For the flat geometry of the diaphragm

\[
dS(l) = dS_c(l) + dS_h(l) = \frac{dQ(l)}{T_c(l)} \left[ 1 - \frac{T_c(l)}{T_h(l)} \right] + \frac{dQ_{fc}(l)}{T_c(l)} + \frac{dQ_{fh}(l)}{T_h(l)} = dS_Q(l) + dS_f(l). \tag{3}
\]
Process heat and mass transfer entropy increment is equal to the sum of local entropy changes of the media participating in heat transfer through surface \(dA(l)\) of an exchanger as well as to the sum of entropy increment \(dS_Q(l)\) resulting from irreversible heat penetration \(dQ(l)\) through the diaphragm and also to entropy increment \(dS_f(l)\) caused by the heat of fluid friction in the passages.

The entropy increment \(dS(l) = 0\) if and only if \(dQ(l) = 0\) and \(dQ_{f\ldots}(l) = 0\) (no thermodynamic transformation). The entropy increments are always positive when \(dQ(l) > 0\) or \(dQ_{f\ldots}(l) > 0\) (for all other thermodynamic transformations).

After complementing Eq. (3) with the temperature distributions on the diaphragm surfaces \((T_{wh}(l), T_{wc}(l))\), obtained from either numerical analyses [7,8] or experimental investigations, we can analyse the entropy increase in the thermal layers on both sides of the heat transfer surfaces and in the contaminated diaphragm itself [9]

\[
dS_Q(l) = \frac{dQ(l)}{T_e(l)} \left[ 1 - \frac{T_c(l)}{T_h(l)} \right] = \\
dQ(l) \left\{ \frac{1}{T_e(l)} \left[ 1 - \frac{T_c(l)}{T_{wc}(l)} \right] + \frac{1}{T_{wc}(l)} \left[ 1 - \frac{T_{wc}(l)}{T_{wh}(l)} \right] \\
+ \frac{1}{T_{wh}(l)} \left[ 1 - \frac{T_{wh}(l)}{T_h(l)} \right] \right\} = \\
dS_{T_{wc},T_c}(l) + dS_{T_{wh},T_{wc}}(l) + dS_{T_h,T_{wh}}(l) = dS_{T_h,T_c}(l) > 0 . \quad (4)
\]

The local entropy increment \(dS_Q(l)\) caused by irreversible heat transfer \(dQ(l)\) through infinitesimal surface element \(dA(l)\) of a surface heat exchanger is equal to the sum of the entropy increment in the thermal layers of the fluids and in the (contaminated) diaphragm itself.

The differential form of the relations Eq. (4) can be used to calculate the entropy increase caused by the heat transfer between the flowing fluids.

In order to obtain relevant heat transfer, the process should be optimised to minimise the entropy increase, which involves minimising of all the terms in the above equation. By integrating the local entropy increases along fluid flow path \(l\) (or through heat transfer surface \(A(l)\)) we can obtain the total entropy increment connected with heat penetration in the entire surface heat exchanger.

In technical analyses we can assume that the dominating effect in surface heat exchangers is heat penetration through the diaphragm. In other
words, the friction heat connected with the fluid flow in the passages is small compared to the effect of heat penetration through the diaphragm. Therefore the entropy increase connected with the irreversible heat penetration through the diaphragm is far larger than the entropy increase from the turbulent fluid flow

$$dS_Q(A) \gg dS_f(A).$$

(5)

According to (5), calculating the entropy increase connected with the irreversible process of heat penetration through the infinitesimal surface element $dA(l)$ requires information on the heated medium’s local specific entropy change

$$\frac{dQ(A)}{T_c(A)} \approx \frac{dQ_c(A) + dQ_{fc}(A)}{T_c(A)} = m_c ds_c(A)$$

(6)

and the local dimensionless thermodynamic potential coefficient expressed by relevant temperature values

$$0 < TPC_{T_c,T_h}(A) = \left[1 - \frac{T_c(A)}{T_h(A)}\right] < 1.$$  

(7)

Knowing the mass flow of heated medium ($m_c$), real distributions $ds_c(A)$, $T_c(A)$ and $T_h(A)$ for $0 \leq A \leq A_i$ (where $A_i$ denotes entire heat exchanger area) we can calculate the entropy increase as a measure of energy degradation in the irreversible process of heat penetration between the fluids in the entire surface heat exchanger.

