Modal Analysis of a Trapezoidal Violin Built after the Description of Félix Savart

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One-dimensional experimental modal analysis of an unvarnished trapezoidal violin built after the description of F. Savart and an anonymous trapezoidal violin on display in the Music Instrument Museum of Brussels is described. The analysis has revealed ten prominent modes. A mode that may potentially play a role of the “tonal barometer” of the instrument is pointed out. The mode shapes are symmetric and of high amplitude, due to the construction of the instrument. Subjective evaluation of the sound quality demonstrated no pronounced difference between the trapezoidal violin and normal violin.

Keywords: trapezoidal violin, modal analysis, mode shapes.

1. Introduction

Modal analysis has been frequently used as an effective tool to describe natural vibrations of many classical string instruments, such as the violin (MARSHALL, 1985; SKRODZKA et al., 2009; 2013; 2014), the guitar (SKRODZKA et al., 2011; TORRES, BOULLOSA, 2009), the cello (FOUILHE et al., 2011), and less commonly used instruments, such as the Russian balalaika (MORRISON, ROSSING, 2001), the kantele (PENTTINEN et al., 2005), and the jarana jarocha (BOULLOSA, GOMEZ, 2014). An interesting instrument, not studied so far, is the Savart trapezoidal violin, or box-fiddle. Félix Savart, who has been well-known for his work related to magnetic fields, was also interested in the physics of the violin. In 1818 he constructed an experimental trapezoidal violin – a simplified instrument without arches, with straight sound holes and a bass bar placed in the middle of the front plate. This inexpensive violin, which is much easier to build than a traditional one, was never adopted for music although in blind-listening tests its sound quality proved to be comparable to instruments of famous Italian masters (SAVART, 1819). A series of experiments led Savart to the following conclusions: (1) the more regular the body of the instrument is, the easier it vibrates, e.g., flat plates vibrate more easily then those fixed by their camber, (2) the quality of sound improves when the plate vibrates in a symmetrical manner. Basing on those findings, he decided to place the bass bar in the middle of the instrument and to make both plates plane on the inside, with a very small arching on the outside, caused only by the difference in thickness between the centre and the edges of the plate (SAVART, 1819).

Nowadays it is obvious that the design has a great influence on the dynamic behaviour of the instrument (SKRODZKA et al., 2011). The trapezoidal violin with its unique design has not been intensively investigated and there is only little information about its dynamic behaviour. FONTANA and SERAFIN (2003) used a three dimensional wave-guide mesh to model the impulse response and the spectrum of the outgoing velocity at the bridge of the instrument. GOUGH (2007) derived some mode shapes of the Savart violin using a finite element shell model. However, in both papers cited above there is no information about the exact values of modal parameters (modal frequencies, modal
damping) and only some examples of modal deformations (mode shapes) are shown. In the present work a replica of Savart trapezoidal violin is investigated by means of an experimental modal analysis technique in the aim of obtaining the most prominent and the best pronounced natural patterns of vibration and their parameters. To the best authors’ knowledge this paper is the first attempt to describe the modes of vibration of the trapezoidal violin.

2. The instrument

The trapezoidal violin was made by the author T. Duerinck after the description of F. Savart in his memoires (Savart, 1819) and an anonymous trapezoidal violin on display in the Music Instrument Museum (MIM) of Brussels (Experimental violin, 1818). The memoires were the primary and most important source of information about the instrument. Since the luthier who made the trapezoidal violin on display in the MIM cannot be known for certain, that violin was considered a secondary source of information. The dimensions of the investigated replica of the Savart violin are given in Table 1. The front and the back plate were plane surfaces on the inside. The bass bar was mounted along the main axis of the top plate without tension, as described by Savart (1819). The soundpost was placed as usual, just below the right foot of the bridge. The instrument was equipped with a medium tension H310 D’Addario Helicore set of strings, and tuned to playing condition, i.e. with strings up to the pitch, damped and without chin or shoulder rest. The instrument was unvarnished. Although modal damping may be slightly reduced for varnished corpuses when compared to the unvarnished instruments (Skrodzka et al., 2013), the lack of varnishing did not influence the violin’s quality, as the damping trends are not robust quality discriminators (Dünnwald, 1999). The front plate was made of natural dried spruce of high quality, as were the bass bar and blocks. The back plate and ribs were made of good quality maple.