As an example of the proposed entropy increase calculation method, the calculations for surface heat exchangers working in a typical Clausius-Rankine cycle are presented. In general, the energy flows between two potentials. The task of the engine is to convert the thermal energy flowing between the sources. The maximal and minimal parameters of the working media in a particular cycle result from the heat transfer between discussed media and the upper and lower heat sources. In the case of the C-R cycle for instance, these parameters are determined by the action of heat exchangers in the boiler and the condenser (Fig. 2).

The entropy increase in the process of irreversible heat transfer between the upper heat source and the working fluid in the boiler’s surface heat exchanger unit is equal to

$$\Delta S_H = \sum_{i=1}^{i=N} \int_0^{A_i} \frac{dQ_H(A_i)}{T_{CH}(A_i)} \left[1 - \frac{T_{CH}(A_i)}{T_H(A_i)}\right] dA_i + \Delta S_{fH},$$

(8)
The entropy increase in the process of irreversible heat transfer between the working fluid and the lower heat source in the condenser passes is equal to

\[ \Delta S_L = \sum_{j=1}^{j=M} \int_0^{A_j} dA_j \left[ \frac{dQ_L(A_j)}{T_L(A_j)} \left( 1 - \frac{T_L(A_j)}{T_{CL}(A_j)} \right) \right] dA_j + \Delta S_{fL}, \]

(10)

\[ \Delta S_{fL} = \sum_{j=1}^{j=M} \int_0^{A_j} dA_j \left[ \frac{dQ_{fCL}(A_j)}{T_{CL}(A_j)} + \frac{dQ_{fL}(A_j)}{T_L(A_j)} \right] dA_j, \]

(11)

where:

- \( \Delta S_{\ldots} \) – is the entropy increase,
- \( Q_H(A_i), Q_L(A_j) \) – distributions of heat penetration through the diaphragms,
- \( Q_{f\ldots}(A_i), Q_{f\ldots}(A_j) \) – distributions of friction heat caused by the turbulent flow of fluids in the passages,
\( T_H(A_i) \) — temperature distribution of heating gas along \( A_i \) surface heat exchanger in the boiler,

\( T_{CH}(A_i) \) — temperature distributions of working fluid along \( A_i \) surface heat exchanger in the boiler,

\( T_{CL}(A_j) \) — temperature distributions of working fluid along \( A_j \) surface heat exchanger in the condenser,

\( T_L(A_j) \) — temperature distributions of cooling water along \( A_j \) surface heat exchanger in the condenser,

\( 1 \leq i \leq N \) — number of the heat exchanger working between the high thermodynamic potential and the cycle,

\( 1 \leq j \leq M \) — number of the heat exchanger working between low thermodynamic potential and the cycle.

On the basis of (5), it can be assumed that entropy increase connected with irreversible heat penetration through the diaphragm is far larger than the entropy increase resulting from the turbulent fluid flow. However, it should be remembered that the fluid flow does contribute to general entropy increase even if this contribution is negligible.

Knowing the functions which describe the heat transfer and temperature distributions on both sides of the heat exchanger diaphragm we can calculate the entropy increases in successive exchangers as a measure of energy degradation in irreversible heat transfer between fluids.

If for the sake of simplicity we can assume:

i) all temperature distributions are constant functions of passage surfaces \( T_H(A_i) = T_H, \ T_{CH}(A_i) = T_{CH} \), for each \( i \) and \( T_{CL}(A_j) = T_{CL}, \ T_L(A_j) = T_L \) for each \( j \) (all heat transfer processes are isothermal);

ii) all energy conversions are isentropic, that is they take place without friction \( (Q_f(A) = 0) \);

then the entropy increases in:

- heat transfer between the upper source and the working medium is

\[
\Delta S_H = \frac{\Delta Q_H}{T_{CH}} \left[ 1 - \frac{T_{CH}}{T_H} \right] = \frac{\Delta Q_H}{T_{CH}} TPC_{T_H,T_{CH}}, \tag{12}
\]

- heat transfer between the working medium and the lower heat source is

\[
\Delta S_L = \frac{\Delta Q_L}{T_L} \left[ 1 - \frac{T_L}{T_{CL}} \right] = \frac{\Delta Q_L}{T_L} TPC_{T_{CL},T_L}. \tag{13}
\]
The efficiency of such an ideal engine is:

\[ \eta_{\text{CYCLE}} = \eta_C = \left( 1 - \frac{T_{CL}}{T_{CH}} \right) = TPC_{TCH,TCL} \]  

(14)

and is equal to Carnot efficiency.

To ensure the maximal efficiency of an engine of a given power output, i.e., to reach

\[ \eta_{\text{CYCLE}} = TPC_{TCH,TCL} \to \max \]

(15)

entropy increases in upper and lower source heat exchangers should be minimal

\[ TPC_{T_H,TCH} \to \min \quad \text{and} \quad TPC_{T_L,TCL} \to \min . \]  

(16)

Additionally, the examined cycle (engine) will be more efficient when the temperature of the upper heat source is higher and that of the lower heat source is lower, i.e.,

\[ TPC_{T_H,T_L} \to \max . \]  

(17)

The results of the above analysis comply with the theory of thermal cycles.

3 Analyzing entropy increment in surface heat exchangers working in a typical Clausius-Rankine cycle

Sample calculations of the entropy increase, being the measure of energy degradation in flows, are presented using industrial surface heat exchanger units working on the C–R cycle principle. All the relations describing entropy increase in irreversible heat flow between the fluids form the basis for the analysis of energy degradation in the most common power plant components in Poland. The exact schematic of the power block can be found in [10], for instance. This typical cycle works between high and low thermodynamic potentials to convert heat into electric energy, which then can be transmitted over long distances. Within the area of the high thermodynamic potential, a system of surface heat exchangers transfers heat between an exhaust gas and water, the latter being the cycle’s working medium. The device in which this process takes place is the OP 650 pulverised fuel boiler. Flowing over the surfaces of the boiler’s seven successive heat exchangers \((1 \leq i \leq N = 7)\), the hot exhaust gas transfers the heat to the water.
delivered by the feed pump. Within the area of low thermodynamic potential, a condenser unit with a cooling water system removes the heat from the cycle to the surroundings [11]. In this case the water flows through two condenser passes \( (1 \leq j \leq M = 2) \). An analysis of how the regeneration system works is important because it also increases the efficiency of the other two systems. A 13K215 steam turbine (nominal electric power 215 MW) works with this cycle.

This article determines the measure of irreversibility, that is energy degradation resulting from the heat flow process in high and low thermodynamic potential systems. Entropy increase can be calculated in the heat transfer from the heating medium (exhaust gas) to the working and cooling medium (water in both cases). The analysis was based on the author’s own thermodynamic parameter measurements of working industrial devices. In real thermal cycle installations we can only obtain thermodynamic parameters recorded at their inlets and outlets. Hence for the purpose of this analysis, the linear distributions of the thermodynamic parameters (temperature and pressure) had been assumed.

### 3.1 Clausius-Rankine cycle calculation input data

Figure 3 presents a typical thermal C-R cycle in a \( T-s \) coordinate system, with the basic components of particular cycle parts. The aforementioned feed water pump works between low- and high-pressure regeneration system components.

![Figure 3. Thermodynamic parameters of the working medium in C-R cycle components.](image-url)
The exact values of the thermodynamic parameters are presented in Tab. 1. The data was recorded by the author and is comparable with [10]. The successive points in Tab. 1 describe parameters measured simultaneously at the inlets and outlets of successive devices.

### Table 1. Values of thermodynamic parameters of working medium (Fig. 3) for calculation of entropy increase in a C-R cycle.

<table>
<thead>
<tr>
<th>Number of points in the cycle Fig. 3</th>
<th>Pressure $p_{C...}$ [kPa]</th>
<th>Temperature $T_{C...}$ [K]</th>
<th>Mass flow $m_C$ [kg/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiler</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>18 500</td>
<td>517.25</td>
<td>180.6</td>
</tr>
<tr>
<td>2</td>
<td>18 500</td>
<td>596.15</td>
<td>180.6</td>
</tr>
<tr>
<td>3</td>
<td>18 500</td>
<td>362.46</td>
<td>180.6</td>
</tr>
<tr>
<td>4</td>
<td>17 303</td>
<td>677.46</td>
<td>180.6</td>
</tr>
<tr>
<td>5</td>
<td>16 107</td>
<td>767.47</td>
<td>180.6</td>
</tr>
<tr>
<td>6</td>
<td>16 107</td>
<td>760.55</td>
<td>180.6</td>
</tr>
<tr>
<td>7</td>
<td>14 910</td>
<td>819.55</td>
<td>180.6</td>
</tr>
<tr>
<td>11</td>
<td>2 786</td>
<td>624.75</td>
<td>156.4</td>
</tr>
<tr>
<td>12</td>
<td>2 611</td>
<td>743.66</td>
<td>156.4</td>
</tr>
<tr>
<td>13</td>
<td>2 435</td>
<td>810.55</td>
<td>156.4</td>
</tr>
<tr>
<td>Condenser</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>7.054</td>
<td>312.30</td>
<td>133.3</td>
</tr>
<tr>
<td>21</td>
<td>7.054</td>
<td>312.30</td>
<td>133.3</td>
</tr>
</tbody>
</table>