Table 1. Dimensions of Savart trapezoidal violin, in millimetres.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top width</td>
<td>84.4</td>
</tr>
<tr>
<td>Bottom width</td>
<td>225</td>
</tr>
<tr>
<td>Length inside to upper block</td>
<td>328.5</td>
</tr>
<tr>
<td>Ribs height</td>
<td>34.5</td>
</tr>
<tr>
<td>Edge thickness of the bottom plate</td>
<td>2.3</td>
</tr>
<tr>
<td>Edge thickness of the top plate</td>
<td>2.3</td>
</tr>
<tr>
<td>Thickest point of the bottom plate</td>
<td>5.1</td>
</tr>
<tr>
<td>Thickest point of the top plate</td>
<td>6.2</td>
</tr>
<tr>
<td>Bottom block width</td>
<td>54</td>
</tr>
<tr>
<td>Bottom block depth</td>
<td>18</td>
</tr>
<tr>
<td>Upper block depth</td>
<td>18</td>
</tr>
<tr>
<td>Distance between the bottom of the instrument and the middle of the soundholes</td>
<td>164.3</td>
</tr>
<tr>
<td>Distance between sound holes</td>
<td>81</td>
</tr>
<tr>
<td>Length of sound holes</td>
<td>69.8</td>
</tr>
<tr>
<td>Bassbar width in the middle</td>
<td>6.8</td>
</tr>
<tr>
<td>Bassbar width at extremity</td>
<td>4.5</td>
</tr>
<tr>
<td>Bridge height</td>
<td>40.6</td>
</tr>
<tr>
<td>Vibrating string length</td>
<td>328.5</td>
</tr>
</tbody>
</table>

The violin was set up on a special mold of significant mass that only touched the instrument at the outline of the back plate at its four corners. Such mounting enabled free vibrations of the top and back plates, as all four contact points between the mold and the instrument were chosen in places were the strengthening wooden blocks were glued to the inside of the instrument body to provide extra gluing surface for the plates. The set up of the trapezoidal violin on the mold is shown in Fig. 1; here the instrument is varnished and with strings.

Fig. 1. a) Mounting of the trapezoidal violin (unvarnished) for the modal experiment; b) The mounting mold details and a view of varnished instrument (not investigated).
3. Modal experiment

Experimental modal analysis describes the dynamics of any vibrating system in terms of modal parameters: natural frequencies and natural damping, as well as deformation patterns (mode shapes) associated with them. The main assumption of modal analysis is that the system under investigation is linear. In reality no mechanical system is linear, but the assumption is not very strict (Skrodzka et al., 2009). As the method was well described in our previous papers (Skrodzka et al., 2009; 2001, 2013; Skrodzka, Sęk, 1998), only the most crucial details are given below. The instrument was excited by an impact hammer in all 288 measuring points, one by one, to provide a broadband excitation in the frequency domain (PCB Impact Hammer 086C05; sensitivity 2.25 mV/N). The acceleration response signal was measured at a fixed measuring point marked as a black circle in Fig. 2. An ONO SOKKI NP-2910 accelerometer, with a mass of 2 grams and sensitivity of 0.3 pC/m/s² was used as a sensor.

Both the excitation and the response signals were measured perpendicularly to the top plate, in the most important direction regarding the vibration of the trapezoidal instrument. The mass of the accelerometer was significantly less than 10% of the mass of the top of the instrument and did not affect the results of measurements. The accelerometer was mounted on the instrument with bee-wax. The position of the accelerometer was chosen experimentally in a preliminary test, such as to avoid the areas of the top plate where the bass bar was attached, with respect to proper course of coherence function and repeatable frequencies of peaks in FRFs. Based on input and output signals Frequency Response Functions (FRFs) were calculated between all successive excitation points and the single fixed response point. Modal parameters extracted from FRFs were calculated by means of a SMS STAR-Modal® package. The FRFs were calculated at all 228 measuring points on the soundboard, separated by 1.2 cm from each other (areas under the bridge and strings were omitted). The distribution of measuring points is shown in Fig. 2.

All FRFs were measured in a frequency range of 0–1600 Hz with 2 Hz spectral resolution and their quality was controlled by the course of the function of coherence. As working functions FRFs were measured in the frequency domain, ten spectral averages were used to reduce the variance of accidental noise in measured signals and to improve the quality of measured FRFs (Lyons, 2000). If the coherence function was not consistently close to 1 the measurement was repeated. An example of an FRF is shown in Fig. 3. Except the first mode, only the modes with slight damping, less than 10% of the critical, were selected. Although such modes are not efficient sound radiators, they are very important since they define how a player describes the “feeling of the instrument” (Fletcher, Rossing, 1997).

![Fig. 2. Geometry of modal analysis measuring mesh. The black circle denotes the fixed position of the accelerometer. Examples of Frequency Response Functions from Fig. 3 were measured between points denoted by black squares and the point where the accelerometer was placed.](image1)

![Fig. 3. Examples of Frequency Response Functions measured between points denoted in Fig. 2 by black squares and the point where the accelerometer was mounted (black circle in Fig. 2). Arrows indicate frequencies of modes described in the text.](image2)

4. Results and discussion

Results of modal analysis of the top plate of the trapezoidal violin are shown in Fig. 4. The most distinctive and the best-pronounced modes are only taken into account in the present paper. As mentioned above, we have selected modes with slight damping, i.e. less than 10% of the critical, except the first mode at a frequency of 234 Hz. As seen in Fig. 4, the values of modal damping are not zero. Thus, complex modes describe the actual vibrational behaviour, similarly to guitars (Skrodzka et al., 2011) and violins (Skrodzka et al., 2009, 2013, 2014).

Ten distinct eigenmodes (Schleske, 2002) at which the plate vibrated in a certain pattern were found in the measured frequency range of 0–1600 Hz. As the strings were damped, their vibrational modes
Fig. 4. Modal parameters of the trapezoidal violin.

could not affect the eigenpatterns of the plate. The only effect they could have is due to the tension they put on the front plate.