### 3.2 Low thermodynamic potential of the Clausius-Rankine cycle

A two-pass $A_j (1 \leq j \leq 2)$ condenser [12,13] converts energy in the lower heat source (low thermodynamic potential) of the C-R cycle. Water flows on both sides of the heat transfer surfaces. This process is presented in a $T$–$s$ coordinate system in Fig. 4.

Analysis requires information on the specific entropy changes in the lower-temperature medium in the exchanger. The values of the parameters which can be used for calculating these changes are given in Tab. 2. The parameters of the working medium, i.e., steam condensed in the condenser, are given in Tab. 1.

### 4 Calculation results

Using the above data, calculations were performed to determine entropy increases in real devices. The results of these calculations are shown in Tab. 3.
Figure 4. Water temperature distribution in the condenser (low thermodynamic potential) of the C-R cycle.

Table 2. Thermodynamic parameters of cooling water in a C-R cycle condenser.

<table>
<thead>
<tr>
<th>Number of points in the coolant system Fig. 5</th>
<th>$p_L$ [MPa]</th>
<th>$T_L$ [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.240</td>
<td>292.15</td>
</tr>
<tr>
<td>41</td>
<td>0.229</td>
<td>297.89</td>
</tr>
<tr>
<td>42</td>
<td>0.217</td>
<td>301.33</td>
</tr>
</tbody>
</table>

Table 3. Entropy increment in particular C-R cycle heat exchangers.

| Number | Heat exchanger       | Heat transfer $|\Delta Q_L$ | Entropy increment $|\Delta S_L|$ |
|--------|----------------------|---------------|-------------|
| $j = 1$ | 1 st pass of condenser | 192 069.80    | 35.91       |
| $j = 2$ | 2 nd pass of condenser | 115 146.10    | 15.55       |
|        | $\Delta Q_L = 307215.90$ | $\Delta S_L = 51.46$ |             |

5 Conclusions

The presented results should be considered as a continuation of the investigations of steam condensation with non-condensable gases [7]. The
model takes into account the heat transfer through the thermal boundary layer which consist of the film of condensate flowing down the heat transfer surface, and the gas part of the layer being the presented steam-air mixture. The discussed model has been improved and tested for free and forced convection on different condenser surfaces. The results of heat transfer distribution as a function of tube length were compared with the cases of real condenser tubes in operation in the installations of 215 MW power units [12,13]. This detailed analysis makes the basis for the definition of entropy increment [14].

In this article, local and integral relations were defined for calculating entropy increases resulting from irreversible heat flow processes in surface heat exchangers. Based on the above relations, the operational analysis of such heat exchangers provides new opportunities for cycle optimisation through:

- Selection of devices. For instance, fluidised bed boilers have smaller entropy increases than pulverised-fuel boilers [14]. Supercritical boilers are likely to have smaller irreversible heat flow entropy increases than subcritical devices.

- Particular methods of heat flow realisation. For instance, the heat flow via radiation in the examined boiler had a larger entropy increase than the heat flow via convection.

- Analysis of local entropy increases in the thermal layers of flowing fluids to optimise device operation. For instance, we can calculate the effect of heat transfer surface pollution on boiler efficiency or the effect of inertial gases on steam condensation in condensers [15,16].

- Cycle configuration.

Further research is required with regard to:

- Acquiring more precise knowledge on the distributions of the thermodynamic parameters of media in surface heat exchangers.

- Determining the effect of friction heat on total entropy increase in heat exchangers.

- The fact that entropy increase in the entire cycle is equal to the sum of the entropy increases in each component device.

- Optimising the construction of surface heat exchangers by minimising the sum of entropy increases.
References