At 234 Hz the vibrations in the top plate are clearly divided into four areas by two nodal lines, one vertical along the joint and one horizontal around the position of the bridge. At this mode the vibrations are strongest at the bottom right part of the plate. At 253 Hz and 399 Hz the top plate vibrates in a similar way with variations on the horizontal nodal line. At 624 Hz a nodal line appears from the left soundhole and passes to the lower end of the joint. At a modal frequency of 698 Hz a very strong vibration occurs, only being separated by a single nodal line passing horizontally along the soundholes. In this mode the vibrations are strongest at the lower end of the plate. Another very strong vibration is present at 937 Hz; like the previously described mode of 234 Hz this vibration is only separated by two nodal lines, one vertical, passing along the joint, and one horizontal in the middle of the soundholes. The vibration in this mode is, however, much stronger then the previous similar ones. It also should be noted that the frequency of 937 Hz is only by 1 Hz higher than the double octave of 234 Hz. At 1077 Hz a more complex pattern is seen, again vibrating very strongly. Two vertical nodal lines are present, approaching each other,
as the instrument narrows in the upper section. In addition, two horizontal lines are seen, one passes right below the soundholes and the other one is less prominent, curving from above the right soundhole in a small angle towards the upper left part of the instrument. Two horizontal node lines are also seen at 1179 Hz. In this mode only one vertical nodal line is present. At 1320 Hz three vertical nodal lines are seen. The last mode, at a frequency of 1576 Hz, is very complex. Two horizontal nodal lines are present and only one small vertical nodal line passes along the left bottom side of the instrument.

As it could be expected for a symmetric instrument, the results show that the mode shapes are more regular and symmetrical compared to normal violins (Skrodzka et al., 2009; 2013; Bissinger, 2008; Dünnwald, 1999). The results also indicate that the amplitudes of vibrations are stronger then in normal violins. From modal results obtained for only one trapezoidal violin it is difficult to tell which mode can be regarded a “tonal barometer” of the sound quality, similarly to mode B(1+) of the violin (Skrodzka et al., 2013; Bissinger, 2008). However, comparing the mode shapes of good violins (Bissinger, 2008) and modal deformations of the trapezoidal violin we suggest that the mode at 698 Hz with a single nodal line passing horizontally along the soundholes may play such a role.

The sound quality of the trapezoidal violin was evaluated subjectively. The jury of the Instrument Building of the Royal Conservatory, Ghent, Belgium described the sound of the instrument as sweet and soft, less brilliant and loud then the sound of a normal violin, and lacking overtones. The instrument does not lack power in the lower notes, the sound being described as vaguely similar to that of a viola. The jury clearly made no distinction between the Savart violin and other violins in terms of better or worse quality. First the trapezoidal violin was assessed by listening to its recorded sound and to the recordings of two normal violins, made by the author T. Duerinck and a German Stradivari model made by Neunbach. The jury was not informed which recording was made on which instrument. Afterwards the trapezoidal violin was played live by members of the jury alongside with two other normal violins made by T. Duerinck.

The bass bar is usually described by luthiers as more important for the lower notes, hence its name. However, the description obtained from the jury members suggests that the different placement of the bass bar did not affect the lower notes as much as the high ones. The result of evaluation suggest that the placement of a normal bass bar does not primarily affect the sound of a violin by disturbing lower frequencies, but rather by disturbing the symmetry of the top plate and forcing it to vibrate in a more complex way which causes more overtones. The notion of this function of the bassbar was already briefly mentioned by Heron-Allen (1885). By producing more overtones this effect could account for the difference in the brilliance and tone character between the Savart violin and a regular violin. Although Savart succeeded in making the violin vibrate more regularly and symmetrically, his instrument did not appear successful. A possible reason for that may be connected with the historical context: Savart made his trapezoidal violins in the era of romanticism. Their soft sweet tone could not make up for the lack of brilliance, quality, and especially power in the higher notes, which were so searched after in that era. The authors do not support the conclusion of F. Savart who claimed that his violin had a better sound then normal violin. It should be, however, kept in mind that the notion of “better sound” is subjective. There is no doubt that the sound of the F. Savart violin is different and has its advantages and disadvantages. Some violin players, when asked which instrument they would prefer to take home after playing both the trapezoidal violin and a Stradivari model made by the same luthier T. Duerinck, chose the trapezoidal instrument.

5. Conclusions

The results of modal analysis of the trapezoidal violin built after the description of Félix Savart lead to the following conclusions.

1. Placing the bar in the middle of the top plate results in a better symmetry of mode shapes compared to a normal violin’s mode shapes.
2. The results confirm that a flat plate vibrates stronger then one fixed by its camber.
3. It may be possible that the mode with a single nodal line passing horizontally along the soundholes, at a frequency of 698 Hz, may be a “tonal barometer” of trapezoidal violins.
4. Subjective evaluation of sound revealed no big differences between the trapezoidal instrument and normal violins.

Acknowledgments

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References